Corruption within a Cooperative Society

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Reihe Ökonomie / Economics Series No. 48

July 1997

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Abstract

In this paper we take up a model of Okada (1996) to describe the possibility of collective cooperation in a $n$-person Prisoners' Dilemma game by means of institutional arrangements. In addition, we introduce the possibility to corrupt the institutional authority by paying him some positive transfer in order not to be punished in case of defection. It is shown that there exists a maximal number of corrupting and defecting agents such that the organization is still formed and the rest of the population cooperates.

Keywords
Corruption, cooperation, Prisoners' Dilemma, organization

JEL-Classifications
C72
Comments

I would like to thank Akira Okada and Arno Riedl for helpful comments on an earlier version of this paper.
1 Introduction

“All power tends to corrupt and absolute power corrupts absolutely.” — Lord Acton¹

Corruption can be observed in different kinds of organizations all over the world. It is one of the major social problems and its actuality hasn’t decreased since the very beginning of human society. A lot of so-called “developing countries”² suffer from corruption but industrial countries also do. However, the dimension of social effects resulting from corruption is much greater in poor countries, than it is in rich. Consequently, in recent years the literature in developing economics has started to point out the major role of this topic. Following these lines, it is not only important to establish a somehow democratic state — a project that itself is often difficult enough if we look at countries like Somalia, Nigeria or Rwanda. It is furthermore necessary to take care of the well-functioning of this state in order to successfully coordinate social and economic activities on the national and international level.

See e.g. Leipold (1994) for a discussion of these problems of organization especially in Black Africa. In fact, the World Bank has already precised the idea of well-functioning of the state by defining the term “governance” as “the manner in which power is exercised in the management of a country’s economic and social resources for development.”³ For a deeper discussion of this subject see also the article of Leisinger (1995).

The purpose of this paper is not to give an answer to the question of what to do against corruption. This will be a practical problem and I think there isn’t even a general answer to this question, since there are a lot of different country-specific motivations that may be the reason why corruption is actually observed. Nevertheless, it can also be argued that there must be some theoretical explanation why corruption is so persistent and why corruptive elements in organizations can

¹Quoted from Friedrich (1972), p.128
²I will use this term for the rest of this paper, although I think that it is quite misleading, since in the deep sense every country is a developing country.
³World Bank (1994)
often be extremely stable. It is this theoretical question that I would like to address in this paper.

In order to do so I choose a game theoretic approach. In fact, the model I use is very similar to the one in Okada (1996). There, Okada presents a model first introduced by Selten (1973) to describe the possibility of collective cooperation in an $n$-person Prisoners' Dilemma game by means of institutional arrangements. Okada shows that even under the assumption that agents do not consider any collective interests but, instead, simply maximize their individual payoffs the society may be able to establish a form of social organization in order to guarantee cooperation. The mechanism is the following: All members of the organization together engage an extern agent (officer) who can enforce cooperation, at least in the organization, by means of punishing every member that defects. The obtained results demonstrate the important role of institutions in connection with the successful coordination of individual and collective interests within a society.

Okada's approach is special in the sense that he considers the members of a society as "social beings" who might be able "to change and reconstruct the rules of their game, if necessary." I think this point of view is extremely realistic, especially if we consider agents who, as members of an organization, see themselves under the rule of a higher authority. In this respect, this paper is, first, an extension of the situation in Okada, by allowing the agents to change even more the rules of their game, in the sense that, after the organization has been formed, every agent can try to corrupt the enforcement agent. Second, this can also be seen as a first attempt to consider the enforcement agent not just as an extern agent but as a strategic player. However, at least for the moment, I am going to assume the strategic considerations of the officer to be very simple, since I am interested in the emergence of corruption from the point of view of the agents in the society. Finally, as the arguments in this model are actually strongly correlated to those of Okada, this paper is also an application of Okada's noncooperative approach in modelling the collective and social effects of individual behavior. Unfortunately, not in the same positive direction.

The paper is organized as follows. In the next section I give a short review of the model in Okada (1996), including some of the main results concerning social

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4 Okada (1996), p.2
cooperation. The introduced notation is also the basis for the model of corrupting agents, that is presented in section 3. Finally, the relevance of the obtained results and also several possibilities to extend the assumptions that are made in this model are discussed.

2 Okadas Model of Social Cooperation

In the model of Okada (1996) the society of agents is initially described by an anarchic state of nature, in which every agent faces an $n$-person Prisoners’ Dilemma game (PD). Let $N = \{1, \ldots, n\}$ be the set of agents where each of them has two possible actions: $C$ (cooperation) and $D$ (defection). Payoffs are given by a function

$$f_i(a_i, h), \quad i \in N, \ a_i \in \{C, D\},$$

$$h = 0, \ldots, n - 1,$$

where $a_i$ is agent $i$'s action and $h$ is the number of other agents cooperating.

I assume here the agents to be homogeneous,\(^5\) therefore $f_i \equiv f$, for all $i \in N$ and, since we are in the situation of a PD, the function $f$ fulfils:

$$f(D, h) > f(C, h), \text{ for all } h = 1, \ldots, n - 1,$$

$$f(C, n - 1) > f(D, 0),$$

$$f(C, h) \text{ and } f(D, h) \text{ are increasing in } h.$$

In order to introduce the above described mechanism to enforce collective cooperation, Okada defines an organization to be given by a quadruple $\varphi = (S, p, i^*, C)$, where $S \subseteq N$ is the set of agents in the organization, $p$ is the amount by which a defecting members payoff is reduced, $i^*$ is the externally engaged officer and $C$ is the cost function depending on the number of members of the organization. Costs include e.g. the wage $w$ paid to the officer.

The model of organization formation is then described by a three stage game. First, all agents decide whether to participate or not to participate in the organization. Second, members of the organization bargain over the actual punishment level

\(^5\)Okada also studies the case of heterogeneous agents.
and the cost allocation. If no agreement is reached, the organization is not formed. Third, all agents in the society play the PD.

Similarly to Okada, for the following I assume the punishment level \( p \) to take only two possible values: 0 and \( p^* \), where \( p^* \) is the efficient punishment level, that is sufficiently high to ensure that for every member of the organization cooperation payoff-dominates defection. Agreement on the punishment level has to be reached by unanimity. Costs of the organization are assumed to be exogenously given and equally distributed among members. I additionally assume that average costs \( \frac{C(s)}{s} \) are decreasing in \( s \).\(^6\)

Now, the crucial point in Okada is that there exists a minimum size of the organization \( s^* \), such that

\[
f(C, s^* - 2) - \frac{C(s^* - 2)}{s^* - 2} < f(D, 0) < f(C, s^* - 1) - \frac{C(s^* - 1)}{s^* - 1},
\]

meaning that (i) cooperation in the organization of size \( s^* \) gives a strictly higher payoff than the noncooperative equilibrium point of the PD, and (ii) this is no longer fulfilled if only one member leaves the organization.

Thus, the success of the organization depends only on its size. If less than \( s^* \) agents participate in the first stage, no agreement is reached in the second stage and, finally, everybody in the society will defect. If the number of participants is greater or equal than \( s^* \), then in the second stage agreement on the efficient punishment level \( p^* \) will be reached. In this case, every member of the organization will cooperate in the third stage, while every agent outside the organization will defect.

Consequently, this leads to the following important result.\(^7\)

**Theorem 2.1** Any action combination in the participation stage is a (strict) equilibrium point iff exactly \( s^* \) agents participate in the organization.

The theorem implies that only a maximal number of agents can be expected to join the organization. This number is equal to the minimal number of agents such that the organization is successful at all. Therefore, if the organization is formed, the society can be described by two distinct groups of agents, those who join the

\(^6\)This fits also into the model of Okada, although it is not explicitly assumed.

\(^7\)Okada (1996), p.8
organization and contribute to the society by cooperating, and those who don’t join the organization and “free-ride”.

If $s^* = n$, full participation in the organization can be supported by a pure strategy equilibrium. Moreover, a mixed strategy equilibrium does not exist. If $s^* < n$, there is a unique symmetric mixed equilibrium and Okada gives also a precise formula to calculate every agent’s probability to participate in the organization. In addition it turns out, that if one assumes some parameters in this formula (e.g. $s^*$) to be constant with respect to the population size $n$, the probability to join the organization decreases and converges to zero as $n$ goes to infinity.

Let us now turn to the model of corrupting agents.

### 3 The Model of Corrupting Agents

In order to make the effects of corruption very clear, I start within the set-up of Okadas model and assume that $s^* = n$, the minimum size of the organization is equal to the population size.\(^8\) I assume furthermore the society to be already in the strict equilibrium point of total cooperation. Thus, the organization has been formed by the whole society and every agent receives the payoff from cooperation

$$f(C, n-1) - \frac{C(n)}{n} > f(D, 0),$$

while the officer $i^*$ gets the (exogenously given) wage $w$, ($w \leq C(n)$).

The next definition shows what is meant in this paper by the term corruption.

**Definition 3.1** *Agent $i$ is said to corrupt the officer $i^*$ if he pays him some positive transfer $T_i$ to ensure that he is not going to be punished in case of defection.*

I assume that the officer does not have any moral scruples to accept a corruptive offer of some agent $i$ in the society. Since he receives a final payoff $w + T_i > w$, he has a strictly positive incentive to accept it. I assume also that he is able to achieve

\(^8\)This is not that crucial as it perhaps seems to be. It only simplifies the following analysis without having any qualitative effects on the results.
his part of the contract, i.e. he can always prevent an agent from being punished. There is no higher authority that takes control over the officers actions. In this respect the officer has the absolute power in the society and thus the officer can be seen in the light of Lord Actons quotation at the very beginning of this paper.

Every agent should be willing to pay any transfer $T_i$, such that

$$0 < T_i < f(D, h) - f(C, h),$$

where $h$ is the number of agents he expects to cooperate in the society. By assumption the difference on the RHS is strictly positive for every $h$. Hence there is always some amount $T_i$ such that the profit from corruption and defection can be positive. In fact, the actual amount $T_i$ that agent $i$ is willing to pay could be highly influential on the success of the corruption. One could think of the officer to accept the bribe if it is positive, but to prevent the agent always from being punished only if the bribe is sufficiently high. If it was too low, then with some positive probability agent $i$ might be detected and punished. Thus agent $i$ had to face an uncertain payoff of corruption. However, to keep things simple I assume that any positive bribe will work to successfully corrupt the officer and that for every agent $T_i$ is exogenously given by $T_i \equiv T > 0$.

It is in the nature of corruption that it can not be observed by anyone else in the society except — obviously — by the officer and the corrupting agent $i$ themselves. Therefore, punishment should be thought of more as a private procedure rather than a public one. For example monetary punishments are private procedures with this respect. Although other members of the society do notice that there is somebody corrupting and defecting, they are not able to make sure that the respective agent is actually being punished. This is under total control of the officer.

Analogously to Okada, the game theoretic set up in this model is also given by a three stage game.

1. **The corruption decision stage:**
   At the beginning every agent has to decide whether to corrupt the officer, or not.

2. **The renegotiation stage:**
   In the second stage all agents renegotiate if they still want to join the organiz-
ation. I assume this renegotiation to be very simple, in the sense that agents unanimously have to agree on further participating in the organization. Otherwise the organization breaks down. This assumption is motivated by the minimality of the existing organization.

3. The action decision stage:

In the last stage every agent has to decide what strategy to play in the PD, either cooperation ($C$) or defection ($D$).

In order to analyze possible equilibria of this game I make use of the backward induction approach. Let $\mathcal{K}$ denote the set of agents who chose corruption in the first stage.

In the last stage agents will choose their actions depending on whether they reached an agreement about the organization in stage two, or not. If unanimous agreement is reached, the punishment mechanism is an effective incentive to cooperate exactly for those agents who did not corrupt in the first stage. Every agent who did corrupt, however, will defect without being punished. Therefore, in this case payoffs are as follows:

$$\pi_i = \begin{cases} 
  f(C, n-k-1) - \frac{C(n)}{n}, & i \notin \mathcal{K} \\
  f(D, n-k) - \frac{C(n)}{n} - T, & i \in \mathcal{K},
\end{cases}$$

where $k = |\mathcal{K}|$ is the number of agents that corrupted in the first stage. Notice that every agent $i \in \mathcal{K}$ also pays his part of the organization costs. This is certainly a necessary condition since otherwise by minimality the organization could not stay formed.

If no agreement is reached in the second stage, then because of no existing punishment mechanism, there is no incentive to cooperate anymore. Every agent has the strictly dominant strategy to defect. Thus in this case agent $i$'s payoff is the following:

$$\pi_i = \begin{cases} 
  f(D, 0), & i \notin \mathcal{K} \\
  f(D, 0) - T, & i \in \mathcal{K}.
\end{cases}$$

Concerning the renegotiation stage the society may be characterized by two distinct interest groups. Those agents who did corrupt in the first stage will want
to join the organization, in order to receive the high payoff of defection against a large number of cooperators. Agents who did not corrupt might also want to participate, because by non-corrupting they already revealed their positive attitude towards social cooperation. However, they might also know that every corrupting agent will certainly defect if the organization stays formed, thereby reducing every cooperators payoff. In consequence, they will participate if and only if the number of agents who corrupted in the first stage is not too large. This is captured by the following proposition.

**Proposition 3.2** There is a maximal number of corrupting agents \( k^* \), such that the following holds. As long as no more than \( k^* \) agents corrupt, every non-corrupting agent is strictly better off by participating in the organization and cooperating. If more than \( k^* \) agents corrupt, every non-corrupting agent has a positive incentive to leave the organization.

**Proof:** Formally, the proposition says that \( \exists k^* \geq 0 \):

\[
\forall k \leq k^* : \quad f(C, n - k - 1) > f(D, 0) + \frac{C(n)}{n},
\]

\[
\forall k > k^* : \quad f(C, n - k - 1) < f(D, 0) + \frac{C(n)}{n}.
\]

Clearly, for every \( h \leq n \)

\[
f(D, 0) < f(D, 0) + \frac{C(h)}{h}. \tag{1}
\]

The RHS is decreasing in \( h \) and attains its minimum at \( h = n \). By definition of the PD,

\[
f(C, 0) < f(D, 0) \text{ and } f(C, h) \text{ is increasing in } h. \tag{2}
\]

Finally, since \( s^* = n \), we have that

\[
f(C, n - 1) > f(D, 0) + \frac{C(n)}{n}. \tag{3}
\]

Now the proposition follows from (1), (2) and (3). \( \square \)

As an immediate result we obtain the following
Corollary 3.3 In the renegotiation stage unanimous agreement to join the organization is reached iff \( k \leq k^* \).

Proof: Follows directly from Proposition 3.2. \( \square \)

In the first stage every agent has to decide whether to corrupt, or not. Let \( a = (a_1, \ldots, a_n) \), with \( a_i \in \{ \text{corrupt, don't corrupt} \} \), denote the decision vector of the society in the corruption decision stage. Then \( \mathcal{K} = \{ i \in N \mid a_i = \text{corrupt} \} \), \( k = |\mathcal{K}| \) and payoffs are as follows:

Case 1: \( k \leq k^* \)

\[
\pi_i = \begin{cases} 
  f(C, n - k - 1) - \frac{c(n)}{n}, & i \notin \mathcal{K} \\
  f(D, n - k) - \frac{c(n)}{n} - T, & i \in \mathcal{K}.
\end{cases}
\]

Case 2: \( k > k^* \)

\[
\pi_i = \begin{cases} 
  f(D, 0), & i \notin \mathcal{K} \\
  f(D, 0) - T, & i \in \mathcal{K}.
\end{cases}
\]

The next proposition already states the main result in this model.

Proposition 3.4 In the corruption decision stage any decision vector \( a \) is an equilibrium iff exactly \( k^* \) agents corrupt.

Proof: We have to prove two directions. Assume that \( k = k^* \). Then, every corrupting agent would be worse off by not corrupting, since he would miss the positive payoff difference \( f(D, n - k^*) - T - f(C, n - k^* - 1) \). On the other side, every non-corrupting agent, who decided to corrupt, would in effect lead to a disagreement in the second stage. In consequence, this would reduce his payoff to the worst case, where he receives only \( f(D, 0) - T \). To prove the other direction, assume first that \( k < k^* \). In this case it follows from Proposition 3.2 that every non-corrupting agent could be better off by corrupting the officer. If \( k > k^* \), every corrupting agent could always be better off if he did not corrupt. Either \( k - 1 > k^* \), then he would still save the bribe \( T \), or \( k - 1 = k^* \), then he could even get more by cooperating in the third stage. \( \square \)
The proposition shows that we can always expect a minimal number of agents in the society to corrupt. Similarly to the result of Okada, but in the opposite direction, this minimal number is equal to the maximal number $k^*$ of corrupting agents such that the organization is still successful.

Certainly, $k^*$ can be equal to zero. This would mean that the only equilibrium is just the decision vector where everybody in the society does not corrupt. In this situation the initial plan of social contract may be expected to be stable. However, if $k^* > 0$, everybody has a positive incentive to corrupt, which finally always leads to a collapse of the idea of social contract. Either the organization breaks down totally and everybody finds himself back in the old time stage of anarchy, where everybody defects again everybody. Or the organization is still joined by all members of the society, but some members successfully undermine the organization and are strictly better off at the expense of everybody else. Unfortunately, I think it is the latter what we observe in many examples of social organization.

If $k^* > 0$ the above results say nothing about who in the society successfully corrupts the officer and who does not. All agents are assumed to be equal and therefore everybody has the same incentive on the one hand to corrupt, on the other hand to hold to the idea of social contract. An alternative approach is to look at the mixed strategy equilibrium. There we see that everybody in the society corrupts with some probability that is greater than zero.

**Proposition 3.5** In the corruption decision stage there exists a unique mixed strategy equilibrium where every agent in the society corrupts with some probability $p > 0$. The solution for $p$ is given by the following formula:

$$
\sum_{k=0}^{k^*-1} \frac{P(k)}{P(k^*)} \frac{f(D, n-k) - f(C, n-k-1)}{f(C, n-k^*-1) - \frac{C(n)}{n} - (f(D, 0) - \frac{T}{p})} = 1,
$$

where $P(k)$ is the probability that $k$ other agents corrupt, given by $P(k) = \binom{n-1}{k} p^k (1-p)^{n-1-k}$.

**Proof:** The solution for $p$ is obtained by calculating the mixed strategy equilibrium condition: for every agent $i$ the expected payoffs for both his pure strategies corrupt
and don't corrupt have to be equal. Formally, this condition looks as follows:

\[
\sum_{k=0}^{k^*-1} P(k) [f(D, n-k) - \frac{C(n)}{n} - T] + \sum_{k=k^*}^{n-1} P(k) [f(D, 0) - T]
\]

\[
= \sum_{k=0}^{k^*} P(k) [f(C, n-k-1) - \frac{C(n)}{n}] + \sum_{k=k^*+1}^{n-1} P(k) f(D, 0),
\]

where \(P(k)\) is as defined in the proposition. Rearranging things yields

\[
\sum_{k=0}^{k^*-1} P(k) [f(D, n-k) - \frac{C(n)}{n}] + P(k^*) f(D, 0) - T
\]

\[
= \sum_{k=0}^{k^*} P(k) [f(C, n-k-1) - \frac{C(n)}{n}],
\]

which is equivalent to

\[
\sum_{k=0}^{k^*-1} P(k) [f(D, n-k) - f(C, n-k-1)]
\]

\[
= P(k^*) [f(C, n-k^*-1) - \frac{C(n)}{n} - f(D, 0) + \frac{T}{P(k^*)}].
\]

From this equation, the formula given in the proposition can now easily be derived. To see that there exists one unique solution for \(p\), first write the formula explicitly as

\[
\sum_{k=0}^{k^*-1} \frac{k^*(k^*-1) \cdots (k+1)}{(n-k-1) \cdots (n-k^*)} \left(\frac{1-p}{p}\right)^{k^*-k}
\]

\[
\frac{f(D, n-k) - f(C, n-k-1)}{f(C, n-k^*-1) - \frac{C(n)}{n} - (f(D, 0) - \frac{T}{\binom{n}{k^*} p^{k^*}(1-p)^{n-k^*-1}})} = 1.
\]

Let \(g(p)\) denote the LHS as a real function of \(p\), for \(p \in (0, 1)\). Since \(\lim_{p \to 0} g(p) = +\infty\) and \(\lim_{p \to 1} g(p) = 0\) and \(g\) is strictly monotonically decreasing, there exists a unique solution such that \(g(p) = 1\).

Notice that the second ratio in the proposition may be similarly interpreted as the incentive ratio in Okada.\(^9\) While the numerator means the incentive to corrupt

\(^9\)Okada (1996), p.9
when \( k \) others already do, the denominator represents the incentive not to corrupt but to cooperate when overall corruption in the organization is at its critical point \( k^* \).

4 Discussion

Starting with a society, described as an \( n \)-person Prisoners' Dilemma game with the additional structure of an institutional arrangement, the results in this paper give a theoretical explanation for how corruption can emerge within this society and, in addition, why it can be extremely stable in the following sense: (i) all members of the organization have an incentive to corrupt. (ii) even in the presence of corruption, members who don't corrupt still hold to the idea of social cooperation and contribute to the wealth of the whole society. The extent of corruption, measured by the value of \( k^* \), depends on the characteristics of the society, that are given by the payoff function of the underlying Prisoners' Dilemma game and the cost function of the organization.

The analysis suggests that even if a society has already overcome the anarchic state of nature by successfully organizing social cooperation, there is still the hazard that some members try to undermine this organization at the expense of everybody else. Clearly, the institutional authority, the officer, plays an important role in this context. If he always refused any form of corruption, things would certainly look different. However, since in general the officer should be considered to be even more strategically thinking than in this model, the results indicate at any rate the strong need for control mechanisms within the organization. To speak in the language of Arrow, we have to “take care of the caretakers”!

There are several possibilities to extend the assumptions that are made in this model. Let me mention three. First, one could introduce the variation that corruption is no longer always successful, either because the officer may refuse the bribe, or because there is a positive probability that the defection is detected. The value of this probability could, e.g., depend on the actual amount the agent paid for the bribe. This might change the considerations of an agent who is willing to corrupt.
Another interesting subject would be the competitive dynamics between two or more officers who might be able to control each other. In this context I think also of the general competition between whole organizations, all of them being able to provide the same public good. (In our case the benefits from social cooperation.)

Finally, the most important point is to look at the effects of corruption not in a static but in a dynamic model. One example how this could be done is already given by Feichtinger and Wirl (1994), although they analyze corruption only from the officers point of view. Other possibilities may come from evolutionary game theory.

In order to conclude this paper, let me say that, if we look at the world, either in so-called “developing” or in industrial countries, the problems of corruption (sometimes) within a cooperative society ask for a strong effort both in research and in practice. Certainly, there are a lot of more issues we have to take into consideration than those mentioned in this paper. However, the ideas presented in this work might help at least as a simple benchmark for a deeper discussion of this subject and those that are related. That this discussion is needed, is something of which I am firmly convinced.
References


