NON-EXPECTED UTILITY THEORY

Mark J. Machina

To appear in *The New Palgrave Dictionary of Economics, 2nd Edition*

edited by Steven N. Durlauf and Lawrence E. Blume,
Macmillan (Basingstoke and New York), forthcoming

Abstract: Beginning with the work of Allais and Edwards in the early 1950's and continuing through the present, psychologists and economists have uncovered a growing body of evidence that individuals do not necessarily conform to many of the key assumptions or predictions of the expected utility model of choice under uncertainty, and seem to depart from this model in systematic and predictable ways. This has led to the development of alternative models of preferences over objectively or subjectively uncertain prospects, which seek to accommodate these systematic departures from the expected utility model while retaining as much of its analytical power as possible.

Although the expected utility model has long been the standard theory of individual choice under objective and subjective uncertainty, experimental work by both psychologists and economists has uncovered systematic departures from the expected utility hypothesis, which has led to the development of alternative models of preferences over uncertain prospects.

The Expected Utility Model

In one of the simplest setting of choice under economic uncertainty, the objects of choice consist of finite-outcome *objective lotteries* of the form $\mathbf{P} = (x_1,p_1;...;x_n,p_n)$, yielding a monetary payoff of x_i with probability p_i, where $p_1 + ... + p_n = 1$. In such a case, the expected utility model of risk preferences assumes (or posits axioms sufficient to imply) that the individual ranks these prospects on the basis of an *expected utility preference function* of the form

$$V_{EU}(\mathbf{P}) \;\equiv\; V_{EU}(x_1,p_1;...;x_n,p_n) \;\equiv\; U(x_1)\cdot p_1 + \ldots + U(x_n)\cdot p_n$$

in the standard economic sense that the individual prefers lottery $\mathbf{P}^* = (x_1^*,p_1^*;...;x_{n^*}^*,p_{n^*}^*)$ over lottery $\mathbf{P} = (x_1,p_1;...;x_n,p_n)$ if and only if $V_{EU}(\mathbf{P}^*) > V_{EU}(\mathbf{P})$, and is indifferent between them if and only if $V_{EU}(\mathbf{P}^*) = V_{EU}(\mathbf{P})$. $U(\cdot)$ is termed the individual's *von Neumann-Morgenstern utility function*, and its various mathematical properties serve to characterize various features of the individual's attitudes toward risk, for example:

- $V_{EU}(\cdot)$ exhibits *first order stochastic dominance preference* (a preference for shifting probability from lower to higher outcome values) if and only if $U(x)$ is an increasing function of x.

- $V_{EU}(\cdot)$ exhibits *risk aversion* (an aversion to all mean-preserving increases in risk) if and only if $U(x)$ is a concave function of x.

- $V_{EU}^*(\cdot)$ is *at least as risk averse as* $V_{EU}(\cdot)$ (in several equivalent senses) if and only if its utility function $U^*(\cdot)$ is a concave transformation of $U(\cdot)$ (that is, if and only if $U^*(x) \equiv \rho(U(x))$ for some increasing concave function $\rho(\cdot)$).

As shown by Bernoulli (1738), Arrow (1965), Pratt (1964), Friedman and Savage (1948), Markowitz (1952) and others, this model admits of a tremendous flexibility in representing attitudes toward risk, and can be applied to many types of economic decisions and markets.

But in spite of its flexibility, the expected utility model has testable implications which hold regardless of the shape of the utility function $U(\cdot)$, since they follow from the *linearity in the probabilities* property of the preference function $V_{EU}(\cdot)$. These implications can be best expressed by the concept of an α:(1–α) *probability mixture* of two lotteries $\mathbf{P} = (x_1,p_1;...;x_n,p_n)$ and $\mathbf{P}^* = (x_1^*,p_1^*;...;x_{n^*}^*,p_{n^*}^*)$, which is defined as the single-stage lottery $\alpha\cdot\mathbf{P} + (1-\alpha)\cdot\mathbf{P}^* = (x_1, \alpha\cdot p_1; \ldots; x_n, \alpha\cdot p_n; x_1^*, (1-\alpha)\cdot p_1^*; \ldots; x_{n^*}^*, (1-\alpha)\cdot p_{n^*}^*)$. The mixture $\alpha\cdot\mathbf{P} + (1-\alpha)\cdot\mathbf{P}^*$ can be thought of as a coin flip yielding lotteries **P** and **P*** with probabilities α:(1–α), where the uncertainty in the coin and in the subsequent lottery is resolved simultaneously. Linearity in the probabilities is equivalent to the following property, which serves as the key foundational axiom of the expected utility model (Marschak, 1950):

> **Independence Axiom**: If lottery **P*** is preferred (indifferent) to lottery **P**, then the probability mixture $\alpha\cdot\mathbf{P}^* + (1-\alpha)\cdot\mathbf{P}^{**}$ is preferred (indifferent) to $\alpha\cdot\mathbf{P} + (1-\alpha)\cdot\mathbf{P}^{**}$ for every lottery **P**** and every mixture probability $\alpha \in (0,1]$.

This axiom can be interpreted as saying 'given an α:(1–α) coin, the individual's preferences for receiving **P*** versus **P** in the event of a head should not depend upon the prize **P**** that would be received in the event of a tail, nor upon the probability α of landing heads (so long as this probability is positive).' The strong normative appeal of this axiom has contributed to the widespread adoption of the expected utility model.

The property of linearity in the probabilities, as well as the senses in which it has been found to be empirically violated, can be illustrated in the special case of preferences over all lotteries $\mathbf{P} = (\bar{x}_1,p_1;\bar{x}_2,p_2;\bar{x}_3,p_3)$ over a fixed set of outcome values $\bar{x}_1 < \bar{x}_2 < \bar{x}_3$. Since we must have $p_2 = 1 - p_1 - p_3$, each such lottery can be completely summarized by its pair of probabilities (p_1, p_3), as plotted in the 'probability triangle' of Figure 1. Since upward movements in the diagram (increasing p_3 for fixed p_1) represent shifting probability from outcome $\bar{x}_2$ up to $\bar{x}_3$, and leftward movements represent shifting probability from $\bar{x}_1$ up to $\bar{x}_2$, such movements constitute first order stochastically dominating shifts and will thus always be preferred. Expected utility indifference curves (loci of constant expected utility) are given by the formula

$$U(\bar{x}_1)\cdot p_1 + U(\bar{x}_2)\cdot[1-p_1-p_3] + U(\bar{x}_3)\cdot p_3 \;=\; \text{constant}$$

and are accordingly seen to be parallel straight lines of slope $[U(\bar{x}_2)-U(\bar{x}_1)]/[U(\bar{x}_3)-U(\bar{x}_2)]$, as indicated by the solid lines in the figure. The dotted lines in Figure 1 are loci of constant expected value, given by the formula $\bar{x}_1\cdot p_1 + \bar{x}_2\cdot[1-p_1-p_3] + \bar{x}_3\cdot p_3 = \text{constant}$, with slope $[\bar{x}_2-\bar{x}_1]/[\bar{x}_3-\bar{x}_2]$. Since northeast movements along the constant expected value lines shift probability from $\bar{x}_2$ down to $\bar{x}_1$ and up to $\bar{x}_3$ in a manner that preserves the mean of the distribution, they represent simple increases in risk (Rothschild and Stiglitz (1970,1971)). When $U(\cdot)$ is concave (i.e., risk averse), its indifference curves will have a steeper slope than these constant expected value lines, and such increases in risk move the individual from more to less preferred indifference curves, as illustrated in the figure. It is straightforward to show that the indifference curves of any expected utility maximizer with a more risk averse (i.e., more concave) utility function $U^*(\cdot)$ will be steeper than those generated by $U(\cdot)$.

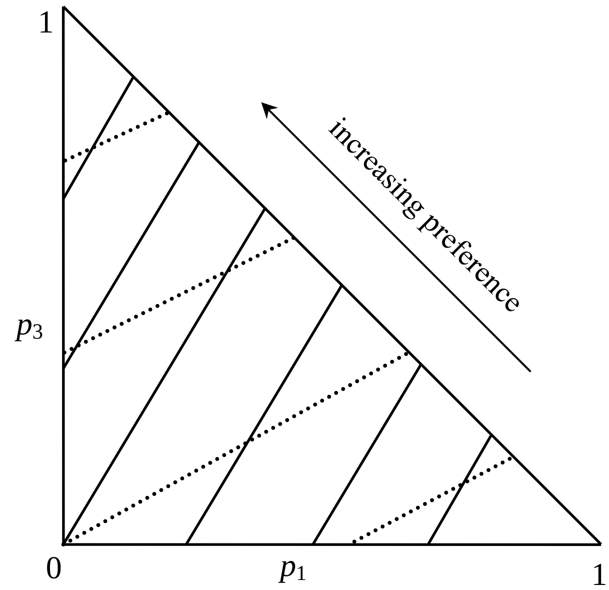

Figure 1
Expected Utility Indifference Curves in the Probability Triangle

Systematic Violations of the Expected Utility Hypothesis

In spite of its normative appeal, researchers have uncovered several types of widespread systematic violations of the expected utility model and its underlying assumptions. These can be categorized into (i) violations of the Independence Axiom (such as the Common Consequence and Common Ratio effects), (ii) violations of the hypothesis of probabilistic beliefs (such as the Ellsberg Paradox) and (iii) violations of the model's underlying assumptions of descriptive and procedural invariance (such as reference-point and response-mode effects).

Violations of the Independence Axiom

The best-known violation of the Independence Axiom is the so-called *Allais Paradox*, in which individuals are asked to rank each of the following pairs of lotteries, where \$1M = \$1,000,000:

$$a_1 : \left\{ 1.00 \text{ chance of } \$1\text{M} \right. \qquad \text{versus} \qquad a_2 : \begin{cases} .10 \text{ chance of } \$5\text{M} \\ .89 \text{ chance of } \$1\text{M} \\ .01 \text{ chance of } \$0 \end{cases}$$

$$a_3 : \begin{cases} .10 \text{ chance of } \$5\text{M} \\ .90 \text{ chance of } \$0 \end{cases} \qquad \text{versus} \qquad a_4 : \begin{cases} .11 \text{ chance of } \$1\text{M} \\ .89 \text{ chance of } \$0 \end{cases}$$

Researchers such as Allais (1953), Morrison (1967), Raiffa (1968), Slovic and Tversky (1974) and others have found that the modal if not majority preference of subjects is for a_1 over a_2 in the first pair of choices and for a_3 over a_4 in the second pair. However, such preferences violate expected utility, since the first ranking implies the inequality $U(\$1\text{M}) > .10 \cdot U(\$5\text{M}) + .89 \cdot U(\$1\text{M}) + .01 \cdot U(\$0)$ whereas the second implies the inconsistent inequality $.10 \cdot U(\$5\text{M}) + .90 \cdot U(\$0) > .11 \cdot U(\$1\text{M}) + .89 \cdot U(\$0)$. By setting $\bar{x}_1 = \$0$, $\bar{x}_2 = \$1\text{M}$ and $\bar{x}_3 = \$5\text{M}$, the lotteries a_1, a_2, a_3 and a_4 are seen to form a parallelogram when plotted in the probability triangle (Figure 2), which explains why the parallel straight line indifference curves of an expected utility maximizer must either prefer a_1 and a_4 (as illustrated for the relatively steep indifference curves of the figure) or else prefer a_2 and a_3 (for relatively flat indifference curves). Figure 3 illustrates *non-expected utility indifference curves* which *fan out*, and are seen to exhibit the typical Allais Paradox rankings of a_1 over a_2 and a_3 over a_4.

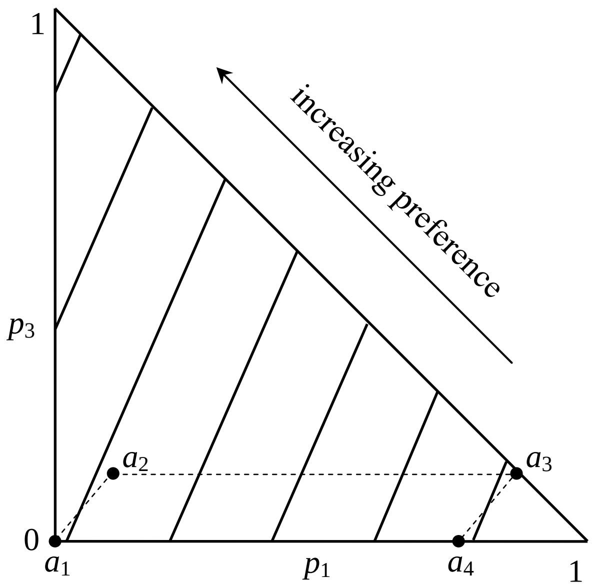

Figure 2
Expected Utility Indifference Curves
and the Allais Paradox Choices

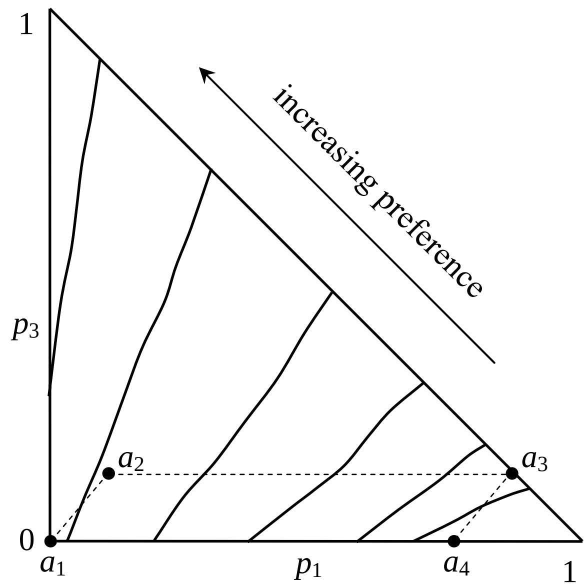

Figure 3
Allais Paradox Choices and
Indifference Curves which 'Fan Out'

Although the Allais Paradox was originally dismissed as an isolated example, subsequent experimental work by psychologists, economists and others have uncovered a similar pattern of violations over a range of probability and payoff values, and the Allais Paradox is now seen to be a special case a widely observed phenomenon known as the *common consequence effect.* This effect involves pairs of prospects (probability mixtures) of the form:

$$b_1 : \begin{cases} \alpha & \text{chance of } x \\ 1-\alpha & \text{chance of } \mathbf{P}^{**} \end{cases} \quad \text{versus} \quad b_2 : \begin{cases} \alpha & \text{chance of } \mathbf{P} \\ 1-\alpha & \text{chance of } \mathbf{P}^{**} \end{cases}$$

$$b_3 : \begin{cases} \alpha & \text{chance of } x \\ 1-\alpha & \text{chance of } \mathbf{P}^{*} \end{cases} \quad \text{versus} \quad b_4 : \begin{cases} \alpha & \text{chance of } \mathbf{P} \\ 1-\alpha & \text{chance of } \mathbf{P}^{*} \end{cases}$$

where the lottery **P** involves outcomes both greater and less than the amount x, and **P**** first order stochastically dominates **P*** (in Allais' example, x = \$1M, **P** = (\$5M,10/11;\$0,1/11), **P*** = (\$0,1), **P**** = (\$1M,1) and α = .11). Although the Independence Axiom clearly implies choices of either b_1 and b_3 (if x is preferred to **P**) or else b_2 and b_4 (if **P** is preferred to x), researchers have found a tendency for subjects to choose b_1 in the first pair and b_4 in the second. When the distributions **P**, **P*** and **P**** are each over a common outcome set $\{\bar{x}_1,\bar{x}_2,\bar{x}_3\}$ with $\bar{x}_2 = x$, the prospects $\{b_1,b_2,b_3,b_4\}$ again form a parallelogram in the (p_1,p_3) triangle, and a choice of b_1 and b_4 again implies indifference curves which fan out.

The intuition behind this phenomenon can be described in terms of the above 'coin-flip' scenario. According to the Independence Axiom, one's preferences over what would occur in the event of a head ought not depend upon what would occur in the event of a tail. However, they *may well* depend upon what would otherwise happen (as Bell (1985) notes, 'winning the top prize of \$10,000 in a lottery may leave one much happier than receiving \$10,000 as the lowest prize in a lottery'). The common consequence effect states that the *better off* individuals would be in the event of a tail (in the sense of stochastic dominance), the *more risk averse* their preferences over what they would receive in the event of a head. That is, if the distribution **P**** in the pair $\{b_1,b_2\}$ involves very high outcomes, one may prefer not to bear further risk in the unlucky event that one doesn't receive it, and hence opt for the sure outcome x over the risky distribution **P** (that is, choose b_1 over b_2). But if **P*** in $\{b_3,b_4\}$ involves very low outcomes, one might be more willing to bear risk in the lucky event that one doesn't receive it, and prefer going for the lottery **P** rather than the sure outcome x (choose b_4 over b_3).

A second type of systematic violation of linearity in the probabilities, also noted by Allais and subsequently termed the *common ratio effect,* involves prospects of the form:

$$c_1:\begin{cases} p & \text{chance of } \$X \\ 1-p & \text{chance of } \$0 \end{cases} \quad \text{versus} \quad c_2:\begin{cases} q & \text{chance of } \$Y \\ 1-q & \text{chance of } \$0 \end{cases}$$

$$c_3:\begin{cases} \alpha\cdot p & \text{chance of } \$X \\ 1-\alpha\cdot p & \text{chance of } \$0 \end{cases} \quad \text{versus} \quad c_4:\begin{cases} \alpha\cdot q & \text{chance of } \$Y \\ 1-\alpha\cdot q & \text{chance of } \$0 \end{cases}$$

where $p > q$, $0 < X < Y$ and $\alpha \in (0,1)$. (The term 'common ratio effect' comes from the common value of prob($X)/prob($Y) in the upper and lower pairs.) Setting $\{\bar{x}_1,\bar{x}_2,\bar{x}_3\} = \{\$0,\$X,\$Y\}$ and plotting these prospects in the probability triangle as in Figure 4, the line segments $\overline{c_1c_2}$ and $\overline{c_3c_4}$ are seen to be parallel, so that the expected utility model again predicts choices of c_1 and c_3 (if the indifference curves are relatively steep) or else c_2 and c_4 (if they are flat). However, experimental studies by MacCrimmon (1968), Tversky (1975), MacCrimmon and Larsson (1979), Kahneman and Tversky (1979), Hagen (1979), Chew and Waller (1986) and others have found a systematic tendency for choices to depart from these predictions in the direction of preferring c_1 over c_2 and c_4 over c_3, which again suggests that indifference curves fan out, as in the figure. For example, Kahneman and Tversky (1979) found that while 86% of their subjects preferred a .90 chance of winning $3,000 to a .45 chance of $6,000, 73% preferred a .001 chance of $6,000 to a .002 chance of $3,000. Kahneman and Tversky (1979) observed that when the positive outcomes $3000 and $6000 in the above gambles are replaced by *losses* of these magnitudes to obtain the lotteries c_1', c_2', c_3' and c_4', preferences typically 'reflect,' to prefer c_2' over c_1' and c_3' over c_4'. Setting $\bar{x}_1$ = –$6000, $\bar{x}_2$ = –$3000 and $\bar{x}_3$ = $0 (to preserve the ordering $\bar{x}_1 < \bar{x}_2 < \bar{x}_3$) and plotting as in Figure 5, such preferences again suggest that indifference curves in the probability triangle fan out. Battalio, Kagel and MacDonald (1985) found that laboratory rats choosing among gambles involving substantial variations in their daily food intake also exhibited this pattern of choices.

One criticism of this evidence has been that individuals whose initial choices violated the Independence Axiom in the above manners would typically 'correct' themselves once the nature of their violations were revealed by an application of the above type of coin-flip argument. Thus, while even Leonard Savage chose a_1 and a_3 when first presented with such choices by Allais, he concluded upon reflection that these preferences were in error. (Savage (1954, pp.101-103)). Although Moskowitz found that allowing subjects to discuss opposing written arguments led to a decrease in the proportion of violations, 73% of the initial fanning-out type choices remained unchanged after the discussions (pp.232-237, Table 6). When written arguments were presented but no discussion was allowed, there was a 93% persistency rate of such choices (p. 234, Tables 4,6). In experiments where subjects who responded to Allais-type problems were then presented with written arguments both for *and against* the expected utility position, neither MacCrimmon (1968), Moskowitz (1974) nor Slovic and Tversky (1974) found predominant net swings toward the expected utility choices.

Further descriptions of these and other violations of the Independence Axiom can be found in Machina (1983,1987), Sugden (1986), Weber and Camerer (1987), Camerer (1989) and Starmer (2000).

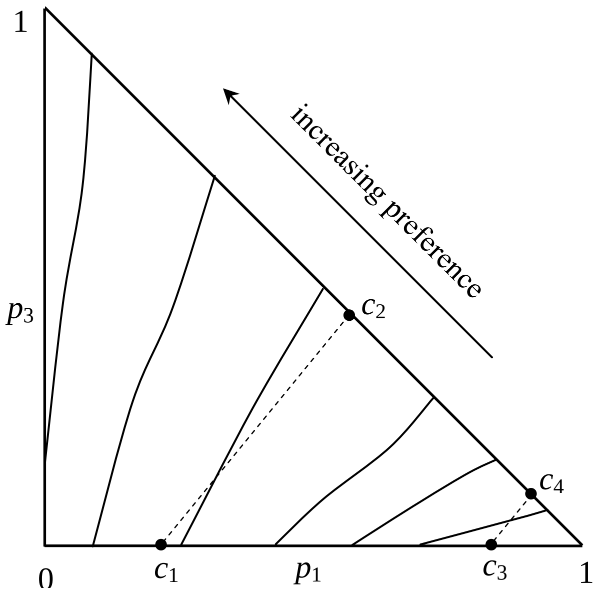

Figure 4
Common Ratio Effect and
Fanning Out Indifference Curves

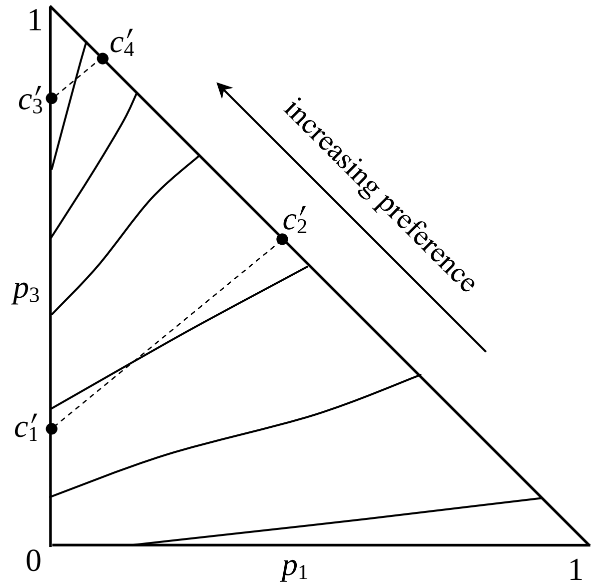

Figure 5
Common Ratio Effect for Losses and
Fanning Out Indifference Curves

Non-Existence of Probabilistic Beliefs

Although the expected utility model was first formulated in terms of preferences over *objective lotteries* $\mathbf{P} = (x_1,p_1;...;x_n,p_n)$ with prespecified probabilities, it has also been applied to preferences over *subjective acts* $f(\cdot) = [x_1 \text{ on } E_1;...;x_n \text{ on } E_n]$, where the uncertainty is represented by a set $\{E_1,...,E_n\}$ of mutually exclusive and exhaustive *events* (such as the alternative outcomes of a horse race) (Savage, 1954). As long as an individual possesses well-defined *subjective probabilities* $\mu(E_1),\ldots,\mu(E_n)$ over these events, their *subjective expected utility* preference function takes the form

$$W_{SEU}(f(\cdot)) \;\equiv\; W_{SEU}(x_1 \text{ on } E_1;...;x_n \text{ on } E_n) \;\equiv\; U(x_1)\cdot\mu(E_1) + \ldots + U(x_n)\cdot\mu(E_n)$$

However, researchers have found that individuals may not possess such well-defined subjective probabilities, in even the simplest of cases. The best-known example of this is the *Ellsberg Paradox* (Ellsberg, 1961), in which the individual must draw a ball from an urn that contains 30 red balls, and 60 black or yellow balls in an unknown proportion, and is offered the following bets based on the color of the drawn ball:

	30 balls	60 balls	
	red	black	yellow
$f_1(\cdot)$	\$100	\$0	\$0
$f_2(\cdot)$	\$0	\$100	\$0
$f_3(\cdot)$	\$100	\$0	\$100
$f_4(\cdot)$	\$0	\$100	\$100

Most individuals exhibit a preference for $f_1(\cdot)$ over $f_2(\cdot)$ and $f_4(\cdot)$ over $f_3(\cdot)$. When asked, they explain that the chance of winning under $f_2(\cdot)$ could be anywhere from 0 to 2/3 whereas under $f_1(\cdot)$ it is known to be exactly 1/3, and they prefer the bet that offers the known probability. Similarly, the chance of winning under $f_3(\cdot)$ could be anywhere from 1/3 to 1 whereas under $f_4(\cdot)$ it is known to be exactly 2/3, so the latter is preferred. However, such preferences are inconsistent with any assignment of subjective probabilities μ(red), μ(black), μ(yellow) to the three events: If the individual *were* to be choosing on the basis of such probabilistic beliefs, the choice of $f_1(\cdot)$ over $f_2(\cdot)$ would 'reveal' that μ(red) > μ(black), but the choice of $f_4(\cdot)$ over $f_3(\cdot)$ would reveal that μ(red) < μ(black). A preference for gambles based on probabilistic partitions such as $\{\text{red}, \text{black}\cup\text{yellow}\}$ over gambles based on subjective partitions such as $\{\text{black}, \text{red}\cup\text{yellow}\}$ is termed *ambiguity aversion*.

In an even more basic example, Ellsberg presented subjects with a pair of urns, the first containing 50 red balls and 50 black balls, and the second with 100 red and black balls in an unknown proportion. When asked, a majority of subjects strictly preferred to stake a prize on drawing red from the first urn over drawing red from the second urn, *and* strictly preferred staking the prize on drawing black from the first urn over drawing black from the second. It is clear that there can exist no subjective probabilities p:(1–p) of red:black in the second urn, including 1/2:1/2, which can simultaneously generate *both* of these strict preferences. Similar behavior in this and related problems has been observed by Raiffa (1961), Becker and Brownson (1964), MacCrimmon (1965), Slovic and Tversky (1974) and MacCrimmon and Larsson (1979).

Violations of Descriptive and Procedural Invariance

Researchers have also uncovered several systematic violations of the standard economic assumptions of stability of preferences and invariance to problem description in choices over risky prospects. In particular, psychologists have found that alternative means of representing or 'framing' probabilistically equivalent choice problems lead to systematic differences in choice. Early examples of this were reported by Slovic (1969), who found that offering a gain or loss contingent on the joint occurrence of four independent events with probability p elicited different responses than offering it on the occurrence of a single event with probability p^4 (all probabilities were stated explicitly). In comparison with the single-event case, making a gain contingent on the joint occurrence of events was found to make it more attractive, and making a loss contingent on the joint occurrence of events made it more unattractive.

One class of framing effects exploits the phenomenon of a *reference point*. According to economic theory, the variable which enters an individual's von Neumann-Morgenstern utility function should be total (i.e., final) wealth, and gambles phrased in terms of gains and losses should be combined with current wealth and reexpressed as distributions over final wealth levels before being evaluated. However, risk attitudes towards gains and losses tend to be more stable than can be explained by a fixed utility function over final wealth, and utility functions might be best defined in terms of changes from the 'reference point' of current wealth. In his discussion of this phenomenon, Markowitz (1952, p.155) suggested that certain circumstances may cause the individual's reference point to temporarily deviate from current wealth. If these circumstances include the manner in which a problem is verbally described, then differing risk attitudes over gains and losses from the reference point can lead to different choices, depending upon the exact description of an otherwise identical problem. A simple example of this, from Kahneman and Tversky (1979), involves the following two questions:

> 'In addition to whatever you own, you have been given 1,000 (Israeli pounds). You are now asked to choose between a 1/2:1/2 chance of a gain of 1,000 or 0 or a sure chance of a gain of 500.'

and

> 'In addition to whatever you own, you have been given 2,000. You are now asked to choose between a 1/2:1/2 chance of a loss of 1,000 or 0 or a sure loss of 500.'

These two problems involve identical distributions over final wealth. But when put to two different groups of subjects, 84% chose the sure gain in the first problem but 69% chose the 1/2:1/2 gamble in the second.

In another class of examples, not based on reference point effects, Moskowitz (1974), Keller (1985) and Carlin (1990) found that the proportion of subjects choosing in conformity with the Independence Axiom in examples like the Allais Paradox was significantly affected by whether the problems were described in the standard matrix form, decision tree form, roulette wheels, or as minimally structured written statements. Interestingly, the form judged the 'clearest representation' by the majority of Moskowitz's subjects (the tree form) led to the lowest degree of consistency with the Independence Axiom, the highest proportion of Allais-type (fanning out) choices, and the highest persistency rate of these choices (pp.234,237-38).

In other studies, Schoemaker and Kunreuther (1979), Hershey and Schoemaker (1980), Kahneman and Tversky (1982,1984), and Slovic, Fischhoff and Lichtenstein (1977) found that subjects' choices in otherwise identical problems depended upon whether they were phrased as decisions whether or not to gamble as opposed to whether or not to insure, whether statistical information for different therapies was presented in terms of cumulative survival probabilities or cumulative mortality probabilities, etc. (see the references in Tversky and Kahneman (1981)).

Whereas framing effects involve alternative *descriptions* of an otherwise identical choice problem, alternative *response formats* have also been found to lead to different choices, leading to what have been termed *response-mode effects*. For example, under expected utility, an individual's von Neumann-Morgenstern utility function can be assessed or elicited in a number of different manners, which typically involve a sequence of prespecified lotteries $\mathbf{P}_1$, $\mathbf{P}_2$, $\mathbf{P}_3$, ..., and ask for (i) the individual's certainty equivalent $CE(\mathbf{P}_i)$ of each lottery $\mathbf{P}_i$, (ii) the *gain equivalent* G_i that would make the gamble (G_i,1/2;\$0,1/2) indifferent to $\mathbf{P}_i$, or (iii) the *probability equivalent* $\wp_i$ that would make the gamble (\$1000,$\wp_i$;\$0,1$-\wp_i$) indifferent to $\mathbf{P}_i$. Although such procedures should generate equivalent assessed utility functions, they have been found to yield systematically different ones (e.g. Hershey, Kunreuther and Schoemaker (1982) and Hershey and Schoemaker (1985)).

In a separate finding now known as the *preference reversal phenomenon*, subjects were first presented with a number of pairs of lotteries and asked to make one choice out of each pair. Each pair of lotteries took the following form:

$$p\text{-bet}:\begin{cases} p & \text{chance of } \$X \\ 1-p & \text{chance of } \$0 \end{cases} \qquad \text{versus} \qquad \$\text{-bet}:\begin{cases} q & \text{chance of } \$Y \\ 1-q & \text{chance of } \$0 \end{cases}$$

where $0 < X < Y$ and $p > q$. The terms 'p-bet' and '\$-bet' derive from the greater probability of winning in the first bet, and greater possible gain in the second bet. Subjects were next asked for their certainty equivalents of each of these bets, via a number of standard elicitation techniques. Standard theory predicts that for each such pair, the prospect selected in the direct choice problem would also be assigned the higher certainty equivalent. However, subjects exhibit a systematic departure from this prediction in the direction of choosing the p-bet in a direct choice, but assigning a higher certainty equivalent to the \$-bet (Lichtenstein and Slovic, 1971). Although this finding initially generated widespread scepticism, it has been replicated by both psychologists and economists in a variety of settings involving real-money gambles, patrons in a Las Vegas casino, group decisions and experimental market trading. By expressing the implied preferences as '\$-bet ~ CE(\$-bet) $\succ$ CE(p-bet) ~ p-bet $\succ$ \$-bet,' some economists have categorized this phenomenon as a violation of transitivity and tried to model it as such (see the ‘regret theory’ model below). However, most psychologists and economists now view it as a response-mode effect: specifically, that the psychological processes of valuation (which generates certainty equivalents) and direct choice are differentially influenced by the probabilities and payoffs involved in a lottery, and that under certain conditions this can lead to choices and valuations which ‘reveal’ opposite preference rankings over a pair of gambles.

Non-Expected Utility Models of Risk Preferences

Non-Expected Utility Functional Forms

Researchers have responded to departures from linearity in the probabilities in two manners. The first consists of replacing the expected utility form $V_{EU}(\mathbf{P}) = U(x_1)\cdot p_1 + \ldots + U(x_n)\cdot p_n$ by some more general form for the preference function $V(\mathbf{P}) = V(x_1,p_1;...;x_n,p_n)$. Several such forms have been proposed (for the Rank Dependent, Dual and Ordinal Independence forms, the payoffs must be labeled so that $x_1 \leq \ldots \leq x_n$, and $G(\cdot)$ must satisfy $G(0) = 0$ and $G(1) = 1$):

Prospect Theory	$\sum_{i=1}^{n} \upsilon(x_i)\cdot\pi(p_i)$	Edwards (1955,1962), Kahneman & Tversky (1979)
Subjectively Weighted Utility	$\sum_{i=1}^{n} \upsilon(x_i)\cdot\pi(p_i) \big/ \sum_{i=1}^{n} \pi(p_i)$	Karmarkar (1978,1979)
Rank-Dependent Expected Utility	$\sum_{i=1}^{n} \upsilon(x_i)\cdot\left[G\left(\sum_{j=1}^{i} p_j\right) - G\left(\sum_{j=1}^{i-1} p_j\right)\right]$	Quiggin (1982)
Dual Expected Utility	$\sum_{i=1}^{n} x_i\cdot\left[G\left(\sum_{j=1}^{i} p_j\right) - G\left(\sum_{j=1}^{i-1} p_j\right)\right]$	Yaari (1987)
Ordinal Independence	$\sum_{i=1}^{n} h\left(x_i, \sum_{j=1}^{i} p_j\right)\cdot\left[G\left(\sum_{j=1}^{i} p_j\right) - G\left(\sum_{j=1}^{i-1} p_j\right)\right]$	Segal (1984) Green & Jullien (1988)
Moments of Utility	$M\left(\sum_{i=1}^{n} \upsilon(x_i)\cdot p_i\,,\ \sum_{i=1}^{n} \upsilon(x_i)^2\cdot p_i, \ldots\right)$	Múnera & de Neufville (1983), Hagen (1979)
Weighted Utility	$\sum_{i=1}^{n} \upsilon(x_i)\cdot p_i \big/ \sum_{i=1}^{n} \tau(x_i)\cdot p_i$	Chew (1983)
Optimism-Pessimism	$\sum_{i=1}^{n} \upsilon(x_i)\cdot g(p_i, x_1, \ldots, x_n)$	Hey (1984)
Quadratic in the Probabilities	$\sum_{i=1}^{n} \sum_{j=1}^{n} K(x_i, x_j)\cdot p_i\cdot p_j$	Chew, Epstein & Segal (1991)
Regret Theory	$\sum_{i=1}^{n} \sum_{j=1}^{n^*} R(x_i, x_j^*)\cdot p_i\cdot p_j^*$	Loomes & Sugden (1982)

Most of these forms have been formally axiomatized, and under the appropriate monotonicity and/or curvature assumptions on their constituent functions $\upsilon(\cdot)$, $G(\cdot)$, etc., most are capable of exhibiting first order stochastic dominance preference, risk aversion, and the above types of systematic violations of the Independence Axiom. Researchers such as Konrad and Skaperdas (1993), Schlesinger (1997) and Gollier (2000) have used these forms to revisit many of the applications previously modeled by expected utility theory, such as asset and insurance demand, to determine which expected-utility-based results are, and which are not, robust to departures from linearity in the probabilities, and which additional properties of risk-taking behavior can be modeled.

Although the form $\sum_{i=1}^{n}\upsilon(x_i)\cdot\pi(p_i)$ was the earliest non-expected utility model to be proposed, it was largely abandoned when it was realized that whenever the weighting function $\pi(\cdot)$ was nonlinear, the generic inequalities $\pi(p_i) + \pi(p_j) \neq \pi(p_i+p_j)$ and $\pi(p_1)+\ldots+\pi(p_n) \neq 1$ implied discontinuities in the payoffs and inconsistency with first order stochastic dominance preference. Both problems were corrected by adopting weights $[G(\sum_{j=1}^{i}p_j)-G(\sum_{j=1}^{i-1}p_j)]$ based on the *cumulative* probability values p_1, p_1+p_2, $p_1+p_2+p_3$, … to obtain the Rank Dependent form. Under the above-mentioned restrictions on this form, these weights necessarily sum to unity, and the Rank Dependent form has emerged as the most widely adopted model in both theoretical and applied analyses. The Dual Expected Utility and Ordinal Independence forms are based on similar weighting formulas.

Unlike the other models, the Regret Theory form dispenses with the assumption of a preference function over lotteries, and instead derives choice from the psychological notions of *rejoice* and *regret* – that is, the reaction to receiving outcome x when an alternative decision would have led to outcome x^*. The primitive of this model is a *regret:rejoice function* $R(x,x^*)$ which is positive if x is preferred to x^*, negative if x^* is preferred to x, zero if they are indifferent, and satisfies the skew-symmetry condition $R(x,x^*) \equiv -R(x^*,x)$. In a choice between lotteries $\mathbf{P} = (x_1,p_1;...;x_n,p_n)$ and $\mathbf{P}^* = (x_1^*,p_1^*;...;x_{n^*}^*,p_{n^*}^*)$ which are realized independently, the individual's *expected rejoice* from choosing **P** over **P*** is given by $\sum_{i=1}^{n}\sum_{j=1}^{n^*}R(x_i,x_j^*)\cdot p_i\cdot p_j^*$, and the individual is predicted to choose **P** if this value is positive, **P*** if it is negative, and be indifferent if it is zero (various proposals for extending this approach beyond pairwise choice have been offered). Since this model specifies choice in pairwise comparisons rather preference levels of individual lotteries, it allows choice to be intransitive, so the individual might select **P** over **P***, **P*** over **P****, and **P**** over **P**. Though some have argued that such cycles allow for the phenomenon of 'money pumps,' it has allowed the model to serve as a proposed solution to the Preference Reversal Phenomenon.

Generalized Expected Utility Analysis

An alternative approach to non-expected utility preferences does not rely upon any specific functional form, but links properties of attitudes toward risk directly to the probability derivatives of a general 'smooth' preference function $V(\mathbf{P}) = V(x_1,p_1;...;x_n,p_n)$. Such analysis reveals that the basic analytics of the expected utility model are in fact quite robust to general smooth departures from linearity in the probabilities. This approach is based on the observations that for the expected utility function $V_{EU}(x_1,p_1;...;x_n,p_n) \equiv U(x_1)\cdot p_1 +\ldots+ U(x_n)\cdot p_n$, the value $U(x_i)$ can be interpreted as the coefficient of p_i, and that many theorems involving a linear function's *coefficients* continue to hold when generalized to a nonlinear function's *derivatives*. By adopting the notation $U(x;\mathbf{P}) \equiv \partial V(\mathbf{P})/\partial \text{prob}(x)$ and the term 'local utility function' for the function $U(\cdot;\mathbf{P})$, standard expected utility characterizations such as those listed at the beginning of this article can be generalized to any smooth non-expected utility preference function $V(\mathbf{P})$ in the following manners (Machina, 1982):

- $V(\cdot)$ exhibits global *first order stochastic dominance preference* if and only if at each lottery **P**, its local utility function $U(x;\mathbf{P})$ is an increasing function of x.
- $V(\cdot)$ exhibits global *risk aversion* (aversion to small or large mean-preserving increases in risk) if and only if at each lottery **P**, its local utility function $U(x;\mathbf{P})$ is a concave function of x.
- $V^*(\cdot)$ is globally *at least as risk averse as* $V(\cdot)$ if and only if at each lottery **P**, $V^*(\cdot)$'s local utility function $U^*(x;\mathbf{P})$ is a concave transformation of $V(\cdot)$'s local utility function $U(x;\mathbf{P})$.

Similar generalizations of expected utility results and characterizations can be obtained for general comparative statics analysis, the theory of asset demand, and the demand for insurance. With regard to the Allais Paradox and other observed violations of the Independence Axiom, it can be shown that the indifference curves of a smooth preference function $V(\cdot)$ will fan out in the probability triangle if and only if $U(x;\mathbf{P}^*)$ is a concave transformation of $U(x;\mathbf{P})$ whenever $\mathbf{P}^*$ first order stochastically dominates $\mathbf{P}$. This analytical approach has been extended to larger classes of preference functionals and distributions by Chew, Karni and Safra (1987), Karni (1987,1989) and Wang (1993), formally axiomatized by Allen (1987), and applied to the analysis of choices under uncertainty by Chew, Epstein and Zilcha (1988), Chew and Nishimura (1992), Dekel (1989), Green and Jullien (1988), Machina (1984,1989,1995) and others.

Non-Expected Utility Preferences under Subjective Uncertainty

Recent years have seen a growing interest in models of choice under subjective uncertainty, with efforts to represent and analyze departures from both expected utility risk preferences and probabilistic beliefs. A non-expected utility preference function $W(f(\cdot)) \equiv W(x_1 \text{ on } E_1;\ldots;x_n \text{ on } E_n)$ over subjective acts $f(\cdot) = [x_1 \text{ on } E_1;\ldots;x_n \text{ on } E_n]$ is said to be *probabilistically sophisticated* if it takes the form $W(f(\cdot)) \equiv V(x_1,\mu(E_1);\ldots;x_n,\mu(E_n))$ for some probability measure $\mu(\cdot)$ over the space of events and some non-expected utility preference function $V(\mathbf{P}) = V(x_1,p_1;\ldots;x_n,p_n)$. Such preferences have been axiomatized in a manner similar to Savage's (1954) axiomatization of the subjective expected utility form $W_{SEU}(f(\cdot)) \equiv U(x_1)\cdot\mu(E_1) + \ldots + U(x_n)\cdot\mu(E_n)$ (Machina and Schmeidler, 1992). Although such preferences can be consistent with Allais-type departures from linearity in (subjective) probabilities, they are not consistent with Ellsberg-type departures from probabilistic beliefs.

Efforts to accommodate the Ellsberg Paradox and the general phenomenon of ambiguity aversion have led to the development of several non-probabilistically sophisticated models of preferences over subjective acts (see the analysis of Epstein (1999) as well as the surveys of Camerer and Weber (1992) and Kelsey and Quiggin (1992)). One such model, the *maximin expected utility* form, replaces the unique probability measure $\mu(\cdot)$ of the subjective expected utility model by a finite or infinite family $\mathcal{M}$ of such measures, to obtain the preference function

$$W_{maximin}(x_1 \text{ on } E_1;\ldots;x_n \text{ on } E_n) \quad \equiv \quad \min_{\mu(\cdot)\in\mathcal{M}}\left[U(x_1)\cdot\mu(E_1)+\ldots+U(x_n)\cdot\mu(E_n)\right]$$

When applied to the Ellsberg Paradox, the family of subjective probability measures $\mathcal{M} = \{(\mu(\text{red}),\mu(\text{black}),\mu(\text{yellow})\} = (1/3,\gamma,2/3-\gamma) \mid \gamma \in [0,2/3]\}$ will yield the typical Ellsberg-type choices of $f_1(\cdot)$ over $f_2(\cdot)$ and $f_4(\cdot)$ over $f_3(\cdot)$ (Gilboa and Schmeidler, 1989).

Another important model for the representation and analysis of ambiguity averse preferences, based on the Rank Dependent form under objective uncertainty, is the *Choquet Expected Utility* form:

$$W_{Choquet}(x_1 \text{ on } E_1;\ldots;x_n \text{ on } E_n) \quad \equiv \quad \textstyle\sum_{i=1}^{n} U(x_i)\cdot\left[C\left(\bigcup_{j=1}^{i} E_j\right) - C\left(\bigcup_{j=1}^{i-1} E_j\right)\right]$$

where for each act $f(\cdot) = [x_1 \text{ on } E_1;\ldots;x_n \text{ on } E_n]$, the payoffs must be labeled so that $x_1 \leq \ldots \leq x_n$, and $C(\cdot)$ is a *nonadditive* measure over the space of events which satisfies $C(\varnothing) = 0$ and $C(\cup_{i=1}^{n} E_i) = 1$. (Gilboa (1987), Schmeidler (1989)). This model has been axiomatized in a manner similar to the Subjective Expected Utility model, and with proper assumptions on the shape of the utility function $U(\cdot)$ and the nonadditive measure $C(\cdot)$ it is capable of demonstrating ambiguity aversion as well as a wide variety of observed properties of risk preferences.

The technique of generalized expected utility analysis under objective uncertainty has also been adopted to the analysis of general non-expected utility/non-probabilistically sophisticated preference functions $W(f(\cdot)) \equiv W(x_1 \text{ on } E_1;...;x_n \text{ on } E_n)$ over subjective acts. So long as such a function is 'smooth in the events' it will possess a 'local expected utility function' (which may be state-dependent) and a 'local probability measure' at each act $f(\cdot)$, and classical results involving expected utility risk preferences and probabilistic beliefs can typically be generalized in the manner described above (Machina, 2005).

See also Allais Paradox; expected utility hypothesis; preference reversals; prospect theory; risk; risk aversion; Savage's subjective expected utility model; uncertainty.

Bibliography

Allais, M. 1953. Le Comportement de l'Homme Rationnel devant le Risque, Critique des Postulats et Axiomes de l'Ecole Américaine, *Econometrica* 21, 503-546.

Allais, M. and O. Hagen (eds.) 1979. *Expected Utility Hypotheses and the Allais Paradox*, D. Reidel Publishing Co., Dordrecht.

Allen, B. 1987. Smooth Preferences and the Local Expected Utility Hypothesis, *Journal of Economic Theory* 41, 340-355.

Arrow, K. 1965. *Aspects of the Theory of Risk Bearing.* Helsinki: Yrjö Jahnsson Säätiö.

Battalio, R., J. Kagel and D. Macdonald. 1985. Animals' Choices over Uncertain Outcomes. *American Economic Review* 75, 597-613.

Becker, S. and F. Brownson. 1964. What Price Ambiguity? Or the Role of Ambiguity in Decision-Making. *Journal of Political Economy* 72, 62-73.

Bell, D. 1985. Disappointment in Decision Making under Uncertainty. *Operations Research* 33, 1-27.

Bernoulli, D. 1738. Specimen Theoriae Novae de Mensura Sortis, *Commentarii Academiae Scientiarum Imperialis Petropolitanae* V, 175-192. English translation: Exposition of a New Theory on the Measurement of Risk, *Econometrica* 22, 1954, 23-36.

Camerer, C. 1989. An Experimental Test of Several Generalized Utility Theories, *Journal of Risk and Uncertainty* 2, 61-104.

Camerer, C. and M. Weber. 1992. Recent Developments in Modelling Preferences: Uncertainty and Ambiguity. *Journal of Risk and Uncertainty* 5, 325–370.

Carlin, F. 1990. Is the Allais Paradox Robust to a Seemingly Trivial Change of Frame? *Economics Letters* 34, 241-244.

Chew, S. 1983. A Generalization of the Quasilinear Mean With Applications to the Measurement of Income Inequality and Decision Theory Resolving the Allais Paradox, *Econometrica* 51, 1065-1092.

Chew, S., L. Epstein and U. Segal. 1991. Mixture Symmetry and Quadratic Utility, *Econometrica* 59, 139-163.

Chew, S., L. Epstein and I. Zilcha. 1988. A Correspondence Theorem Between Expected Utility and Smooth Utility, *Journal of Economic Theory* 46, 186-193.

Chew, S., E. Karni and Z. Safra. 1987. Risk Aversion in the Theory of Expected Utility with Rank Dependent Probabilities. *Journal of Economic Theory* 42, 370-381.

Chew, S. and N. Nishimura. 1992. Differentiability, Comparative Statics, and Non-Expected Utility Preferences. *Journal of Economic Theory* 56, 294-312.

Chew, S. and W. Waller. 1986. Empirical Tests of Weighted Utility Theory. *Journal of Mathematical Psychology* 30, 55-72.

Dekel, E. 1989. Asset Demands without the Independence Axiom. *Econometrica* 57, 163-169.

Edwards, W. 1955. The Prediction of Decisions Among Bets, *Journal of Experimental Psychology* 50, 201-214.

Edwards, W. 1962. Subjective Probabilities Inferred from Decisions, *Psychological Review* 69, 109-135.

Ellsberg, D. 1961. Risk, Ambiguity, and the Savage Axioms. *Quarterly Journal of Economics* 75, 643–669.

Epstein, L. 1999. A Definition of Uncertainty Aversion. *Review of Economic Studies* 66, 579–608.

Friedman, M. and L. Savage. 1948. The Utility Analysis of Choices Involving Risk, *Journal of Political Economy* 56, 279-304.

Gilboa, I. 1987. Expected Utility with Purely Subjective Non-Additive Probabilities. *Journal of Mathematical Economics* 16, 65–88.

Gilboa, I. and D. Schmeidler. 1989. Maxmin expected utility with a non-unique prior. *Journal of Mathematical Economics* 18, 141–153.

Gollier, C. 2000. Optimal Insurance Design: What Can We Do Without Expected Utility, in G. Dionne (ed.) *Handbook of Insurance*. Kluwer Academic Publishers, Boston.

Green, J. and B. Jullien. 1988. Ordinal Independence in Non-Linear Utility Theory. *Journal of Risk and Uncertainty* 1, 355-387.

Hagen, O. 1979. Towards a Positive Theory of Preferences under Risk, in Allais and Hagen (1979).

Hershey, J., H. Kunreuther and P. Schoemaker. 1982. Sources of Bias in Assessment Procedures for Utility Functions. *Management Science* 28, 936-954.

Hershey, J. and P. Schoemaker. 1980. Risk-Taking and Problem Context in the Domain of Losses – An Expected Utility Analysis. *Journal of Risk and Insurance* 47, 111-132.

Hershey, J. and P. Schoemaker. 1985. Probability Versus Certainty Equivalence Methods in Utility Measurement: Are They Equivalent? *Management Science* 31, 1213–1231.

Hey, J. 1984. The Economics of Optimism and Pessimism: A Definition and Some Applications. *Kyklos* 37, 181-205.

Kahneman, D. and A. Tversky. 1979. Prospect Theory: An Analysis of Decision Under Risk. *Econometrica* 47, 263–291.

Kahneman, D. and A. Tversky. 1982. The Psychology of Preferences. *Scientific American* 246, 160-173.

Kahneman, D. and A. Tversky. 1984. Choices, Values and Frames. *American Psychologist* 39, 341-350.

Karmarkar, U. 1978. Subjectively Weighted Utility: A Descriptive Extension of the Expected Utility Model. *Organizational Behavior and Human Performance* 21, 61-72.

Karmarkar, U. 1979. Subjectively Weighted Utility and the Allais Paradox. *Organizational Behavior and Human Performance* 24, 67-72.

Karni, E. 1987. Generalized Expected Utility Analysis of Risk Aversion with State-Dependent Preferences. *International Economic Review* 28, 229-240.

Karni, E. 1989. Generalized Expected Utility Analysis of Multivariate Risk Aversion, *International Economic Review* 30, 297-305.

Keller, L. 1985. The Effects of Problem Representation on The Sure-Thing and Substitution Principles. *Management Science* 31, 738-751.

Kelsey, D. and J. Quiggin 1992. Theories of Choice Under Ignorance and Uncertainty. *Journal of Economic Surveys* 6, 133–153.

Konrad, K. and S. Skaperdas. 1993. Self-Insurance and Self-Protection: A Nonexpected Utility Analysis, *Geneva Papers on Risk and Insurance Theory* 18, 131-146.

Lichtenstein, S. and P. Slovic. 1971. Reversals of Preferences Between Bids and Choices in Gambling Decisions. *Journal of Experimental Psychology* 89, 46–55.

Loomes, G. and R. Sugden 1982. Regret Theory: An Alternative Theory of Rational Choice Under Uncertainty. *Economic Journal* 92, 805–824.

MacCrimmon, K. 1965. *An Experimental Study of the Decision Making Behavior of Business Executives.* Doctoral Dissertation, University of California, Los Angeles.

MacCrimmon, K. 1968. Descriptive and Normative Implications of the Decision-Theory Postulates, in K. Borch, and J. Mossin (eds.). *Risk and Uncertainty: Proceedings of a Conference Held by the International Economic Association.* London: Macmillan and Co.

MacCrimmon, K. and S. Larsson. 1979. Utility Theory: Axioms Versus 'Paradoxes', in Allais and Hagen (1979).

Machina M. 1982. 'Expected Utility' Analysis Without the Independence Axiom. *Econometrica* 50, 277–323.

Machina, M. 1983. Generalized Expected Utility Analysis and the Nature of Observed Violations of the Independence Axiom, in Stigum and Wenstøp (1983).

Machina, M. 1984. Temporal Risk and the Nature of Induced Preferences. *Journal of Economic Theory* 33, 199-231.

Machina, M. 1987. Choice Under Uncertainty: Problems Solved and Unsolved, *Journal of Economic Perspectives* 1, 121-154.

Machina, M. 1989. Comparative Statics and Non-Expected Utility Preferences. *Journal of Economic Theory* 47, 393-405.

Machina, M. 1995. Non-Expected Utility and the Robustness of the Classical Insurance Paradigm, *Geneva Papers on Risk and Insurance Theory* 20, 9-50.

Machina, M. 2005. 'Expected Utility/Subjective Probability' Analysis without the Sure-Thing Principle or Probabilistic Sophistication. *Economic Theory* 26 , 1-62.

Machina, M. and D. Schmeidler. 1992. A More Robust Definition of Subjective Probability. *Econometrica* 60, 745-780.

Markowitz, H. 1952. The Utility of Wealth, *Journal of Political Economy* 60, 151-158.

Marschak, J. 1950. Rational Behavior, Uncertain Prospects, and Measurable Utility. *Econometrica* 18, 111–141.

Morrison, D. 1967. On the Consistency of Preferences in Allais' Paradox. *Behavioral Science* 12, 373-383.

Moskowitz, H. 1974. Effects of Problem Representation and Feedback on Rational Behavior in Allais and Morlat-Type Problems. *Decision Sciences* 5, 225-242.

Múnera, H. and R. de Neufville. 1983. A Decision Analysis Model When the Substitution Principle is Not Acceptable, in Stigum and Wenstøp (1983)

Pratt, J. 1964. Risk Aversion in the Small and in the Large, *Econometrica* 32, 122-136.

Quiggin, J. 1982. A Theory of Anticipated Utility, *Journal of Economic Behavior and Organization* 3, 323-343.

Raiffa, H. 1961. Risk, Ambiguity, and the Savage Axioms: Comment. *Quarterly Journal of Economics* 75, 690-694.

Raiffa, H. 1968. *Decision Analysis: Introductory Lectures on Choices Under Uncertainty*. Reading, Mass.: Addison-Wesley.

Rothschild, M. and J. Stiglitz. 1970. Increasing Risk: I. A Definition, *Journal of Economic Theory* 2, 225-243.

Rothschild, M. and J. Stiglitz. 1971. Increasing Risk: II. Its Economic Consequences, *Journal of Economic Theory* 3, 66-84.

Savage L. 1954. *The Foundations of Statistics.* New York: John Wiley and Sons. Revised and Enlarged Edition, New York: Dover Publications. 1972.

Schlesinger, H. 1997. Insurance Demand Without the Expected Utility Paradigm, *Journal of Risk and Insurance* 64, 19-39.

Schmeidler, D. 1989. Subjective Probability and Expected Utility Without Additivity. *Econometrica* 57, 571–587.

Schoemaker, P. and H. Kunreuther. 1979. An Experimental Study of Insurance Decisions. *Journal of Risk and Insurance* 46, 603-618.

Segal, U. 1984. Nonlinear Decision Weights with the Independence Axiom, manuscript, University of California, Los Angeles.

Slovic, P. 1969. Manipulating the Attractiveness of a Gamble Without Changing its Expected Value. *Journal of Experimental Psychology* 79, 139–145.

Slovic, P., B. Fischhoff and S. Lichtenstein. 1977. Behavioral Decision Theory. *Annual Review of Psychology* 28, 1-39.

Slovic, P. and A. Tversky. 1974. Who Accepts Savage's Axiom? *Behavioral Science* 19, 368-373.

Starmer, C. 2000. Developments in Non-Expected Utility Theory: The Hunt for a Descriptive Theory of Choice under Risk. *Journal of Economic Literature* 38, 332-382.

Stigum, B. and F. Wenstøp (eds.) *Foundations of Utility and Risk Theory with Applications,* Dordrecht : D. Reidel Publishing Co.

Sugden, R. 1986. New Developments in the Theory of Choice Under Uncertainty. *Bulletin of Economic Research* 38, 1-24.

Tversky, A. 1975. A Critique of Expected Utility Theory: Descriptive and Normative Considerations. *Erkenntnis* 9, 163-173.

Tversky, A. and D. Kahneman. 1981. The Framing of Decisions and The Psychology of Choice. *Science* 211, 453-458.

von Neumann, J. and O. Morgenstern. 1944. *Theory of Games and Economic Behavior*. Princeton: Princeton University Press. (2nd Ed. 1947; 3rd Ed. 1953).

Wang, T. 1993. L_p-Fréchet Differentiable Preference and 'Local Utility' Analysis, *Journal of Economic Theory* 61, 139-159.

Weber, M. and C. Camerer. 1987. Recent Developments in Modelling Preferences under Risk. *OR Spektrum* 9, 129–151.

Yaari, M. 1987. The Dual Theory of Choice under Risk, *Econometrica* 55, 95-115.