Status Preference, Wealth, and Dynamics in the Open Economy

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April 2001
Founded in 1963 by two prominent Austrians living in exile – the sociologist Paul F. Lazarsfeld and the economist Oskar Morgenstern – with the financial support from the Ford Foundation, the Austrian Federal Ministry of Education and the City of Vienna, the Institute for Advanced Studies (IHS) is the first institution for postgraduate education and research in economics and the social sciences in Austria. The Economics Series presents research done at the Department of Economics and Finance and aims to share “work in progress” in a timely way before formal publication. As usual, authors bear full responsibility for the content of their contributions.
Abstract

The implications of status preference in a simple open economy model will be investigated in this paper. The open economy is modeled as a continuum of identical representative agents who have preferences over consumption and status. In the paper status is identified as relative wealth, which takes the form of relative holdings international financial assets. A symmetric macroeconomic equilibrium is derived in which status is the source of transitional dynamics for consumption and the current account balance. This result illustrates another way to generate transitional dynamics in the small open economy model, which, as is well-known, does not have well-defined dynamics under perfect capital mobility. The role of status plays in influencing the open economy's adjustment to various macroeconomic shocks is also considered.

Keywords
Status-preference, open economy dynamics

JEL Classifications
E21, F41
Comments
A previous version of this paper has been presented at the 2001 American Economic Association meeting in New Orleans, LA. I thank the participants of the session for their constructive comments and, in particular, Ted Palivos. I also would like to acknowledge the very useful suggestions of Farhad Rassekh and Franz X. Hof. Finally, I wish to thank of the Jubilaeumsfonds of the Austrian National Bank (OeNB), project No. 8701, for its generous financial support.
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1. Introduction

An individual’s utility is usually stated in terms of absolute of goods and services. This standard specification is intuitively appealing and adequate to study many economic problems. There is evidence, however, some of which is provided by Easterlin (1974, 1995) to indicate that an individual’s economic well being depends on his position, or status, in society, along with his absolute consumption, income, or wealth. The idea that individuals are motivated by status considerations is a very old one in economics and can be traced back to thinkers such as David Hume (1978) and Thorstein Veblen (1899). The purpose of this paper is to consider the influence of status-preference in a small open economy model.

While authors such as Frank (1985) have modelled the influence of status-preference in a static setting, there are an increasing number of researchers who study status-preference in a dynamic macroeconomic or endogenous growth context. These include Galí (1994), Corneo and Jeanne (1997), Rauscher (1997), Futagami and Shibata (1998), and Fisher and Hof (2000). Their work illustrates two ways in which status can be modelled in a macroeconomic setting. The approach adopted by Galí (1994), Rauscher (1997) and Fisher and Hof (2000) specifies that status-preference derives from relative consumption. In contrast, Corneo and Jeanne (1997) and Futagami and Shibata (1998) represent status-preference as arising from relative wealth, which these researchers measure in terms of physical capital. In this paper we will also identify status as relative wealth, but will model it in terms of international assets rather than in terms of physical capital.\(^1\) Furthermore, while all these authors model the role of status-preference in a closed economy context, we will, as indicated, consider this question in an open economy framework. We believe this is an important extension of this line of research due to the increasing integration of the world economy and the greater role of international assets in wealth accumulation. Our work

\(^1\)It is sometimes argued that because it is easier to “see” relative consumption compared to relative wealth, the former is more appropriate way to model status. With, however, employee stock-options plans becoming more widespread, it may be easier to observe relative wealth than was previously the case.
is, in addition, related to the recent Spirit of Capitalism literature, which is exemplified by authors such as Cole, Mailath, and Postlewaite (1992), Zou (1994, 1998) and Bakshi and Chen (1996), who seek to explain growth, savings, and asset pricing behavior. This research, based on the ideas of Max Weber (1958), views the wealth accumulation as the means to achieving social status, which itself enables agents to acquire goods that are regarded as “prizes” by society at large. These models can then be interpreted as reduced-form versions of explicit microeconomic models in which agents contend to gain such goods.

A further motivation for our approach is to offer an alternative solution to a long-standing issue in open economy macroeconomics: the fact that the small open economy under the assumption of perfect capital mobility does not have a well-defined transitional dynamic equilibrium. This arises because if the small open economy can borrow or lend at the given world interest rate, it must be the case that the domestic rate of time preference equals the world interest rate in order for the economy to attain an interior equilibrium. This, in turn, which rules out the possibility of well-defined transitional dynamics, since this relationship also fixes a given stock of physical capital or assets. Turnovsky (1997) discusses several ways in which transitional dynamics can be restored in the small open economy, representative agent context. One approach that has been extensively used, see, for instance, Brock (1988), Sen and Turnovsky (1989a, 1989b, 1990), and Frenkel, Razin, and Yuen (1996), is to incorporate into the small open economy model a convex installation cost function for investment demand. While the equality between the domestic rate of time preference and the world interest rate must still be imposed in this framework, the domestic capital stock (and output) will adjust sluggishly to maintain the equality between the rates of return of domestic and foreign assets. The adjustment cost approach is, nevertheless, not without its critics, who point out that it is difficult to

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2 If, on the other hand, the small open economy rate of time preference exceeds the world interest rate, then agents eventually mortgage all of their capital and labor income. In contrast, the most patient country acquires over time the wealth of all other countries. See Barro and Sala-i-Martin (1995), chapter 3 for a discussion of these counterfactual cases.

3 See Turnovsky (1997), chapters 2 and 3. He also points out that this issue does not arise in the overlapping generations framework of which the Blanchard (1985) model is an infinite horizon variant.
reconcile covariance properties of postwar U.S. data with those of macroeconomic models that incorporate plausible values of the adjustment cost parameter.\(^4\)

Another, though more controversial, approach to restoring transitional dynamics in the small open economy model is to specify that agents possess Uzawa-type, time-dependent, endogenous rates of time preference. Authors such as Blanchard and Fischer (1989), Barro and Sala-i-Martin (1995), and Turnovsky (1997) find this formulation intuitively unappealing, however, because a necessary condition in the infinite horizon context to generate saddlepoint dynamics is that the rate of time preference increases with the level of consumption. Obstfeld (1990), drawing on the work of Epstein (1987), among others, offers a defense of this approach and points out its usefulness in relaxing the assumption of time-additive preferences, which is also restrictive.

Two additional, and related, approaches to addressing this issue are to incorporate costs of holding foreign bonds and to specify an upward-sloping supply curve of debt. Both approaches attempt to model, in a certainty equivalent framework, the macroeconomic implications of imperfect substitutability between domestic and foreign assets. The first approach was taken by Turnovsky (1985), while the second has been adopted by authors such as Bhandari, Haque, and Turnovsky (1990). Using these specifications, interior solutions are obtained without imposing equality between the rate of time preference and the world interest rate. Because asset stocks will, as a consequence, adjust slowly according to international arbitrage, economies that incorporate these specifications exhibit transitional dynamics. One issue that arises in these two cases is the fact that the existence of transitional or degenerate dynamics often depends on the macroeconomic shock in question.\(^5\) In addition, it can be argued that it is better to model the implications of international capital market imperfections, which depend, in part, on the risk characteristics of domestic and foreign assets, in an explicitly stochastic setting. In this paper, we will show that our specification of status-preference leads to a transitional dynamic

\(^4\)This is point is made by Kydland and Prescott (1982).

\(^5\)Fisher (1995) and Fisher and Terrell (2000) show that the upward-sloping debt specification is useful, however, in calculating the implications of world interest rate disturbances.
equilibrium and argue that it generates, at least in our simple framework, more sensible predictions concerning the comparative dynamic response to selected macroeconomic shocks than do some of the other approaches discussed here.\(^6\)

In order to concentrate on the influence of status-preference, we will adopt a very simple open economy specification. We will assume that agents receive a given endowment of real income, consume a single real good tradeable at a unitary world price, and lend (or borrow) at an exogenously determined world interest rate.\(^7\) Using an intertemporal optimizing, perfect-foresight framework, we will derive a symmetric equilibrium in which identical agents act in the same way. We will show that the open economy’s transitional dynamics arise from agents’ preference for relative wealth. That is, our formulation of status-preference results in an “effective” domestic rate of return that is endogenously determined outside of steady state equilibrium. This, in turn, results in transitional dynamics for consumption and the current account balance. We will also incorporate a public sector into our model in order to determine how the open economy with status-preference responds to fiscal and initial public debt shocks. Overall, we will consider income (endowment), government expenditure, initial public debt, and world interest rate shocks.

The structure of the paper is as follows. The next section, section 2, describes the model and derives the symmetric, intertemporal equilibrium. Section 3 calculates the effects of income (endowment), government expenditure, initial public debt, and world interest rate shocks on the intertemporal dynamics of the small open economy. In this section we will employ phase diagrams to describe the transitional adjustment of consumption and national debt in response to these macroeconomic disturbances. We close in section 4 with concluding remarks and some suggestions for future work. A brief mathematical appendix is also included.

\(^6\)Since, as we will describe below, output is fixed in our model, our results will not be directly comparable to those derived from the specification that uses an installation cost of investment function.

\(^7\)See Obstfeld and Rogoff, chapters 1 and 2, for a recent exposition of the small open economy endowment model.
2. The Model and Intertemporal Equilibrium

We begin by assuming that there are a large number of representative agents, each of whom has the following preferences over own consumption, $c$, and status, $s$, which we identify with relative wealth, $a/A$

$$U = U(c, s) = U(c, a/A)$$  \hspace{1cm} (1a)

where $a$ is individual wealth and $A$ is aggregate, or average, wealth and all variables in the model are denominated in real terms.\(^8\) We further specify that (1a) is increasing in both consumption and status and strictly concave:

$$U_c > 0, U_s > 0, U_{cc} < 0, U_{ss} < 0, U_{cc}U_{ss} - U_{cs}^2 > 0.$$  \hspace{1cm} (1b)

In order to focus on the influence of status-preference, we make simple assumptions regarding the open economy’s technological and financial market possibilities. Specifically, we assume that the agent receives a given endowment of income, denoted by $\bar{y}$, which is traded internationally at a fixed, unitary world price. In addition, we specify that agents can lend (or borrow) at the exogenously given world interest rate, equal to $r^*$, and can accumulate wealth in the form of domestic government bonds or international assets, which are perfect substitutes.\(^9\) (The domestic public sector will be introduced below.) Employing an infinite horizon, perfect foresight framework, the agent’s maximization problem is formulated as follows

$$\max \int_0^\infty U(c, s)e^{-rt}dt$$  \hspace{1cm} (2a)

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\(^8\)We will assume that the economy achieves an interior solution in which $c > 0$, $a/A > 0$, $a > 0$, $A > 0$. The conditions under which this holds will be derived below. This formulation rules out the possibility that private agents are net debtors. See, however, Hof and Wirl (2001), who analyze this case.

\(^9\)For a variable $x$, $\dot{x}$ denotes a non-policy variable fixed at a constant level, while $\dot{x}$ indicates that its time derivative is being taken. The partial derivatives of functions will be denoted by subscripts. Note that in general we suppress a variable’s time dependence.
subject to
\[ \dot{a} = r^*a + \bar{y} - c - \tau, \quad a(0) = a_0 > 0 \] (2b)

where \( \rho \) is the exogenous rate of pure time preference, \( \tau \) is the given level of lump-sum taxes, and \( a_0 \) is the agent’s initial endowment of wealth. Solving this optimization problem, we obtain the following first order conditions for the representative agent:

\[ U_c(c, a/A) = \lambda \] (3a)

\[ \dot{\lambda} = (\rho - r^*) \lambda - A^{-1} U_s(c, a/A) \] (3b)

\[ \lim_{t \to \infty} \lambda a e^{-\rho t} = 0. \] (3c)

where \( \lambda \) the current costate variable. Equation (3a) is the necessary condition for consumption, while (3b) describes the evolution of the shadow value if individual asset accumulation is chosen optimally. In performing the maximization, the agent ignores the effect of his choice of \( a \) on \( A \). Observe that the evolution of \( \lambda \) depends not only on individual and aggregate asset stocks, but also on the agent’s attitude toward status, which is embodied in the partial derivative \( U_s(c, a/A) \). Note also that equation (3c) represents the transversality condition that constrains the dynamics of private sector asset accumulation.

The next step is to derive the intertemporal macroeconomic equilibrium. Following the standard procedure for this type of problem, we assume that each individual acts in the same way and normalize the population of agents to unity. This implies \( a = A \) and results in the following symmetric private sector equilibrium:

\[ U_c(c, 1) = \lambda, \quad c = c(\lambda), \quad c_\lambda = 1/U_{cc} < 0 \] (4a)

\[ \dot{\lambda} = (\rho - r^*) \lambda - a^{-1} U_s[c(\lambda), 1] \] (4b)

\[ \dot{a} = ra + \bar{y} - c(\lambda) - \tau \] (4c)
together with the transversality condition (3c).

We now incorporate the domestic public sector into the equilibrium and state the following flow budget constraint

\[ \dot{b} = r^* b + g - \tau \]  

(5a)

where \( b \) is the stock of government debt, \( g \) represents government spending, and, as indicated above, \( \tau \) is the level of lump-sum taxes. The government can borrow from the domestic private sector or from abroad at the prevailing world interest rate \( r^* \). We further assume that the accumulation of government debt is subject to the following No-Ponzi condition:

\[ \lim_{t \to \infty} be^{-r^* t} = 0. \]

(5b)

To obtain the overall macroeconomic equilibrium, we introduce the variable \( z = b - a \), which is equal to the economy’s net (or national) indebtedness. This is done in order to permit the small open economy as a whole be a net debtor, while, at the same time, maintaining our assumption that the private sector is a net creditor, which we believe to be a realistic situation. Subtracting (2a) from (5a), we obtain the following expression for the accumulation of net debt, which is equal to the (negative) of the current account balance:

\[ \dot{z} = r^* z + c(\lambda) + g - \bar{y}. \]

(6)

Substituting for \( z \) in equation (4b), we can state the open economy’s equilibrium dynamics as

\[ \dot{\lambda} = (\rho - r^*) \lambda - (b - z)^{-1} U_s [c(\lambda), 1] \]

(7a)

\[ \dot{z} = r^* z + c(\lambda) + g - \bar{y} \]

(7b)

\[ \dot{b} = r^* b + g - \tau \]

(7c)

where the expressions for \( \dot{z} \) and \( \dot{b} \) are repeated for expositional purposes. In order to sim-

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10For the remainder of the paper, lower-case variables will refer to their economy-wide aggregates.
plify further the economy’s dynamic structure, we assume that the fiscal policy authorities maintain a continuously balanced budget, i.e.,

$$\dot{b} = 0, \quad \tau = g + r\dot{b}, \quad \forall t \geq 0$$

(8)

where $b(t) \equiv \bar{b}$ is the constant stock of government debt, which we can identify, without loss of generality, with its exogenous initial stock, $b_0$. Under this fiscal policy rule, the dynamics of the open economy collapse to equations (7a, b), with $b$ replaced by $b_0$ in (7a), and the private sector transversality condition (3c).

As an aid to intuition, it is useful to rewrite the equilibrium dynamics in terms of the control variable, $c$, consumption. To do so, we take the time derivative of (3a), evaluated in the symmetric equilibrium, equate the resulting expression to (7a), and divide by (3a). This procedure results in the following Euler equation for $\dot{c}$

$$\dot{c} = \sigma^e(c) c \left[ r^* + (b_0 - z)^{-1} \frac{U_s(c, 1)}{U_c(c, 1)} - \rho \right] = \sigma^e(c) c \left[ r^e(r^*, b_0, z, c) - \rho \right]$$

(9a)

where

$$\sigma^e(c) = -\frac{U_c(c, 1)}{c U_{cc}(c, 1)}, \quad r^e(r^*, b_0, z, c) = r^* + (b_0 - z)^{-1} \frac{U_s(c, 1)}{U_c(c, 1)}$$

(9b)

are, respectively, the effective intertemporal elasticity of substitution, $\sigma^e(c)$, and the effective domestic, or internal, rate of return on assets, $r^e(r^*, b_0, z, c)$. Observe that the effective rate of return is equal to the sum of the world interest rate and the marginal rate of substitution between status and consumption in the symmetric equilibrium. The latter term represents the additional gain received by domestic agents due to status-preference. Moreover, because status takes the form of relative wealth in our framework, the rate of return $r^e$ is not fixed parametrically, which will imply that the small open economy has (potentially) saddlepoint stable transitional dynamics. To see this, assume, instead, that agents have standard concave preferences solely over own consumption, so that (1a)
equals $U = U(c)$, $U' > 0$, $U'' < 0$. The Euler equation (9a) then collapses to

$$\dot{c} = \sigma(c) c [r^* - \rho]$$

(10)

where $\sigma(c) = -U''/U'$ is the standard elasticity of substitution. Since $r^*$ and $\rho$ are both fixed constants, the equation (10) implies that $r^* = \rho$ must hold in order for the open economy to reach a stable, interior equilibrium. As we discussed in the introduction, this means that the small open economy exhibits no transitional dynamics, since $\dot{c} = \dot{z} = 0$ for all $t \geq 0$ in this case. We would also obtain this result if status corresponded to relative consumption rather than to relative wealth, i.e., if (1a) were specified as $U = U(c, s) = U(c, c/C)$, where $C$ would represent aggregate, or average, consumption. With this formulation, the only the static levels of consumption and its marginal utility would be affected by status-preference.

In moving to a characterization of the economy’s dynamic and equilibrium properties, it is convenient to introduce the following constant elasticity parameterization of preferences:

$$U(c, s) = \frac{1}{1 - \alpha} \left[ (c^{1-\delta} s^{\delta})^{1-\alpha} - 1 \right], \quad s = a/A, \quad \alpha > 0, \quad 0 < \delta < 1.$$  

(11)

Using this specification of preferences, the Euler equation (9a) becomes

$$\dot{c} = \frac{c}{\delta + \alpha(1-\delta)} \left[ r^* + (b_0 - z)^{-1} \frac{\delta c}{1-\delta} - \rho \right]$$

(12a)

where the effective intertemporal elasticity of substitution and the effective rate of return

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11See, however, the model of Fisher and Hof (2000) in which relative consumption does influence equilibrium dynamics in a closed economy, endogenous growth context.

12The static marginal utility is derived from the relative consumption version of equation (3a), which is given by $U_c(c, 1) + c^{-1}U_s(c, 1) = \lambda$ in the symmetric equilibrium.

13The partial derivatives of (11) are stated in the appendix. Using these expressions, we can show that (11) satisfies the curvature conditions we imposed on $U(c, s)$ in (1b).
on assets are, respectively, equal to:

\[ \sigma^e = [\delta + \alpha(1 - \delta)]^{-1}, \quad r^e(r^*, b_0, z, c) = r^* + (b_0 - z)^{-1} \frac{\delta c}{1 - \delta}. \]  \hspace{1cm} (12b)

Clearly, both \( \sigma^e \) and \( r^e \) depend on the parameter \( \delta \), which represents the private sector’s attitude toward status in equation (11). It is straightforward to show that an increase in \( \delta \) raises (resp. lowers) \( \sigma^e \) if \( \alpha > 1 \) (resp. \( \alpha < 1 \)) and that a rise in \( \delta \) increases \( r^e \). In addition, because both higher levels of consumption and national debt raise the marginal rate of substitution between status and consumption, both changes (in partial equilibrium) increase the effective rate of return.\(^{14}\)

Letting \( c = \dot{z} = 0 \) in equations (7b) and (12b), we can calculate the following parameterized steady state solutions for consumption, national debt, and the effective rate of return

\[ \ddot{c} = \frac{(1 - \delta)(\rho - r^*)(\bar{y} - g - r^*b_0)}{(1 - \delta)\rho - r^*} \]  \hspace{1cm} (13a)

\[ \ddot{z} = \frac{-\delta(\bar{y} - g) + (1 - \delta)(\rho - r^*)b_0}{(1 - \delta)\rho - r^*} \]  \hspace{1cm} (13b)

\[ \ddot{r}^e = \rho \]  \hspace{1cm} (13c)

where \( \ddot{x} \) denotes the steady state value of a variable \( x \). Using the definition \( \ddot{a} \equiv b_0 - \ddot{z} \), the long-run solution for the private sector’s asset position is, in turn, given by:

\[ \ddot{a} = \frac{\delta(\bar{y} - g - r^*b_0)}{(1 - \delta)\rho - r^*}. \]  \hspace{1cm} (13d)

In order to guarantee an interior solution for steady state consumption and private sector asset holdings, we assume \( (\bar{y} - g - r^*b_0) > 0 \) and \( [(1 - \delta) \rho - r^*] > 0 \). The first condition requires that the (fixed) level of output exceed government expenditure plus interest service on the inherited stock of public debt, while the second restricts the values that the

\(^{14}\)Consequently, the positive relationship in our model between the effective rate of return and the stock of national debt arises from status-preference rather than from capital market imperfections. In addition, a rise in the world interest rate leads to a one-for-one increase in the effective rate of return, while an increase in the initial stock of government debt lowers \( r^e \) in partial equilibrium.
status parameter $\delta$ can take.\(^\text{15}\) Since $0 < \delta < 1$, observe that the second condition is also sufficient for $\rho > r^*$, and will be, as we will show, a necessary condition for the existence of a saddlepoint equilibrium for the linearized versions of (7b) and (12a). Equations (13a, d), show that $\tilde{c}$ and $\tilde{a}$ are larger the greater is (fixed) output, net of government expenditure and public sector interest service. Noting the numerator of (13b), the steady state solution for $\tilde{z}$, national debt, can be either positive or negative. It is clear that the greater is the initial stock of public debt, $b_0$, the larger is the level of national indebtedness, while the opposite is the case, the greater is the level of (fixed) output net of government expenditure, $(\bar{y} - g)$. For expositional purposes, we will assume subsequently that the small open economy is a net debtor in steady state equilibrium, i.e., $\tilde{z} > 0$.\(^\text{16}\) Finally, equation (13d) states that the long-run value of the effective rate of return is pinned-down by the exogenous rate of time preference.

Taking first-order approximations of (7b) and (12a) around the steady state solutions (13a, b), we derive the following matrix differential equation

$$
\begin{pmatrix}
\dot{c} \\
\dot{z}
\end{pmatrix} =
\begin{pmatrix}
\theta_{11} & \theta_{12} \\
1 & r^*
\end{pmatrix}
\begin{pmatrix}
c - \tilde{c} \\
z - \tilde{z}
\end{pmatrix}
$$

(14a)

where

$$
\theta_{11} = \frac{\rho - r^*}{\delta + \alpha(1 - \delta)} > 0, \quad \theta_{12} = \frac{(1 - \delta)(\rho - r^*)^2}{\delta[\delta + \alpha(1 - \delta)]} > 0.
$$

(14b)

The stability properties of this system are determined by the signs of the trace and

\(^{15}\)The first condition can be rewritten as $b_0 < (\bar{y} - g)/r^*$, which says that the initial stock of public debt cannot exceed the present discounted value of output net of government expenditure. We can also show that the second condition is equivalent to the restriction that $\delta$ not exceed the ratio $(\rho - r^*)/\rho$.

\(^{16}\)Using equation (13b), the condition for $\tilde{z} > 0$ is:

$$
b_0 > \frac{\delta(\bar{y} - g)}{(1 - \delta)(\rho - r^*)}.
$$

In addition, it is easy to show by differentiating (13b) with respect to $\delta$ that $\tilde{z}$ declines the higher is the status parameter $\delta$. 

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determinant of the coefficient matrix of (14). These are given, respectively, by

\[ \mu_1 + \mu_2 = \theta_{11} + r^* > 0, \quad \mu_1 \mu_2 = \theta_{11}r^* - \theta_{12} \]  

(15)

where \( \mu_1, \mu_2 \) are the corresponding eigenvalues of the dynamic system. For the equilibrium of (14a) to be a saddlepoint, \( \mu_1 \mu_2 \) must be negative. Using the expressions for the matrix elements \( \theta_{11}, \theta_{12} \) in (14b), we can show that the determinant of the coefficient matrix of (14b) equals:

\[ \mu_1 \mu_2 = -\sigma e^\epsilon \frac{(\rho - r^*)[(1 - \delta)\rho - r^*]}{\delta} = -\frac{(\rho - r^*)[(1 - \delta)\rho - r^*]}{\delta [\delta + \alpha(1 - \delta)]}. \]  

(16)

Since this expression is negative under our assumption \( [(1 - \delta)\rho - r^*] > 0 \), the equilibrium of (14a) is a saddlepoint, with \( \mu_1 < 0, \mu_2 > 0, |\mu_1| < \mu_2 \). Using standard methods, we can then obtain the following solutions for consumption and national debt along the stable saddlepath:

\[ c = \tilde{c} + (r^* - \mu_1)(\tilde{z} - z_0)e^{\mu_1 t} = \tilde{c} + \frac{\theta_{12}}{\theta_{11} - \mu_1}(\tilde{z} - z_0)e^{\mu_1 t} \]  

(17a)

\[ z = \tilde{z} - (\tilde{z} - z_0)e^{\mu_1 t} \]  

(17b)

where \( z \) adjusts from an exogenous initial stock, \( z_0 \).\(^{17}\) Combining the solutions (17a, b), we obtain the saddlepath that describes the stable co-movements of consumption and net debt

\[ (c - \tilde{c}) = -(r^* - \mu_1)(z - \tilde{z}) = \frac{-\theta_{12}}{\theta_{11} - \mu_1} (z - \tilde{z}). \]  

(17c)

The graph of this locus has a negative slope, so that consumption and national debt adjust in opposite directions along the saddlepath, i.e., \( \text{sgn}(\dot{c}) = -\text{sgn}(\dot{z}) \). We can now derive the phase diagram, illustrated by Figure 1, of equation (14a). Using equations (7b) and (12a), the \( \dot{c} = 0 \) and \( \dot{z} = 0 \) loci are described, respectively, by:

\[ c = \frac{1 - \delta}{\delta}(\rho - r^*)(b_0 - z) \]  

(18a)

\(^{17}\)Because \( \mu_1 \) is an eigenvalue of coefficient matrix of (14a), \( (r^* - \mu_1) = \theta_{12}/(\theta_{11} - \mu_1) \).
\[ c = -r^* z + \bar{y} - g. \] (18b)

The \( c = 0 \) locus has a negative slope that depends on the size of the status parameter \( \delta \). For instance, the greater weight the private sector attaches to status, i.e., the greater is \( \delta \), the flatter is the \( c = 0 \). The \( z = 0 \) locus, in contrast, is independent of consumer preferences and has a slope equal to \(-r^*\). Note from the first equality of (17c) that the slope of \( z = 0 \) is smaller in absolute value than that of the stable saddlepath. The intersection of the \( c = 0 \) and \( z = 0 \) loci is illustrated by point \( A \) in Figure 1 and determines the steady state values of \( c \) and \( z \).

We can also use the solutions for consumption and national debt to solve for the path of the effective rate of return, \( r^e \). First, we linearize the parameterized expression for the effective rate of return, equation (12b), about its steady state value of \( \rho \). This yields:

\[ r^e = \rho + \frac{\delta}{(1 - \delta)(b_0 - \bar{z})} (c - \bar{c}) - \frac{\bar{c}}{(b_0 - \bar{z})}(z - \bar{z}) \]. (19a)

Substituting the second equality of (17a) and the solution for \( z \) given in (17b) into (19a), we obtain the following expression for the path of \( r^e \):

\[ r^e = \rho + \frac{\delta}{(1 - \delta)(b_0 - \bar{z})} \left( \frac{\theta_{12}}{\theta_{11} - \mu_1} - \frac{\bar{c}}{(b_0 - \bar{z})} \right) (\bar{z} - z_0) e^{\mu_1 t}. \] (19b)

This solution can be simplified by substituting for \( \theta_{11}, \theta_{12} \) and using the fact that

\[ \frac{\bar{c}}{(b_0 - \bar{z})} = \frac{\bar{c}}{\bar{a}} = \frac{(1 - \delta)(\rho - r^*)}{\delta} \] (19c)

where the latter expression is calculated using equations (13a, d).\(^{18}\) Making these substitutions in (19b) and evaluating the resulting expression, we can show the stable solution for the effective rate of return equals:

\[ r^e = \rho + \frac{(\rho - r^*)\mu_1}{\bar{a}(\theta_{11} - \mu_1)} (\bar{z} - z_0) e^{\mu_1 t}. \] (19d)

\(^{18}\)Observe that the ratio \( \bar{c}/\bar{a} \) is a negative function of the status parameter \( \delta \).
The solution for $r^e$ states that if $z_0 < \tilde{z}$ (resp. $z_0 > \tilde{z}$), then $r^e < \rho$ (resp. $r^e > \rho$) along the stable saddle path. Consequently, it is the case that $sgn (r^e) = sgn (\dot{z})$ during the transition to steady state equilibrium.

3. Comparative Dynamics

In this section we will describe the dynamic response of consumption and national debt in the small open economy to the following permanent shocks: income (endowment), government expenditure, initial public debt, and world interest rate shocks. To calculate the steady state effects of these disturbances, we employ the steady solutions, equations (13a, b), derived in the previous section. The transitional response of the small economy will be calculated using stable saddlepath solutions given above in equations (17a, b) and will be illustrated with diagrams based on Figure 1.

3.1. Income Shocks

An increase in the economy’s endowment of income has the following steady state impact on consumption and national debt:

$$\frac{d\tilde{c}}{dy} = \frac{d\tilde{c}}{dg} = \frac{(1 - \delta)(\rho - r^*)}{(1 - \delta)\rho - r^*} > 1$$  \hspace{1cm} (20a)

$$\frac{d\tilde{z}}{dy} = \frac{d\tilde{z}}{dg} = -\frac{\delta}{(1 - \delta)\rho - r^*} < 0.$$  \hspace{1cm} (20b)

In response to the rise in $\bar{y}$, consumption increases and national debt declines in long-run equilibrium. In terms of Figure 2a, this corresponds to a shift in the $\tilde{z} = 0$ locus to the right, which results in a new intersection with the $\tilde{c} = 0$ locus at point $C$. Observe from (20a) that the rise in $\bar{y}$ increases $\tilde{c}$ more than proportionately. In other words, income

\[19\] The changes in the long-run stock of private sector assets $\bar{a}$ are calculated using the identity $a \equiv b_0 - z$.

\[20\] The shocks are, therefore, permanent unanticipated shocks.

\[21\] Note from equations (13) that this disturbance is equivalent to a decrease in government expenditure.
(and government expenditure) shocks have a multiplier effect on steady state consumption in our model of status-preference. This result is easily explained using Figure 2a. Using the first equality of (17a), the initial response of consumption equals:

\[
\frac{dc(0)}{dy} = -\frac{dc(0)}{dy} = 1 + \frac{\delta \mu_1}{(1 - \delta) \rho - r^*} < 1. 
\] (21)

In terms of Figure 2a, this corresponds to an initial jump in \( c(0) \) to point \( B \), which lies on the saddle path corresponding to the new steady state equilibrium at point \( C \). From equation (21), it is evident that since the rise in consumption at \( t = 0 \) is less than the increase in income, this disturbance also raises private savings. This reflects the fact that status—in the form of relative wealth—and own consumption are both normal goods in this economy. Consequently, the private sector accumulates assets, which, in the symmetric equilibrium, leads to an improvement in the economy’s aggregate asset position that corresponds to a decline in \( z \). It is this long-run decline in the economy’s net indebtedness that permits steady state consumption to rise by more than the rise in income. In Figure 2a, this process is depicted by the adjustment of net debt and consumption along the new stable locus from point \( B \) to point \( C \) in which \( \dot{z} < 0, \dot{c} > 0 \). These results stand in sharp contrast to those that would obtain if consumer preferences were a function solely of own consumption, i.e., if \( U = U(c) \), or if status depended on relative consumption rather than on relative wealth, i.e., if \( U = U(c, c/C) \). Because the Euler relationship in both of these cases collapses to equation (10), there are no transitional dynamics subsequent to the increase in \( \bar{y} \), only an immediate one-for-one increase in consumption with no change in the equilibrium stock of national debt.

We can also describe impact of this shock on the effective rate of return, \( r^e \). Using the solution for \( r^e \) given above in (18d), we can show that the initial response of effective

\footnote{Using the second equality of (17a), it is straightforward to show that the initial increase in \( c(0) \) is unambiguously positive.}

\footnote{Indeed, the long-run response of consumption and net debt are greater, the larger is the status parameter \( \delta \). In terms of Figure 2a, a larger value of \( \delta \) corresponds to a “flatter” \( \dot{c} = 0 \) locus. This produces, in turn, greater changes in \( \ddot{c} \) and \( \ddot{z} \) for a given shift in the \( \dot{z} = 0 \) locus.}
return equals:

\[ \frac{dr^c(0)}{d\hat{y}} = \frac{\delta}{(1 - \delta) \bar{\dot{a}}} \frac{dc(0)}{d\hat{y}} = -\frac{\delta(\rho - r^*)\mu_1}{[(1 - \delta)(\rho - r^*)](\theta_{11} - \mu_1)} > 0. \]  

(22)

Thus, the effective rate of return initially rises subsequent to the increase in \( \hat{y} \). This reflects the fact that initial jump in consumption raises the marginal rate of substitution between status and consumption. After \( t = 0 \), the effective rate of return then declines, (along with national debt), toward its steady state value of \( \rho \).

We are now in a position to ask how the response of the economy to the income shock would differ if, instead, agents possessed Uzawa-type preferences, leaving the rest of the economic structure unchanged.\(^{24}\) We can show that in response to an increase in \( \hat{y} \) that the long-run stock of debt \( \tilde{z} \) rises, while the steady state level of consumption \( \tilde{c} \) is unchanged. This is brought about by an initial increase in consumption that exceeds the rise in income. Agents with Uzawa-type preferences then dissave during the entire transition to the steady state equilibrium, which raises \( \tilde{z} \) and brings consumption back to its original, pre-shock level. If, on the other hand, the economy is characterized by an upward-sloping debt function and preferences depend solely on own consumption, then we can show that consumption immediately rises one-for-one with the increase in \( \hat{y} \).\(^{25}\) Since there are no transitional dynamics in this case, the stock of national debt is completely unaffected by the income shock.\(^{26}\) In our view, the dynamic response of

\(^{24}\)With time dependent preferences, equation (2a) becomes

\[ \max \int_0^\infty U(c)e^{-\int_0^t \beta(s)ds}dt \]

where \( \beta(s) = \beta[U(c)] \), \( \beta'(U) > 0 \), \( \beta''(U) > 0 \), \( \beta(U) - U(c)\beta'(U) > 0 \). The assumption \( \beta'(U) > 0 \) is required for the economy to have saddlepoint dynamics. In addition, our subsequent discussion applies equally well to the case of fiscal shocks.

\(^{25}\)In the context of our model, the upward-sloping debt function would be equal to:

\[ r(z) = r^* + \psi(z), \psi' > 0, \psi'' > 0. \]

We can also show that the results for the model in which there are costs to adjusting foreign bonds are qualitatively similar to those derived in the upward-sloping debt function approach.

\(^{26}\)As indicated in the introduction, world interest rate shocks will, however, lead to transitional dynamic responses for \( c \) and \( z \).
the small open economy under our specification of status-preference is more plausible, compared to these alternative approaches, because the burden of long-run adjustment falls on both consumption and national debt.

3.2. Fiscal Shocks

For expositional purposes, we next include a phase diagram, Figure 2b, that illustrates the adjustment of the economy subsequent to a permanent, balanced-budget increase in government expenditure. The adjustment of the economy in this case is, thus, the mirror image of that depicted in Figure 2a. In Figure 2b the \( \dot{z} = 0 \) locus shifts to the left. This results in a new intersection with the \( \dot{c} = 0 \) locus at point \( C' \), which involves a greater steady state stock of national debt and a lower steady state level of consumption. We infer from equation (20a) that the long-run fall in consumption exceeds the rise in government expenditure. This is due to the fact that agents also reduce asset holdings during the transitional phase. Combined with the increase in lump-sum taxation to support the higher level of \( g \), this results in proportionately fewer resources for consumption in the long-run. Equally, however, this result, which arises from the status-preferring behavior of agents, implies that the fall in initial consumption from point \( A' \) to point \( B' \) is less than the rise in government expenditure.\(^{27}\)

3.3. Government Debt Shocks

We turn now to an analysis of the effects of an increase in the inherited stock of government debt, \( b_0 \). This is a negative wealth effect from the perspective of the private sector. In this case, it is the \( \dot{c} = 0 \) locus that shifts to the right, while the \( \dot{z} = 0 \) locus remains in its original position. This results in a new steady state equilibrium at point \( E \) in Figure 2c, which is associated with a higher stock of national debt and a lower level of consumption. The fall in \( \dot{c} \) is a consequence of the increase in lump-sum taxes required to finance the

\(^{27}\)Using equation (22), it is clear that the increase in \( g \) causes a fall in \( r^e \). The effective rate of return subsequently approaches its long-run value of \( \rho \) from below.
higher stock of $b_0$ and a reduction in private sector asset income. From equations (13a, b), the expressions for the changes in $\tilde{c}$ and $\tilde{z}$ are given by:

\[
\frac{d\tilde{c}}{db_0} = -\frac{(1 - \delta)(\rho - r^*)r^*}{(1 - \delta)(\rho - r^*)} < 0 \tag{23a}
\]

\[
\frac{d\tilde{z}}{db_0} = \frac{(1 - \delta)(\rho - r^*)}{(1 - \delta)(\rho - r^*)} > 1. \tag{23b}
\]

Observe from (23b) that a rise in the initial stock of public debt increases the stock of national debt $\tilde{z}$ more than proportionately. Ultimately, this is because agents treat relative wealth as a normal good in the utility function. This is reflected in the fact that the private sector reduces its net asset position during the transition to the new steady state equilibrium. Using the identity $a \equiv b_0 - z$ and equation (23b), we can further show that fall in private sector asset holdings equals:

\[
\frac{d\tilde{a}}{db_0} = -\frac{\delta r^*}{(1 - \delta)(\rho - r^*)} < 0. \tag{23c}
\]

In addition, we can show, using the first equality of (17a), that initial private sector dissaving is sufficiently great so that $c(0)$ rises to a point such as $D$. The expression for the change in $c(0)$ is equal to

\[
\frac{dc(0)}{db_0} = \frac{d\tilde{c}}{db_0} + (r^* - \mu_1)\frac{d\tilde{z}}{db_0} = -\mu_1\frac{d\tilde{z}}{db_0} > 0 \tag{24}
\]

where we have used equations (23) to obtain the last equality of (24).\footnote{According to the solution for the effective rate of return in equation (19d), the increase in $b_0$ causes an initial decline in $r^e$. Thus, the direct negative effect of the rise in $b_0$ on $r^e$ is stronger than the positive effect of the increase in $c(0)$ on $r^e$.} This means that the new stable locus in Figure 2c lies to the right of its original position, implying that consumption is greater than $\tilde{c}_0$—its value before the increase in $b_0$—during the first phase of its adjustment toward the new steady state equilibrium at $E$. It is this initial dissaving on the part of the private sector that causes $\tilde{z}$ to increase by more than the rise in $b_0$.}
3.4. World Interest Shocks

Finally, we consider the dynamic impact of an increase in the world interest rate, \( r^* \). Unlike the long-run effects of the domestic shocks, a rise in the world interest rate has an ambiguous impact on the steady state values of consumption and national debt. Differentiating equations (13a, b) with respect to \( r^* \), we obtain the following expressions for the changes in \( \tilde{c} \) and \( \tilde{z} \):

\[
\frac{d\tilde{c}}{dr^*} = \frac{(1 - \delta)}{[(1 - \delta)\rho - r^*]^2} \left\{ \delta \rho (\bar{y} - g - r^*b_0) - (\rho - r^*) [(1 - \delta)\rho - r^*] b_0 \right\}
\] (25a)

\[
\frac{d\tilde{z}}{dr^*} = \frac{-\delta [\bar{y} - g - (1 - \delta)\rho b_0]}{[(1 - \delta)\rho - r^*]^2}.
\] (25b)

The restrictions we imposed earlier to guarantee \( \tilde{c} > 0, \tilde{a} > 0 \), are insufficient to sign the expressions (25a, b). Nevertheless, two cases can be identified that depend on the relative shifts in the \( \tilde{c} = 0 \) and \( \tilde{z} = 0 \) loci. It is straightforward to show that the rise in \( r^* \) causes both the \( \tilde{c} = 0 \) locus and the \( \tilde{z} = 0 \) to shift to the left.\(^{29}\) In the first case, depicted in Figures 2d, the horizontal shift in the \( \tilde{c} = 0 \) locus exceeds that of the \( \tilde{z} = 0 \) locus and results in a new equilibrium, illustrated at point \( G \), that corresponds to a higher value of \( \tilde{c} \) and a lower value of \( \tilde{z} \).\(^{30}\) In contrast, the horizontal shift in the \( \tilde{z} = 0 \) locus exceeds that of the \( \tilde{c} = 0 \) locus in Figure 2e, which implies that long-run consumption—along with national debt—falls. The new long-run equilibrium in this case is illustrated by point \( I \). In both instances consumption falls at \( t = 0 \) before rising along the new saddle paths.\(^{31}\) This is depicted in Figure 2d by the movement from point \( F \) to point \( G \) and in Figure 2e by the movement from point \( H \) to point \( I \). Intuitively, the difference between

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\(^{29}\)Evaluated in the steady state equilibrium, the horizontal shifts in the \( \tilde{c} = 0 \) and \( \tilde{z} = 0 \) loci are given by \(-b_0/\rho \) and \(-\tilde{z}/r^* \), respectively. We can show that the shift in the \( \tilde{c} = 0 \) locus exceeds (resp. is less than) the shift in the \( \tilde{z} = 0 \) locus as \( b_0 \) exceeds (resp. is less than) \( \rho \tilde{z}/r^* \).

\(^{30}\)This result would unambiguously obtain if, instead, the economy possessed the Uzawa-type preferences discussed above. This is due to the fact that since the rate of time preference depends positively on consumption, it must be the case that \( \tilde{c} \) increases if \( r^* \) rises. For the upward-sloping debt specification, we can show that while the response of \( \tilde{c} \) to the increase in \( r^* \) can either be positive or negative, the stock of national debt unambiguously declines.

\(^{31}\)Even though initial consumption falls, the direct effect of a higher world interest rate will increase the effective, domestic rate of return \( r^* \).
the two cases is that in the first case, Figure 2d, the accumulation of private sector assets and, thus, interest income, is sufficiently great to allow for a higher level of long-run consumption, while in the second case, Figure 2e, this effect is insufficiently strong. In general, the impact of higher asset income will be greater the larger is (fixed) output net of government expenditure, \((\bar{y} - g)\), and the smaller is the inherited stock of government debt, \(b_0\). Finally, we can identify the case, illustrated in Figure 2f, in which a rise in \(r^*\) leads to an *increase* in national debt as well as to a decline in consumption. As in Figure 2e, the horizontal shift in the \(\dot{z} = 0\) locus exceeds that of the \(\dot{c} = 0\) locus. The shift in the \(\dot{z} = 0\) locus is, however, sufficiently great in Figure 2f so that the new point of intersection now corresponds to point \(I'\), which involves both higher \(\bar{z}\) and lower \(\bar{c}\). In this case, agents decumulate assets along the saddlepath \(H'I'\), due to the relatively large increase in lump-sum taxes required to support the rise in public sector debt service. This is caused by the fact that the economy inherits in this case a relatively large stock of government debt, \(b_0\), compared to the level of (fixed) output net of government expenditure, \((\bar{y} - g)\). Using equation (25b), the condition that the rise in \(r^*\) increases \(\bar{z}\) to given by:

\[
b_0 > \frac{(\bar{y} - g)}{(1 - \delta)\rho}.
\]

4. Conclusions and Extensions

The goal of this paper is to study the implications for status, modelled as relative wealth, in a small open economy framework. In order to focus on the role of status-preference, we selected a simple one-good model in which output was fixed and agents could lend (and borrow) at the given world interest rate. We derived a symmetric dynamic equilibrium in which status-preference was the source of transitional dynamics. This is the case, because our specification results in an effective domestic rate of return that is endogenously deter-
mined outside of long-run equilibrium. Our approach thus offers an alternative method of generating transitional dynamics in a small open economy framework with perfect capital mobility. The influence of status was investigated in considering four macroeconomic shocks: income (endowment), government expenditure, initial public debt, and world interest rate shocks. The notable results we derived were that income and government expenditure shocks have a multiplier effect on the steady state level of consumption, while initial public debt shocks cause the long-run stock of national debt to respond more than proportionately. Such multiplier effects arise because agents with a preference for relative wealth respond to macroeconomic disturbances by adjusting both consumption and asset accumulation, which tends to magnify the long-run impact of these disturbances. In contrast, we showed that world interest rate disturbances have, in general, an ambiguous effect on the steady state values of consumption and national debt, although we were able to distinguish several cases.

Since the present structure of the model is quite simple, many extensions are possible. One obvious extension is to allow agents to accumulate physical capital as well as financial assets. Not only would this introduce investment and output dynamics into the small open economy framework, but would also imply that status-preference would depend on the relative holdings of physical capital as well as on relative financial wealth. The preference weights agents place on the two forms of relative wealth would then influence the overall structure of asset accumulation and growth in the open economy. The production structure can be further modified by permitting an endogenous employment decision. This would extend the work of Hof (1999), who has studied the issue of labor supply and status in a closed economy context. Another avenue in which our model can be extended is in its goods structure. In the present specification, agents consume a single traded good at a fixed unitary price. One possible modification is to assume that agents consume nontraded as well as traded goods and that status, for example, depends on the relative consumption of imported goods. This may be particularly relevant in studying the role of status in developing economies in which goods that generate status are imported from abroad. Finally, the issue of the optimal public policy response to status-preference
can be explored. The question would be to investigate whether status-preference leads to distortions in the decentralized equilibrium and then to calculate the optimal taxes or subsidies that would reproduce the first-best, or social, optimum.

5. Appendix

We state here the first and second partial derivatives of the constant elasticity utility function, equation (11), used to derive the parameterized solution of the model:

\[ U_c(c, s) = (1 - \delta)c^{-\alpha(1-\delta)}s^{\delta(1-\alpha)} > 0, \quad U_s(c, 1) = \delta c^{(1-\delta)(1-\alpha)}s^{\delta(1-\alpha)-1} > 0 \]

\[ U_{cc}(c, s) = -(1 - \delta) [\delta + \alpha(1 - \delta)] c^{-\alpha(1-\delta)-(1+\delta)}s^{\delta(1-\alpha)} < 0 \]

\[ U_{ss}(c, s) = -\delta [1 - \delta(1 - \alpha)] c^{(1-\alpha)(1-\delta)}s^{\delta(1-\alpha)-2} < 0 \]

\[ U_{sc}(c, s) = \delta(1 - \delta)(1 - \alpha)c^{-\alpha(1-\delta) - \delta}s^{\delta(1-\alpha)-1}. \quad (A1) \]

Using the expressions for \( U_{cc}, U_{ss}, \) and \( U_{sc}, \) we show that strict concavity, \( U_{cc}U_{ss} - U_{sc}^2 > 0, \) holds. With the expressions for \( U_c, U_{cc}, \) and \( U_s \) evaluated in the symmetric equilibrium in which \( s = 1, \) we calculate the relationships in (12b) describing the effective intertemporal elasticity of substitution, \( \sigma^e, \) and the effective rate of return, \( r^e. \)

References


Figure 1: Phase Diagram

Figure 2a: Increase in (Fixed) Income
Figure 2b: Increase in Government Expenditure

Figure 2c: Increase in Initial Public Debt
Figure 2d: Increase in the World Rate

Figure 2e: Increase in the World Rate
Figure 2f: Increase in the World Rate

\[ \dot{c}' = 0 \]

\[ \dot{c} = 0 \]

\[ H' \]

\[ z' = 0 \]

\[ z = 0 \]

\[ z_0 \]

\[ z_1 \]