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Abstract

Using data from the Vienna Stock Exchange we investigate three different types of consumption based capital asset pricing models: the well known two state model of Mehra and Prescott, the model of Rietz, which includes also a crash state, and an own four state model. The aim of this additional state is to take the super normal returns on the Vienna Stock Exchange during the 1980s into account. For all the models we calculate the risk premium in order to see whether the models could explain the empirically observed risk premium. For the calculation of risk premia we use estimators generated by the General Method of Moments.

Keywords
Consumption based capital pricing models, GMM, equity premium puzzle

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Comments
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1 Introduction

In this paper we estimate risk premia of different types of Consumption Based Capital Asset Pricing Models (C-CAPM) using Austrian data. The Mehra-Prescott [10] and the Rietz model [14] are analyzed. The model Mehra and Prescott used did fail to explain the risk premia generated by market activities. They argued that frictionless models could hardly depict reality. Rietz extended their model in that he included a third state in the Markov chain. We extend this model further arguing that the asymmetric setup is logically not satisfying and calculate risk premia for a C-CAPM that includes also a fourth, a gold rush, state.

This paper was triggered by two reasons: first, we wanted to apply standard versions of C-CAPM to Austrian data and compare the results to the experience with the US data. Especially we were interested if there exists an equity premium puzzle in Austria, too. The equity premium puzzle was first described by Mehra and Prescott [10] who noted that the return on risky assets (equity) exceeded the return on risk free debt. In their model they could not explain the difference between these two assets, the risk premium – for the U. S. they calculated some 6 % and their model generated only 0.35 %. The second reason were the supernormal profits that the Vienna Stock Exchange generated during the 1980s, which can easily be seen in figure 3. We argue that such supernormal profits make it necessary to extend the standard models to account for the probability of such profits.

Our finding is that although the data set is different to that the above mentioned authors used (both in length and assets) we reproduce the same patterns as they did. For the simple two state model of Mehra and Prescott we found an equity premium puzzle for Austria. Additionally we could "solve" this puzzle by the introduction of the crash state but the criticism of Mehra and Prescott [11] of the Rietz model remains valid. Namely, that the introduction of the third, crash state does not "explain" the puzzle, the high risk aversion parameter seems to be more important.

For the estimation of the free parameters of the models we use the Generalized Method of Moments (GMM). The basic idea behind the GMM approach is the same as with the usual calibration approach - one estimates the free parameters of a theoretical model by matching conditions on the empirical moments. These conditions or restrictions on the moments are derived from the theoretical model. In contrast to the calibration approach GMM takes account of the uncertainty that is caused by estimates of moment conditions and provides us with test statistics.

The paper is organized as follows. In section 2 we first describe the general C-CAPM. We restrict the model to two assets: a risky and a riskless. We derive the moment conditions that are implied by the C-CAPM and are
needed to apply the General Methods of Moments to calculate estimators for the parameters of the assumed economies. Section 3 briefly introduces GMM. The basic idea of the GMM is presented and we show how we can use the general framework of GMM to estimate the parameters of our different specifications of the C-CAPM. Finally, a few practical problems with the application of GMM are discussed. In section 4 we describe the data sets and how we customized them to be used in our calculations. In section 5 we describe the Mehra/Prescott setup, calculate risk premia with the estimated parameters. The same calculations are carried out for the Rietz setup in section 6 and for the gold rush economy in section 7. For all these set-ups we compare the risk premia we obtain via the GMM. In the last section we state our conclusions.

2 Consumption Based Capital Asset Pricing

In this section a brief introduction to the so called Consumption Based Capital Asset Pricing Model (C-CAPM, e.g. Lucas [9], is given. In contrast to models of asset pricing, which are used in finance – Arbitrage Pricing Theory [15], Capital Asset Pricing Model, and Mean Variance Analysis – this economic model starts with an explicit formulation of an agent's utility-maximization problem under a budget constraint which reflects the trade-off between consumption and investment in assets. The utility function is assumed to be time-separable and thus the maximization problem is given by:

$$E_0[\sum_{t=0}^{\infty} \beta^t U(c_t)] \to \max$$

(1)

s.t.

$$c_t + \sum_{j=1}^{N} p_{jt} q_{jt} \leq \sum_{j=1}^{N} (p_{jt} + y_{jt}) q_{jt-1}$$

(2)

where $E_0[\cdot]$ denotes the conditional expectation operator at time $t = 0$, $U(\cdot)$ is the utility function, which is assumed to be a strictly concave function.

We have $N$ assets, $p_{jt}$ denotes the price, $q_{jt}$ the quantity, $y_{jt}$ the dividend of asset $j$ at time $t$, $c_t$ is the consumption at time $t$, and $\beta$ is a constant discount factor of utility. For the utility function we use the common CRRA-type utility function, which is given by (3) where $\gamma$ is the constant relative

\footnote{A good introduction into different models of asset pricing and their connections can be found in [5].}
risk aversion parameter – the reciprocal value of the inter-temporal rate of substitution.

\[
U(c_t) = \begin{cases} 
\frac{c_t^{1-\gamma}-1}{1-\gamma} & \text{if } \gamma \neq 1 \\
\ln c_t & \text{if } \gamma = 1
\end{cases}
\] (3)

The above constrained dynamic maximization problem can be solved by setting up the Bellman equation, yielding the set of \( N \) first order conditions:

\[
E_t\left[m_{t+1}R_{jt+1}\right] = 1 \quad j = 1, \ldots, N
\] (4)

where

\[
m_{t+1} = \beta \left(\frac{c_{t+1}}{c_t}\right)^{-\gamma}
\] (5)

is the time varying stochastic discount factor of return and

\[
R_{jt+1} = \frac{p_{jt+1} + y_{jt+1}}{p_t}
\] (6)

is the gross return of asset \( j \) at time \( t = t+1 \). Equation (4) can be formulated for virtually any asset pricing model where the differences lie in the particular expressions for \( m_{t+1} \) [5].

In the case of two assets, one risk free and the other risky we have:

\[
\begin{align*}
E_t\left[m_{t+1}R_{t+1}^e\right] &= 1 \\
E_t\left[m_{t+1}R_{t+1}^f\right] &= 1
\end{align*}
\] (7)

where the superscripts \( e \) and \( f \) stand for equity (risky) and risk free. Next define the excess return as the difference between return on risky and on risk free assets: \( r_t^e = R_t^e - R_t^f \). The expected excess return - the risk premium - is thus given by \( E_t\left[r_{t+1}^p\right] = E_t\left[R_{t+1}^e\right] - R_{t+1}^f \). Then equations (7) can be summarized as:

\[
E_t\left[m_{t+1}r_{t+1}^p\right] = 0.
\] (8)

By using equations (6) and (7) and by observing that there are no dividends paid for the risk free asset and that the price of the risk free asset is known to be one at time \( t+1 \) we get the following expressions for prices:

\[
\begin{align*}
p_t^e &= E_t\left[m_{t+1}(p_{t+1}^e + y_{t+1}^e)\right] \\
p_t^f &= E_t\left[m_{t+1}\right],
\end{align*}
\] (9)
Now it will be assumed that the growth rate of the production level $y_t$ follows an ergodic $n$-state Markov process. In contrast to this approach, which follows [10], in the original Lucas economy [9] it was the level which was assumed to follow a Markov process. In equilibrium we have:

$$y_{t+1} = x_{t+1}y_t = c_{t+1}$$

(10)

where $x_{t+1} \in \{\lambda_1, \ldots, \lambda_n\}$ is the growth rate of production or consumption respectively and $\{\lambda_1, \ldots, \lambda_n\}$ is the set of states of the Markov Process. In the following the $(n \times n)$ matrix of transition probabilities, whose elements are given by $\phi_{ij} = P(x_{t+1} = \lambda_j \mid x_t = \lambda_i)$ will be denoted by $\Phi$ and the $(n \times 1)$ vector of stable probabilities $\Pi$, where $\pi_i = P(x_{t+1} = \lambda_i)$.

In equilibrium prices depend only on the current state $(y_t, x_t)$ and therefore they may be expressed as a function of $(y_t, x_t)$. Hence using equation (5) in equilibrium equations (9) can be rewritten as:

$$\begin{align*}
p^e_t(y_t, x_t) &= E_t \left[ \beta \left( \frac{y_{t+1}}{y_t} \right)^{-\gamma} (p^e_t(y_{t+1}, x_{t+1}) + y_{t+1}) \right] \\
p^f_t(y_t, x_t) &= E_t \left[ \beta \left( \frac{y_{t+1}}{y_t} \right)^{-\gamma} \right].
\end{align*}$$

(11)

Furthermore in a Markov process forecasts depend only on $y_t$ and $x_t$ and hence one can drop the time index and re-define the state as $(c, i)$ if $y_t = c$ and $x_t = i$. Now rewrite (11):

$$\begin{align*}
p^e(c, i) &= \beta \sum_{j=1}^n \phi_{ij} \left( \frac{\lambda_j c}{c} \right)^{-\gamma} (p^e(\lambda_j c, j) + \lambda_j c) \\
p^f(c, i) &= \beta \sum_{j=1}^n \phi_{ij} \left( \frac{\lambda_j c}{c} \right)^{-\gamma}
\end{align*}$$

(12)

which yields:

$$\begin{align*}
p^e(c, i) &= w_i c \\
p^f(c, i) &= \beta \sum_{j=1}^n \phi_{ij} \lambda_j^{-\gamma}
\end{align*}$$

(13)

where

$$w_i = \beta \sum_{j=1}^n \phi_{ij} \lambda_j^{1-\gamma} (w_j + 1).$$

(14)

This can be rewritten in matrix notation as

$$\begin{align*}w &= A(w + e) \\
&= (I - A)^{-1} Ae,
\end{align*}$$

(15)
where \( w \) is the \((n \times 1)\) vector formed by the elements \( w_i \) given by equation (14), 
\( e \) is a vector of ones, \( I \) is the unity matrix, and \( A \) is a \((n \times n)\) matrix whose elements are given by \( a_{ij} = \beta \phi_{ij} \lambda^{1-\gamma} \). Note that the stability of the matrix \( A \) is a necessary and sufficient condition for the existence of the expected utility given in the maximization problem (1) [14].

Note that prices (eq. 13) are homogeneous of degree one in output; a necessary condition as a result of the specification of the utility function, which is assumed to have a constant relative risk aversion parameter.

The expectations of the future returns, given that the current state is \((c, i)\), for all future states \((\lambda_j, c, j)\) can be calculated as follows:

\[
\begin{align*}
E_i[R^c_j] &= \frac{e^{\gamma (\lambda_j, c, j) + \gamma}}{p^j(c, i)} = \frac{\lambda_j(w_j+1)}{w_i} \\
E_i[R^f_j] &= \frac{1}{p^j(c, i)}.
\end{align*}
\]

(16)

The conditional expectations of the future returns, independent of the future state are:

\[
\begin{align*}
E_i[R^c] &= \sum_{j=1}^{n} \phi_{ij} E_i[R^c_j] \\
E_i[R^f] &= \frac{1}{p^j(c, i)}.
\end{align*}
\]

(17)

Therefore the unconditional expectations of the future returns are:

\[
\begin{align*}
E[R^c] &= \sum_{i=1}^{n} \pi_i E_i[R^c] \\
E[R^f] &= \sum_{i=1}^{n} \pi_i E_i[R^f].
\end{align*}
\]

(18)

From the above equations the risk premium can easily be calculated as the difference between expected return on risky and on risk free assets.

3 The Generalized Method of Moments

The model described in the previous section contains two parameters, which describe preferences — the discount factor \( \beta \) and the relative risk aversion parameter \( \gamma \). Additionally some technology defining parameters will be introduced by specifying the possible realizations of the Markov Process (see sections (5) to (7) for the different specifications of the Markov process and the corresponding parameters). We will estimate some of the parameters — for given values of the remaining ones — by the Generalized Method of Moments (GMM, [7]). In a calibration approach one sets the parameters such that the moments which are implied by the economic model are equal to the sample moments. This ignores a source of uncertainty which arises from the fact that the sample moments are estimators for the population moments [3].
The basic idea behind the GMM is similar to the calibration approach: Derive restrictions on the moments of some observable random variables from a theoretical model and then estimate the free parameters of the model such that the theoretical moment restrictions are fulfilled for the sample "as good as possible". But in contrast to the calibration approach the GMM takes the uncertainty which arises from the estimation of the moments into account from which test statistics for the estimated parameters can be derived.

In the following a short introduction, which follows [13] and [6], to the GMM will be given. Let \( \{X_t : t = 1, 2, \ldots\} \) be a sequence of \((k \times 1)\) random vectors where it is assumed that \(X_t\) is strictly stationary. Furthermore let \(\theta_0\) be a \((p \times 1)\) vector of parameters to be estimated and \(f(X_t, \theta_0)\) a \((q \times 1)\) vector of functions \(\mathbb{R}^{k \times p} \rightarrow \mathbb{R}^q\) where \(q \geq p\). This vector of functions can be interpreted as the disturbance \(u_t = f(X_t, \theta_0)\) of GMM. Consider unconditional moment restrictions of the form

\[
E[f(X_t, \theta_0)] = 0
\tag{19}
\]

and suppose that a law of large numbers can be applied to \(f(X_t, \theta)\), so that the sample mean \(\bar{f}(\theta)\) of \(f(X_t, \theta)\)

\[
\bar{f}(\theta) = \frac{1}{T} \sum_{t=1}^{T} f(X_t, \theta)
\tag{20}
\]

converges almost surely to its population mean with \(T \rightarrow \infty\). The basic idea of GMM is to get an estimator \(\hat{\theta}\) by choosing \(\theta\) such that the following quadratic form is minimized

\[
J_T(\theta) = \bar{f}(\theta)'W_T\bar{f}(\theta) \rightarrow \min_{\theta}
\tag{21}
\]

where the so called distance matrix \(W_T\) is assumed to be positive definite and to converge almost surely to a positive definite matrix \(W_0\) with \(T \rightarrow \infty\). It can be shown that the resulting estimator \(\hat{\theta}\) is consistent for arbitrary choices of the distance matrix under fairly general conditions and also (asymptotically) efficient for an optimal choice of \(W_T\). Such an optimal choice for the distance matrix is the inverse of the long run the covariance matrix \(\Omega\) of the disturbances \(u_t\): \(W_0 = \Omega^{-1}\). If \(u_t\) is serially uncorrelated then \(\Omega = E[u_t u_t']\); if it is serially correlated

\[
\Omega = \sum_{j=-\infty}^{\infty} E[u_t u_{t-j}']
= \lim_{T \rightarrow \infty} E[\bar{f}(\theta) \bar{f}(\theta)'].
\tag{22}
\]
The covariance matrix \( \Omega \) will be estimated by the Newey-West covariance estimator [12], which is heteroscedasticity and autocorrelation consistent and is essentially the Bartlett estimate of the spectral density at frequency zero. This estimator can be constructed in the following way. Let

\[
\hat{\Omega}_j = \frac{1}{T} \sum_{t=1}^{T-j} f(X_t, \hat{\theta}) f(X_t, \hat{\theta})'
\]

(23)

and

\[
\omega_{j,m} = 1 - \frac{j}{m + 1}
\]

(24)

then the estimator of the covariance matrix is given by:

\[
\hat{\Omega} = \hat{\Omega}_0 + \sum_{j=0}^{m} \omega_{j,m}(\hat{\Omega}_j + \hat{\Omega}_j')
\]

(25)

where \( m \) is usually set to \( T^{1/4} \).

Unfortunately it is necessary to have an estimate \( \hat{\theta} \) of the parameter vector to estimate the covariance matrix of the disturbances and hence the optimal distance matrix. Therefore we use an iterative GMM-procedure which starts by choosing the identity matrix as the distance matrix in order to get a first estimate of the parameter vector by minimizing the quadratic form (21). These estimates are then used to calculate a better estimate of the distance matrix, which itself is then used to get a new estimate of the parameter vector. This can be repeated until the changes of the estimated distance matrices become very small.

In the empirical analysis we will estimate four parameters of three models: The discount rate \( \beta \) and three parameters defining technology \( \mu, \delta, \phi \) (see sections 5 to 7 for the meaning of these parameters). Therefore we have \( \theta = (\beta, \mu, \delta, \phi)' \) and furthermore we let \( X_t = (x_t, R_t, R_t^f)' \). By taking the unconditional expectations of equations (7) and by using the first and second moments of \( x_t \) and \( x_t x_{t-1} \) we construct the following vector of moment

\[^2\text{We decided to estimate only the the discount rate } \beta \text{ for given values of } \gamma \text{ because we were not able to get economically reasonable estimates for both parameters simultaneously.}\]
restrictions:

\[
E[f(X_t, \theta)] = E \begin{bmatrix}
    m_t R_t^\ell - 1 \\
    m_t R_t^{\ell - 1} \\
    x_t - \sum_{i=1}^{n} \pi_i \lambda_i \\
    x_t^2 - \sum_{i=1}^{n} \pi_i \lambda_i^2 \\
    x_t x_{t-1} - \sum_{i=1}^{n} \sum_{j=1}^{n} \pi_i \lambda_i \phi_{ij} \lambda_j \\
    x_t^2 x_{t-1}^2 - \sum_{i=1}^{n} \sum_{j=1}^{n} \pi_i \lambda_i^2 \phi_{ij} \lambda_j^2
\end{bmatrix} = 0 \tag{26}
\]

The system is clearly overidentified as we have four parameters to estimate and six moment restrictions.

For the empirical analysis we used the original GMM code\(^3\) written in GAUSS by Hansen, Heaton, and Ogaki which was also used in [7]. We extended the code by the previously described Newey-West covariance estimator.

Following [2] we also investigated into the relevance of changing the number and types of moments to be matched for the results. Our finding was that the GMM estimators depend highly on the number and types of the moment restrictions used. All the results reported in sections 5 to 7 refer to the moment restrictions (26) because with this setup we got the "most reasonable" results. For example when we additionally used higher moments, \(E[x_t^i]\), \(i > 2\), or moments of the second lag, \(E[x_t x_{t-2}]\), the GMM produced very implausible or even inadmissible estimates for some parameters. However this is also true in some cases for the moment conditions (26) (e.g. see table 7 or 9 and the corresponding explanations). The same problem comes up if one uses additional moment restrictions involving the stochastic discount factor of returns \(m_t\), for instance \(E[m_t - 1/R_{t-1}^\ell] = 0\), which should hold as follows from equation (9).

Another problem with the practical application of GMM which is related to the above is the fact that there is no way to restrict estimates of parameters to a certain range. This can lead to estimators which are economically implausible or even simply impossible - like probabilities greater than one or negative variances.

\(^3\)This code is available in the internet at the WWW address gopher://gopher.american.edu/11/academic.depts/cas/econ/software/gauss
4 Description of Data

For the empirical analysis we use Austrian data as it is provided by the data bank of the WIFO ⁴. In the next three chapters we will examine different specifications of C-CAPM models with two assets, a risky and a risk free. In order to do so we need three real valued time series: changes in consumption per capita of non-durable goods plus services, $c_{t+1}/c_t$, the gross return on a risky assets, $R_t^r$, and the gross return on a risk free asset, $R_t^f$.

Table 1: Description of raw data.

<table>
<thead>
<tr>
<th>Code</th>
<th>Description</th>
<th>Range</th>
<th>Basis</th>
</tr>
</thead>
<tbody>
<tr>
<td>CINSGR</td>
<td>Total real valued consumption with services</td>
<td>01Q64–01Q89</td>
<td>1976</td>
</tr>
<tr>
<td>CINSGR</td>
<td>Total real valued consumption with services</td>
<td>01Q76–03Q94</td>
<td>1986</td>
</tr>
<tr>
<td>CDAKGR</td>
<td>Real valued consumption of durables</td>
<td>01Q64–01Q89</td>
<td>1976</td>
</tr>
<tr>
<td>CDAKGS</td>
<td>Real valued consumption of durables</td>
<td>01Q76–03Q94</td>
<td>1986</td>
</tr>
<tr>
<td>ABEVMG</td>
<td>Population</td>
<td>1964–1994</td>
<td></td>
</tr>
<tr>
<td>FAKBKS</td>
<td>Index of the Vienna stock market</td>
<td>01Q68–03Q94</td>
<td>1968</td>
</tr>
<tr>
<td></td>
<td>Return on dividends in percent</td>
<td>1969–1994</td>
<td></td>
</tr>
<tr>
<td>FRSMAN</td>
<td>Nominal net return on government bonds</td>
<td>01Q68–03Q94</td>
<td></td>
</tr>
<tr>
<td>PVPIGS</td>
<td>Consumer price index</td>
<td>01Q66–03Q94</td>
<td>1966</td>
</tr>
</tbody>
</table>

Table 1 describes the raw data and in the following it will be explained how we derive the three time series from this data. Return on dividends is taken from yearly publication of the Austrian stock exchange market by the Austrian bank Girosentrale.

We have two time series for total consumption and two for consumption of durables, where in each case the first one is based on prices of 1976 and ranges from the first quarter of 1964 to the first quarter of 1989. The second is based on prices of 1986 and ranges from the first quarter of 1976 to the third quarter of 1989. We connect these time series by calculating the corresponding factors for the overlapping interval resulting in two time series for the whole range based on prices of 1986. Next we subtract the series of consumption of durables from the series of total consumption and divide the result by the population⁵ so that we finally arrive at the time series of real valued consumption per capita of non-durable goods plus services, $c_t$. From this $c_{t+1}/c_t$ can easily be formed ⁶.

⁴ Wirtschaftsforschungsinstitut, the Austrian Institute of Economic Research.
⁵ Yearly data is transformed into quarterly data by simply holding the values constant within a year.
⁶ In fact we form $c_{t+1}/c_t$ in order to get quarterly observations of yearly changes of consumption. Analogously the gross returns are computed. The period of our model is therefore one year which implies that assets are held for one year - as in [10]. However
The real valued gross return on the risky asset, $R_t^y$, is calculated in the following way: first we set the index of the Vienna Stock Exchange and the consumer price index both on the basis of 1986=100. Then we can calculate the changes in the consumer price index - the inflation rate - to derive a real valued index of the Vienna stock market which corresponds to the price $p_t^f$ of the risky asset. Next we can calculate real valued dividends $\delta y_t$ in index points as the corresponding percentage the price index. Then $R_t^f$ can be calculated according to equation (6).

Finally we calculate the real valued gross return on the risk free asset $R_t^f$ by subtracting the inflation rate from the nominal net return and adding one. For the nominal net return the return on government bonds on the secondary market is used.

Now we have three real valued time series from the first quarter of 1969 to the third quarter of 1989 (103 observations) based on prices of 1986. Table 2 shows the basic statistics of the three time series and additionally of the risk premium.

<table>
<thead>
<tr>
<th></th>
<th></th>
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<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_{t+1}/c_t$</td>
<td>1.025</td>
<td>0.021</td>
<td>1.110</td>
<td>0.986</td>
<td>0.419</td>
</tr>
<tr>
<td>$R_t^f$</td>
<td>1.035</td>
<td>0.017</td>
<td>1.067</td>
<td>0.993</td>
<td>0.866</td>
</tr>
<tr>
<td>$R_t^e$</td>
<td>1.075</td>
<td>0.318</td>
<td>2.291</td>
<td>0.707</td>
<td>0.930</td>
</tr>
<tr>
<td>$\delta^f$</td>
<td>0.040</td>
<td>0.315</td>
<td>1.388</td>
<td>-0.341</td>
<td>0.861</td>
</tr>
</tbody>
</table>

The series of consumption growth rates is different to the Mehra/Prescott case as our data exhibit strong positive autocorrelation while US data exhibits slightly negative autocorrelation.

Figures 1 to 3 show the plot of the time series of the growth rates of consumption, risk free return, and risky return.

we have four observations per year on yearly growth rates of consumption and rates of returns. The advantage of using yearly rates is that these series do not show seasonal cycles as the quarterly rates do.
Figure 1: Growth rate of consumption

Figure 2: Risk Free Return

Figure 3: Risky Return
5 Mehra-Prescott setup

The motivation for this setup lies therein that "historically the average return on equity has far exceeded the average return on short-term virtually default-free debt" [10]. They find that the risk premium is approximately 6 % for the U.S.A., where their model generated only 0.35 %. The question Mehra and Prescott searched to answer was whether a large differential can be generated by frictionless Arrow-Debreu models. The basic setup is that of a Lucas economy (as described in section 2). Since the model in this and the following section are identical to the original models the reader searching for more detailed exposition is referred to the original articles.

In the Mehra/Prescott economy the \( \lambda_i \) are possible realizations of the following

\[
\lambda_1 = 1 + \mu + \delta \\
\lambda_2 = 1 + \mu - \delta.
\]

The \( \mu \) represent the average growth rate and the \( \delta \) the variability of consumption.

The transition probabilities to these states are determined by the matrix

\[
\Phi = \begin{pmatrix}
\phi & 1 - \phi \\
1 - \phi & \phi
\end{pmatrix}.
\]

The stable probabilities are given by the vector:

\[
\Pi' = (0.5, 0.5).
\]

These probabilities can be used in equations (11) to (18) in order to calculate the risk premium and are incorporated in the vector of moment restrictions (26) to get the GMM estimates of \( \beta \) and the technology defining parameters.

Mehra and Prescott specified their model with both two and four state specification. They used the method of moments to estimate the parameters of the consumption process matching the moments of their sample (mean, variance, and first-order autocorrelation). They restricted \( \gamma \) to lie in the interval \( (0,10] \) and \( \beta \) to be an element of \( (0,1] \) that should yield both reasonable risk-free returns as well as a reasonable risk premium.

In our calculations we use here only the two state specification, a specification with four different states of the world is presented in section 7.

With our data we calculated an empirical risk premium of 4.02 %, while the model generated risk premia as given in table 4, far too low to reflect the value of real market activities.
5.1 Results

The GMM estimates of the parameters are given in table 3. For $\gamma$ ranging from 1 to 10 we get reasonable estimators for $\mu$, $\delta$, and $\phi$. We also observe that the estimators for the technology defining parameters, $\hat{\mu}$, $\hat{\delta}$ and $\hat{\phi}$, do not vary with increasing $\gamma$. Though for $\gamma$ larger than one we get estimators for $\hat{\beta}$ which exceed unity. This does not necessarily imply that consumers prefer future to present consumption as has been demonstrated by as [8]. He shows that $\beta > 1$ is not inconsistent with growing economies and that an equilibrium with positive interest rates may exist in such an economy. The important thing here is that the stochastic discount factor on returns, $m_t$, is smaller than one for all $t$, because $m_t$ greater than one would imply that investors prefer future to present returns. This, of course, would be economically implausible. For all the models we estimated (including those in sections six and seven) we calculated an average stochastic discount factor on return of 0.97.

Furthermore we observe that the values of $\hat{\beta}$ are independent of $\mu$, $\delta$, and $\phi$ (and of $\eta$ and $\zeta$ of section 6 and 7).

The chi-square statistics and the corresponding p-values are also displayed in table 3. The p-value is the probability that a $\chi^2(d)$ distributed random variable is less than the value of the computed test statistic. Under the null hypothesis that the overidentifying moment restrictions (19) are satisfied the test statistic follows such a $\chi^2(d)$ distribution. Here the degree of freedom, $d$, is the difference of the number of moment restrictions $q$ and the number of parameters $p^7$. Hence we reject our model at the 95% level if the p-value is greater than 0.95. As can be seen from table 3 we reject none of our models at the 95% level although the p-values are fairly high. Furthermore we observe that the test provides greater evidence against the model as $\gamma$ increases. The relationship between $\gamma$ and $\beta$ is as expected: if people are more risk averse they would want to save more and consume less today. The test statistics are the same for all our model economies and therefore we report them only once.

\footnote{For a derivation of this test statistic see [7].}
Table 3: GMM estimators for the Mehra-Precott economy.

<table>
<thead>
<tr>
<th>$\gamma$</th>
<th>$\hat{\mu}$</th>
<th>$\hat{\delta}$</th>
<th>$\hat{\phi}$</th>
<th>$\hat{\beta}$</th>
<th>$\chi^2$</th>
<th>p-val</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.0250</td>
<td>0.0203</td>
<td>0.7137</td>
<td>0.9906</td>
<td>0.5080</td>
<td>0.7757</td>
</tr>
<tr>
<td></td>
<td>(0.0031)</td>
<td>(0.0025)</td>
<td>(0.0446)</td>
<td>(0.0056)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.0250</td>
<td>0.0203</td>
<td>0.7138</td>
<td>1.0144</td>
<td>0.4964</td>
<td>0.7802</td>
</tr>
<tr>
<td></td>
<td>(0.0031)</td>
<td>(0.0025)</td>
<td>(0.0446)</td>
<td>(0.0084)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0.0250</td>
<td>0.0203</td>
<td>0.7138</td>
<td>1.0385</td>
<td>0.4849</td>
<td>0.7847</td>
</tr>
<tr>
<td></td>
<td>(0.0031)</td>
<td>(0.0025)</td>
<td>(0.0446)</td>
<td>(0.0115)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>0.0250</td>
<td>0.0203</td>
<td>0.7138</td>
<td>1.0628</td>
<td>0.4735</td>
<td>0.7892</td>
</tr>
<tr>
<td></td>
<td>(0.0031)</td>
<td>(0.0025)</td>
<td>(0.0446)</td>
<td>(0.0147)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>0.0250</td>
<td>0.0203</td>
<td>0.7138</td>
<td>1.0872</td>
<td>0.4623</td>
<td>0.7936</td>
</tr>
<tr>
<td></td>
<td>(0.0031)</td>
<td>(0.0025)</td>
<td>(0.0446)</td>
<td>(0.0179)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>0.0250</td>
<td>0.0203</td>
<td>0.7138</td>
<td>1.1118</td>
<td>0.4513</td>
<td>0.7980</td>
</tr>
<tr>
<td></td>
<td>(0.0031)</td>
<td>(0.0025)</td>
<td>(0.0446)</td>
<td>(0.0213)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>0.0250</td>
<td>0.0203</td>
<td>0.7138</td>
<td>1.1366</td>
<td>0.4404</td>
<td>0.8024</td>
</tr>
<tr>
<td></td>
<td>(0.0031)</td>
<td>(0.0025)</td>
<td>(0.0446)</td>
<td>(0.0248)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>0.0250</td>
<td>0.0203</td>
<td>0.7139</td>
<td>1.1615</td>
<td>0.4296</td>
<td>0.8067</td>
</tr>
<tr>
<td></td>
<td>(0.0031)</td>
<td>(0.0025)</td>
<td>(0.0446)</td>
<td>(0.0284)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>0.0250</td>
<td>0.0203</td>
<td>0.7139</td>
<td>1.1866</td>
<td>0.4189</td>
<td>0.8110</td>
</tr>
<tr>
<td></td>
<td>(0.0031)</td>
<td>(0.0025)</td>
<td>(0.0446)</td>
<td>(0.0321)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>0.0250</td>
<td>0.0203</td>
<td>0.7139</td>
<td>1.2118</td>
<td>0.4083</td>
<td>0.8150</td>
</tr>
<tr>
<td></td>
<td>(0.0031)</td>
<td>(0.0025)</td>
<td>(0.0446)</td>
<td>(0.0359)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Standard errors are given in parentheses.

We list in table 4 the obtained risk premia (in percent). The values are far too low to reflect the empirical values for Austria (4.02 %), we do obtain even negative risk premia.

Table 4: Risk Premia for the Mehra-Precott economy (GMM).

<table>
<thead>
<tr>
<th>$\gamma$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{r}^p$</td>
<td>0.03</td>
<td>0.02</td>
<td>-0.05</td>
<td>-0.16</td>
<td>-0.32</td>
<td>-0.53</td>
<td>-0.79</td>
<td>n/a</td>
<td>n/a</td>
<td>n/a</td>
</tr>
</tbody>
</table>

Risk premia in percent, n/a indicates that risk premia could not be computed because the matrix $A$ is not stable.

The values indicate a Equity Premium Puzzle, similar to the one described by [10].

14
6 Rietz economy

There were a number of "solutions" to the Equity Premium Puzzle\textsuperscript{8}, the most prominent was Rietz (1988). Rietz introduced to the Mehra/Prescott setup a third state. States 1 and 2 are normal growth states while in state 3 a crash can happen, although only with a low probability $\eta$ ($\eta << \phi$). If $\lambda_3$ is low consumption is (drastically) reduced, this affects all future consumption levels:

$$\begin{align*}
\lambda_1 &= 1 + \mu + \delta \\
\lambda_2 &= 1 + \mu - \delta \\
\lambda_3 &= \psi(1 + \mu)
\end{align*}$$

and $\psi$ is a fraction of the other parameters such that $0 < \psi < 1$ and $\lambda_3 < \lambda_2 < \lambda_1$. The resulting transition matrix in the Rietz economy is (where the entries in the $i$-th row and $j$-th column denote the probability of going from state $i$ to state $j$):

$$\Phi = \begin{pmatrix}
\phi & 1 - \phi - \eta & \eta \\
1 - \phi - \eta & \phi & \eta \\
1/2 & 1/2 & 0
\end{pmatrix}. $$

There are doubts whether it is reasonable to assume a return to "high" growth after a depression, resp. to whether a transition is more likely to go from a very low to an intermediate level before returning to high level of growth (see also [14]).

The corresponding probabilities are then

$$\Pi' = \begin{pmatrix}
1 \\
2(1 + \eta) \\
2(1 + \eta)
\end{pmatrix} \quad \begin{pmatrix}
1 \\
2(1 + \eta) \\
(1 + \eta)
\end{pmatrix}. $$

As in the Mehra/Prescott setup we derive the moment expressions from the specification of the three state Markov process via the equations (11) to (18) and the vector of moment restrictions (26).

6.1 Results

Applying the Generalized Methods of Moments, we derive estimators and risk premia which we list in table 5 and table 6, with $\psi$ constant to 0.50 (this corresponds to Rietz's example number one, table 1). We give here

---

\textsuperscript{8}Solutions include habit formation [4], the introduction of non-expected utility [17], and the introduction of transaction costs [1].
only the GMM-estimators for $\gamma = 10$ and $\psi = 0.5$ in table 5, but the pattern is the same for all different combinations of $\gamma$ and $\psi$ we have calculated (i.e. $\gamma = 1, \ldots, 10$ and varying $\psi$ between 0 and 1). $\hat{\phi}$ is increasing in $\eta$, $\hat{\delta}$ decreases in $\eta$.

Table 5: GMM estimators for the Rietz economy for $\gamma = 10$.

<table>
<thead>
<tr>
<th>$\eta$</th>
<th>$\hat{\mu}$</th>
<th>$\hat{\delta}$</th>
<th>$\hat{\phi}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0001</td>
<td>0.0251</td>
<td>0.0197</td>
<td>0.7283</td>
</tr>
<tr>
<td>(0.0031)</td>
<td>(0.0026)</td>
<td>(0.0483)</td>
<td></td>
</tr>
<tr>
<td>0.0002</td>
<td>0.0251</td>
<td>0.0190</td>
<td>0.7449</td>
</tr>
<tr>
<td>(0.0031)</td>
<td>(0.0027)</td>
<td>(0.0530)</td>
<td></td>
</tr>
<tr>
<td>0.0003</td>
<td>0.0252</td>
<td>0.0183</td>
<td>0.7640</td>
</tr>
<tr>
<td>(0.0031)</td>
<td>(0.0028)</td>
<td>(0.0591)</td>
<td></td>
</tr>
<tr>
<td>0.0004</td>
<td>0.0252</td>
<td>0.0176</td>
<td>0.7864</td>
</tr>
<tr>
<td>(0.0031)</td>
<td>(0.0029)</td>
<td>(0.0673)</td>
<td></td>
</tr>
<tr>
<td>0.0005</td>
<td>0.0253</td>
<td>0.0168</td>
<td>0.8130</td>
</tr>
<tr>
<td>(0.0031)</td>
<td>(0.0030)</td>
<td>(0.0785)</td>
<td></td>
</tr>
</tbody>
</table>

Standard errors are given in parentheses.

For the Rietz economy and the subsequent gold rush economy we report only the estimators for the technology defining parameters because the estimators for $\beta$ are the same as in the Mehra/Prescott setup (see table 3). The same is true for the statistics (SE($\hat{\beta}$), $\chi^2$, p-val) which can also be read from table 3. If one looks at the moment restrictions (26) it becomes clear why $\hat{\beta}$ varies only with $\gamma$ and why the estimators for the technology defining parameters are independent of $\gamma$ but vary with $\eta$. The two first moment restrictions in equation (26) are the same for all our model economies and involve only $\beta$ and $\gamma$ whereas the the remaining four refer only to the technology defining parameters.

Table 6: Risk Premia for the Rietz economy (GMM).

<table>
<thead>
<tr>
<th>$\eta$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0001</td>
<td>0.04</td>
<td>0.03</td>
<td>-0.02</td>
<td>-0.10</td>
<td>-0.18</td>
<td>-0.24</td>
<td>-0.17</td>
<td>0.16</td>
<td>1.06</td>
<td>3.00</td>
</tr>
<tr>
<td>0.0002</td>
<td>0.04</td>
<td>0.03</td>
<td>0.00</td>
<td>-0.04</td>
<td>-0.05</td>
<td>0.05</td>
<td>0.42</td>
<td>1.35</td>
<td>3.32</td>
<td>7.01</td>
</tr>
<tr>
<td>0.0003</td>
<td>0.04</td>
<td>0.04</td>
<td>0.02</td>
<td>0.02</td>
<td>0.08</td>
<td>0.33</td>
<td>1.00</td>
<td>2.47</td>
<td>5.37</td>
<td>10.37</td>
</tr>
<tr>
<td>0.0004</td>
<td>0.04</td>
<td>0.05</td>
<td>0.05</td>
<td>0.07</td>
<td>0.21</td>
<td>0.60</td>
<td>1.55</td>
<td>3.53</td>
<td>7.21</td>
<td>13.19</td>
</tr>
<tr>
<td>0.0005</td>
<td>0.04</td>
<td>0.06</td>
<td>0.07</td>
<td>0.12</td>
<td>0.33</td>
<td>0.86</td>
<td>2.07</td>
<td>4.50</td>
<td>8.84</td>
<td>15.51</td>
</tr>
</tbody>
</table>

Risk premia in percent.

As can be seen in table 6 risk aversion parameters between 7 and 9 yield
equity premia which mimic the empirical value. Comparing the Rietz results with those from the Mehra/Prescott economy we note that the increase in $\gamma$ seems to play a more important role in increasing the $r^p$ than the introduction of the crash state. The higher $\gamma$ the greater is the impact of the crash state on the risk premia, $r^p$. Furthermore the criticism of the the Rietz solution to the equity premium puzzle by [11] can also be applied to our Austrian data. They argue that such large values of the risk aversion parameter are inconsistent with virtually all applied general equilibrium studies as well as with the findings of [7], who found that $\gamma$ is near one. In addition to this at risk aversion parameters higher than 7 we get estimates for $\beta$ which are greater than 1.13 (see table 3). As has already been noted in section 5.1 $\beta > 1$ is not necessarily inconsistent with growing economies. Nevertheless $\beta > 1.13$ seems to be a fairly high value and again this result is in contradiction with [7] who found that $\beta$ is slightly smaller than one. We can match $\beta$ in a more plausible range only for $\gamma$ between one and two - but with this setup we cannot match the empirically observed risk premium as can be seen in table 6.

Adding a crash state changes the values of $r^p$ for higher $\gamma$ more than for lower $\gamma$.

7 The gold rush

The Rietz setup has been criticized by [11] for not displaying the same patterns in the consumption structure, not dealing adequately with unexpected inflation, and for (unreasonable) high parameter values. Criticism has also been brought forward by [16], who argued that the crash state severely understates the volatility of asset returns.

Our motivation for criticizing the Rietz setup is the following. It is not convincing that actors should only value supernormal losses and pay no attention to supernormal profits.

The feature of supernormal profits can be seen in the returns at the Vienna Stock Exchange in the 1980s (see figure 3). Agents would not regard this as a shift in the "normal" growth rate, but, as we argue, rather define these as temporary supernormal returns and evaluate their future consumption accordingly.

We re-specify the above economy, adding a fourth gold rush type state. In this state the returns are supernormal, $\lambda_4 = \kappa(1 + \mu)$ where $\kappa$ is larger than
one,

\[
\begin{align*}
\lambda_1 &= 1 + \mu + \delta \\
\lambda_2 &= 1 + \mu - \delta \\
\lambda_3 &= \psi(1 + \mu) \\
\lambda_4 &= \kappa(1 + \mu),
\end{align*}
\]

therefore the following transition matrix:

\[
\Phi = \begin{pmatrix}
\phi & 1 - \phi - \eta - \zeta & \eta & \zeta \\
1 - \phi - \eta - \zeta & \phi & \eta & \zeta \\
1/2 & 1/2 & 0 & 0 \\
1/2 & 1/2 & 0 & 0
\end{pmatrix}.
\]

The stable probabilities for this process are:

\[
\Pi' = \begin{pmatrix}
\frac{1}{2(1+\eta+\zeta)} & \frac{1}{2(1+\eta+\zeta)} & \frac{\eta}{1+\eta+\zeta} & \frac{\zeta}{1+\eta+\zeta}
\end{pmatrix}.
\]

These probabilities can be used in equations (11) to (18) and incorporated in the vector of moment restrictions (26).

We do realize that adding this fourth economy makes the process symmetric, again. But it is this symmetry that appeals most to us — with risk averse agents it is only plausible that they take (potential) losses more into account than (potential) gains. But Rietz’s agents do take the average growth path into account and, simultaneously, deal with a situation where supernormal losses can occur. But, by the same reasoning, why shouldn’t they regard a gold rush (or the like) as unlikely as a stock market crash?

### 7.1 Results

For this gold rush set-up we use three different scenarios. In the first we assume that the probability of a crash is as likely as the probability of a gold rush. In the two following scenarios one state is more likely than the other. The factor by which normal returns are diminished in the crash state, \(\psi\), and the factor by which normal returns are increased in the gold rush state, \(\kappa\), are set to \(\psi = 0.5\) and \(\kappa = 1.5\) in all of the following experiments.

The estimators for \(\beta\) (and the statistics) are the same as before (and not reproduced here, see table 3).

Due to misspecification of the model we calculate estimators for \(\phi\) that are greater than one. This means that the model can’t be fitted to the data.
Table 7: GMM estimators for the Gold Rush economy ($\eta = \zeta$).

<table>
<thead>
<tr>
<th>\zeta</th>
<th>\hat{\mu}</th>
<th>\hat{\delta}</th>
<th>\hat{\phi}</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0001</td>
<td>0.0250</td>
<td>0.0190</td>
<td>0.7450</td>
</tr>
<tr>
<td>(0.0031)</td>
<td>(0.0027)</td>
<td>(0.0053)</td>
<td></td>
</tr>
<tr>
<td>0.0002</td>
<td>0.0250</td>
<td>0.0176</td>
<td>0.7866</td>
</tr>
<tr>
<td>(0.0031)</td>
<td>(0.0029)</td>
<td>(0.0067)</td>
<td></td>
</tr>
<tr>
<td>0.0003</td>
<td>0.0250</td>
<td>0.0160</td>
<td>0.8454</td>
</tr>
<tr>
<td>(0.0031)</td>
<td>(0.0032)</td>
<td>(0.0094)</td>
<td></td>
</tr>
<tr>
<td>0.0004</td>
<td>0.0250</td>
<td>0.0143</td>
<td>0.9346</td>
</tr>
<tr>
<td>(0.0031)</td>
<td>(0.0035)</td>
<td>(0.1513)</td>
<td></td>
</tr>
<tr>
<td>0.0005</td>
<td>0.0250</td>
<td>0.0123</td>
<td>1.0857</td>
</tr>
<tr>
<td>(0.0031)</td>
<td>(0.0041)</td>
<td>(0.2908)</td>
<td></td>
</tr>
</tbody>
</table>

Standard errors are given in parentheses.

7.1.1 $\eta = \zeta$

In table 7 we list the estimators derived via GMM for this setup. 
\hat{\delta} is strongly decreasing in \zeta. Note the perverse result for \hat{\phi} when \zeta is equal to 0.0005. The \hat{\beta} are the same as described in table 3.

The risk premia generated by this model are listed in table 8.

Table 8: Risk Premia for the Gold Rush economy ($\eta = \zeta$) (GMM).

<table>
<thead>
<tr>
<th>\zeta</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0001</td>
<td>0.03</td>
<td>0.02</td>
<td>-0.03</td>
<td>-0.11</td>
<td>-0.21</td>
<td>-0.27</td>
<td>-0.21</td>
<td>0.11</td>
<td>0.99</td>
<td>2.92</td>
</tr>
<tr>
<td>0.0002</td>
<td>0.03</td>
<td>0.02</td>
<td>-0.02</td>
<td>-0.08</td>
<td>-0.10</td>
<td>-0.02</td>
<td>0.32</td>
<td>1.22</td>
<td>3.15</td>
<td>6.79</td>
</tr>
<tr>
<td>0.0003</td>
<td>0.03</td>
<td>0.02</td>
<td>-0.01</td>
<td>-0.05</td>
<td>-0.02</td>
<td>0.18</td>
<td>0.78</td>
<td>2.14</td>
<td>4.86</td>
<td>9.61</td>
</tr>
<tr>
<td>0.0004</td>
<td>0.03</td>
<td>0.02</td>
<td>-0.01</td>
<td>-0.05</td>
<td>-0.05</td>
<td>0.07</td>
<td>0.40</td>
<td>1.10</td>
<td>2.41</td>
<td>4.92</td>
</tr>
<tr>
<td>0.0005</td>
<td>n/a</td>
<td>n/a</td>
<td>n/a</td>
<td>n/a</td>
<td>n/a</td>
<td>n/a</td>
<td>n/a</td>
<td>n/a</td>
<td>n/a</td>
<td>n/a</td>
</tr>
</tbody>
</table>

Risk Premia in percent, n/a indicates that risk premia could not be computed because the \phi are greater than one.

Introducing the gold rush lowers the $r^p$ for the whole set of combinations in comparison to the Rietz setup. Interesting is the feature that for higher $\eta$ and $\zeta$ the $r^p$ decreases for a given $\gamma$ whereas in the Rietz economy it increases.
7.1.2 $\eta > \zeta$

One might argue that the probability of running into the brick wall of a crash is higher than finding gold on the shores of Avalon. For such a situation we obtain the following estimators, table 9.

Table 9: GMM estimators for the Gold Rush economy ($\eta = 0.0005$).

<table>
<thead>
<tr>
<th>$\zeta$</th>
<th>$\hat{\mu}$</th>
<th>$\hat{\delta}$</th>
<th>$\hat{\phi}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0001</td>
<td>0.0252</td>
<td>0.0160</td>
<td>0.8451</td>
</tr>
<tr>
<td></td>
<td>(0.0031)</td>
<td>(0.0032)</td>
<td>(0.0943)</td>
</tr>
<tr>
<td>0.0002</td>
<td>0.0252</td>
<td>0.0152</td>
<td>0.8846</td>
</tr>
<tr>
<td></td>
<td>(0.0031)</td>
<td>(0.0033)</td>
<td>(0.1172)</td>
</tr>
<tr>
<td>0.0003</td>
<td>0.0251</td>
<td>0.0143</td>
<td>0.9344</td>
</tr>
<tr>
<td></td>
<td>(0.0031)</td>
<td>(0.0035)</td>
<td>(0.1512)</td>
</tr>
<tr>
<td>0.0004</td>
<td>0.0251</td>
<td>0.0133</td>
<td>0.9988</td>
</tr>
<tr>
<td></td>
<td>(0.0031)</td>
<td>(0.0038)</td>
<td>(0.2041)</td>
</tr>
<tr>
<td>0.0005</td>
<td>0.0250</td>
<td>0.0123</td>
<td>1.0857</td>
</tr>
<tr>
<td></td>
<td>(0.0031)</td>
<td>(0.0041)</td>
<td>(0.2908)</td>
</tr>
</tbody>
</table>

Standard errors are given in parentheses.

Here we see $\hat{\mu}$ decreasing with a higher probability of receiving supernormal gains, this is even more puzzling since the volatility, $\hat{\delta}$, is decreasing as well. This feature is due to increasing $\hat{\phi}$, starting at a very high value indeed — so that the Markov chain leads our economy with higher probabilities for $\eta$ and $\zeta$ to the "normal" growth paths, the cases Mehra and Prescott investigated.

Table 10: Risk Premia for the Gold Rush economy ($\eta = 0.0005$) (GMM).

<table>
<thead>
<tr>
<th>$\zeta$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0001</td>
<td>0.94</td>
<td>0.05</td>
<td>0.05</td>
<td>0.10</td>
<td>0.29</td>
<td>0.80</td>
<td>1.96</td>
<td>4.32</td>
<td>8.54</td>
<td>15.06</td>
</tr>
<tr>
<td>0.0002</td>
<td>0.04</td>
<td>0.04</td>
<td>0.04</td>
<td>0.07</td>
<td>0.23</td>
<td>0.69</td>
<td>1.76</td>
<td>3.92</td>
<td>7.81</td>
<td>13.87</td>
</tr>
<tr>
<td>0.0003</td>
<td>0.04</td>
<td>0.03</td>
<td>0.02</td>
<td>0.02</td>
<td>0.09</td>
<td>0.33</td>
<td>0.88</td>
<td>1.94</td>
<td>3.85</td>
<td>7.25</td>
</tr>
<tr>
<td>0.0004</td>
<td>0.03</td>
<td>0.38</td>
<td>0.38</td>
<td>0.80</td>
<td>1.66</td>
<td>3.35</td>
<td>6.62</td>
<td>12.68</td>
<td>23.99</td>
<td>38.95</td>
</tr>
<tr>
<td>0.0005</td>
<td>n/a</td>
<td>n/a</td>
<td>n/a</td>
<td>n/a</td>
<td>n/a</td>
<td>n/a</td>
<td>n/a</td>
<td>n/a</td>
<td>n/a</td>
<td>n/a</td>
</tr>
</tbody>
</table>

Risk Premia in percent, n/a indicates that risk premia could not be computed because the $\phi$ are greater than one.

The risk premia, table 10, in this setup are higher than those of the Rietz economy and the last setup with the exception for the combination $\zeta = 0.0003$ and $\gamma > 6$. $r^p$ are matched with $\gamma$ between 6 and 8, but the estimators for $\beta$ are greater than one (see table 3).
7.1.3 \( \eta < \zeta \)

What does change if we live in a booming world? I.e. the probability of finding gold is higher than loosing your assets through a crash of the stock exchange. The estimators are listed in table 11.

Table 11: GMM estimators for the Gold Rush economy \((\eta = 0.0001)\).

<table>
<thead>
<tr>
<th>(\zeta)</th>
<th>(\hat{\mu})</th>
<th>(\hat{\delta})</th>
<th>(\hat{\phi})</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0001</td>
<td>0.0250</td>
<td>0.0190</td>
<td>0.7450</td>
</tr>
<tr>
<td></td>
<td>(0.0031)</td>
<td>(0.0027)</td>
<td>(0.0530)</td>
</tr>
<tr>
<td>0.0002</td>
<td>0.0249</td>
<td>0.0183</td>
<td>0.7642</td>
</tr>
<tr>
<td></td>
<td>(0.0031)</td>
<td>(0.0028)</td>
<td>(0.0592)</td>
</tr>
<tr>
<td>0.0003</td>
<td>0.0249</td>
<td>0.0176</td>
<td>0.7868</td>
</tr>
<tr>
<td></td>
<td>(0.0031)</td>
<td>(0.0029)</td>
<td>(0.0674)</td>
</tr>
<tr>
<td>0.0004</td>
<td>0.0248</td>
<td>0.0168</td>
<td>0.8135</td>
</tr>
<tr>
<td></td>
<td>(0.0031)</td>
<td>(0.0030)</td>
<td>(0.0787)</td>
</tr>
<tr>
<td>0.0005</td>
<td>0.0248</td>
<td>0.0160</td>
<td>0.8457</td>
</tr>
<tr>
<td></td>
<td>(0.0031)</td>
<td>(0.0032)</td>
<td>(0.0945)</td>
</tr>
</tbody>
</table>

Standard errors are given in parentheses.

The values of the estimators are lower than those in the Rietz economy, the \(\hat{\delta}\) are lower than in the setup where the values of \(\eta\) and \(\zeta\) are the same. The \(\hat{\mu}\) are lower than those of all previous setups and are declining as \(\zeta\) increases. The \(\hat{\phi}\) are slightly larger than those in the Rietz economy, but lower than those of the two previous gold rush setups.

The \(\hat{\phi}\) are lower than one but will increase steadily as \(\zeta\) increases, to be greater than one when \(\zeta\) will be equal to 0.0009.

Table 12: Risk Premia for the Gold Rush economy \((\eta = 0.0001)\) (GMM).

<table>
<thead>
<tr>
<th>(\zeta)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0001</td>
<td>0.03</td>
<td>0.02</td>
<td>-0.03</td>
<td>-0.11</td>
<td>-0.21</td>
<td>-0.27</td>
<td>-0.21</td>
<td>0.11</td>
<td>0.99</td>
<td>2.22</td>
</tr>
<tr>
<td>0.0002</td>
<td>0.03</td>
<td>0.02</td>
<td>-0.04</td>
<td>-0.13</td>
<td>-0.23</td>
<td>-0.30</td>
<td>-0.26</td>
<td>0.05</td>
<td>0.91</td>
<td>2.83</td>
</tr>
<tr>
<td>0.0003</td>
<td>0.03</td>
<td>0.01</td>
<td>-0.05</td>
<td>-0.15</td>
<td>-0.26</td>
<td>-0.34</td>
<td>-0.32</td>
<td>-0.03</td>
<td>0.82</td>
<td>2.70</td>
</tr>
<tr>
<td>0.0004</td>
<td>0.03</td>
<td>0.01</td>
<td>-0.06</td>
<td>-0.17</td>
<td>-0.29</td>
<td>-0.39</td>
<td>-0.38</td>
<td>-0.12</td>
<td>0.69</td>
<td>2.52</td>
</tr>
<tr>
<td>0.0005</td>
<td>0.03</td>
<td>-0.00</td>
<td>-0.08</td>
<td>-0.19</td>
<td>-0.33</td>
<td>-0.45</td>
<td>-0.47</td>
<td>-0.24</td>
<td>0.50</td>
<td>2.22</td>
</tr>
</tbody>
</table>

Risk premia in percent.

The risk premia, table 12, are as might have been expected — lower than in the Rietz setup. It is surprising, though, that for intermediate values of
we get negative risk premia while with a higher risk aversion we receive positive values for \( r^p \). In this setup the empirical value for the Austrian \( r^p \) is not matched for the values we use, though a risk aversion parameter between 10 and 11 is able to match it.

8 Conclusion

In this paper we calculate the equity premium for Austria using the Generalized Method of Moments (GMM). We used three different set-ups of C-CAPMs, the Mehra-Prescott economy, the Rietz specification, and an own specification that allows additionally for supernormal profits.

With our Austrian data we get results which correspond qualitatively with the US based Mehra/Prescott and Rietz findings. The Mehra/Prescott setup generates a risk premium which is far too low to reflect the empirically observed risk premium. The Rietz economy is able to "solve" the equity premium puzzle, but at a very high risk aversion parameter. The results from the Rietz setup are not changed dramatically if we introduce a fourth, gold rush state. The \( r^p \) are lowered by the introduction of the possibility of supernormal gains with the exception when the probability of crashes are higher than those of the gold rushes. Although the Rietz economy is not satisfying for logical reasons (the consideration of crashes alone while neglecting gold rushes) the results of the gold rush economy are not of the sort we would consider satisfactory in explaining the puzzle.

The criticism of [11] is confirmed by our findings: for both the Rietz and the gold rush economies the higher risk aversion parameters, \( \gamma \), seem to play a more important role in "explaining" the puzzle than the introduction of additional states. We also estimated the discount factor of consumption, \( \beta \), and found that the estimated values of the discount factors are greater than one. Although in a growing economy this does not necessarily imply that consumers prefer future consumption to present as has been shown by [8] our estimated discount factor is fairly high and at least inconsistent with other empirical work (e.g. see [7]). On the other hand we get estimates for the discount factor within a reasonable range (smaller or only slightly larger than one) for risk aversion parameters which lie between one and two - what would be in agreement with applied general equilibrium studies as well as with other empirical studies. But with this parametrization the risk premium generated by the model is too small to match the empirically observed. We interpret this result as a confirmation of Mehra and Prescott’s criticism of the Rietz "solution" to the equity premium puzzle, namely that the risk aversion parameters are too high.

Finally we would like to note some difficulties with the practical application of GMM. Given our data the GMM estimates depend highly on the number and
type of moment restrictions as well as on the number and type of parameters to be estimated. In particular we were not able to estimate the discount factor of consumption $\beta$ and the relative risk aversion parameter $\gamma$ simultaneously and therefore we preferred to estimate $\beta$ for given values of $\gamma$.

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The equity premium puzzle and the risk-free rate puzzle.
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