Optimal Day-Ahead Bidding of Electricity Storage using Approximate Dynamic Programming

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Outline

Dispatch of Electricity Storage

- Optimal dispatch is a stochastic-dynamic decision problem
- Bidding decisions on the day-ahead real-time market
- Decision about storage level
- Risk aversion or price response to avoid excessive use of balancing market

Method

- Stochastic dynamic programming (SDP)
- Instantaneous profit calculated by two stage dynamic stochastic problem
- Sampling based approach
- Solved using Approximate Dynamic Programming using post decision logic

Numerical Results
Decision Process

**Inter-day decision process**
- Daily decisions
- Optimize storage levels over time
- State transition is a Markov process
- State variables: daily wind production, mean temperatures and weekday
- Approximate Dynamic Programming (ADP)

**Intra-day decision problem**
- Hourly decisions
- Optimize day-ahead bidding decisions
- Day-ahead & real-time price uncertainty dependent on state
- Stochastic Programming
Decision Process

state = \{\text{day, wind, temperature, reservoir}\}

- **day-ahead price scenarios**
  - submit bids
  - day-ahead market is closed
    - day-ahead price realization
      - real-time price scenarios
        - balancing and storage operation
          - final reservoir level

reservoir level at 100%
reservoir level at 50%
reservoir level at 0%
Bidding Curves

- Bids to day ahead (DA) market are submitted one day in advance
- Bidding curves are piecewise constant
- Random prices $\rightarrow$ random bids $\rightarrow$ random storage levels
- Excess bidding on DA market is balanced on real time (RT) market
Define $r_t$ as the function that maps every vector of price realizations to a final storage state. We write

$$V(S_t, r_t) = \max_{r_{t+1}} \max_{x \in \mathcal{X}} \mathbb{E} \left( \pi(x, r_t, r_{t+1}) + \gamma V(S_{t+1}, r_{t+1}) | S_t \right)$$

$$= \max_{r_{t+1}} \max_{x \in \mathcal{X}} \mathbb{E} \left( \pi(x, r_t, r_{t+1}) | S_t \right) + \gamma \mathbb{E} \left( V(S_{t+1}, r_{t+1}) | S_t \right)$$

$$= \max_{r_{t+1}} R(S_t, r_t, r_{t+1}) + \gamma \mathbb{E} \left( V(S_{t+1}, r_{t+1}) | S_t \right)$$

where $\pi$ is the instantaneous profit, $x$ is the intraday decision and

$$R(S_t, r_t, r_{t+1}) = \max_{x \in \mathcal{X}} \mathbb{E}(\pi(x, r_t, r_{t+1}) | S_t)$$

**Proposition 1.** The function $R(.,.,.)$ is concave in $r_t$ and $r_{t+1}$.

**Proposition 2.** $V(S_t, .)$ is concave.
Algorithmic Strategy

- Partition the state space into multi-dimensional grid, i.e.

\[
S = \bigcup_k G_k
\]

where the \( G_k \) are the disjoint grid cells.

- Define function \( G(S) \) which maps every state into grid cell

- Approximate \( V(S, .) \) by piecewise linear concave function \( V(G(S), .) \)

- Iteratively update the sampled post decision value function

\[
\bar{V}^r(S_t, r_{t+1}) = \mathbb{E}(V(S_{t+1}, r_{t+1}) \mid S_t)
\]

\[
\bar{V}^r(S_{t-1}, r_t) = \mathbb{E}\left( \max_{r_{t+1}} R(S_t, r_t, r_{t+1}) + \gamma \bar{V}^r(S_t, r_{t+1}) \mid S_{t-1} \right)
\]
Generate a sample of state transitions: $S_1, ..., S_T$

Define a set of initial reservoir levels as breakpoints: $B_1, ..., B_L$

For $t=2$ to $T$ do

Sample price realizations dependent on $S_t$

For $l=1$ to $L$ do

$$w_{G(S_t), l} \leftarrow \max_{r_t} R(S_t, B_l, r_t) + \gamma \tilde{V}^r(G(S_t), r_t)$$

$$\tilde{V}^r(G(S_{t-1}), B_l) \leftarrow U(w_{G(S_t), l})$$

End

End

Return value function

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Expected Profit (in $1,000)$

Reservoir Level (in MWh)
### Stochastic Programming Formulation (Intra-Day)

**Objective:** maximize risk adjusted expected profit

\[
\max \frac{1}{S} \sum_{s=1}^{S} \left( \sum_{h=1}^{24} \left( p_{hs}^d x_{hs}^d - \hat{p}_{hs}^b x_{hs}^b \right) + \gamma \tilde{V}_r (r_{24s}) \right) - \rho \text{CVaR}_\alpha (P)
\]

- **Day-ahead price response:**
  \[
p_{hs}^d = \hat{p}_{hs}^d - \beta_h^d x_{hs}^d
\]

- **Real-time price response:**
  \[
  \bar{p}_{hs}^b = \hat{p}_{hs}^d - \beta_h^d x_{hs}^d + \beta_h^b x_{hs}^b
  \]

Subject to...
- Min./max. capacity constraints (binary)
- Market and reservoir balance equations
- Price dependent, piecewise-constant bidding curves

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**Symbols:**

- \( x_{hs}^d \): realized day-ahead bids
- \( x_{hs}^b \): real-time balancing decisions
- \( r_{hs} \): realized reservoir levels
- \( \gamma \): discount factor
- \( \beta_{h}^{d,b} \): estimated slope coefficients
- \( \hat{p}_{hs}^d \): day-ahead price scenarios
- \( S \): number of day-ahead price scenarios
- \( \text{CVaR}_\alpha (P) \): conditional value at risk

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*universität wien*
Bidding Curve

**Piecewise-constant bidding curve**: 

\[ x_{hs}^d = \begin{cases} 
  y_{h1} & \text{if } \hat{p}_{hs}^d < q_{h1}^d \\
  \ldots & \\
  y_{hk} & \text{if } q_{hi-1}^d \leq \hat{p}_{hs}^d < q_{hi}^d \\
  \ldots & \\
  y_{hK} & \text{if } \hat{p}_{hs}^d \geq q_{hK}^d 
\end{cases} \]

**Linear formulation**

- Price segments are given by samples
- Equal number of scenarios per price segment
  - price segments with flexible width
  - price segments with equal probability


\[ x_{hs}^d : \text{realized day-ahead bid} \]
\[ y_{hk} : \text{bidding curve coefficient} \]
\[ q_{hi}^d : \text{pre-defined price segment} \]
\[ \hat{p}_{hs}^d : \text{day-ahead price scenario} \]
Bounding the Value Function

\[ \overline{V}(k, r) \]

- **upper bound**
- **lower bound**

\[ w_{k3} \]

\[ w_{k2} \]

\[ w_{k1} \]

\[ u_1 \]

\[ u_2 \]

\[ u_3 \]

\[ r_t \]
PJM Market

- Day ahead market
  - Daily clearing
  - Bidding curves
  - Low volatility
  - High market depth

- Balancing market
  - Intra day trading
  - High volatility
  - Low market depth
**Econometric Model**

**State Transition Model**
- Trigonometric regression
- Detrended VAR(1) model → *inter-day state transition*
- Markov process of wind power, temperature and day length

**Electricity Price Model**
- 48 semi-log models
- Stepwise backward-forward regression
- t-distributed error terms → *intra-day price scenarios*
State Transition Model

Data Sources: PJM Energy Market, Wolfram Mathematica WeatherData
Electricity Price Model

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Value Function Comparison

- **Parameters**
  - 10,000 training iterations
  - 20 day ahead price scenarios for the intraday problem
  - Bidding functions with 10 segments
  - $\gamma = 0.999$, $\rho = 0.05$, 11 breakpoints for value functions

- **Evaluated over 100,000 state transitions**
  - Simulate intra-day prices scenarios
  - Execute bids based on the value function
  - Evaluate decision
  - Calculate average profit

- **Lower Bound**: deterministic optimization based on average prices
Benchmark Policies

- Clairvoyant: future state transitions and prices are known
- Parametric policy

Optimal parameters found with CMA-ES optimizer (Hansen 2006)
Result: Value Function Comparison

The graph compares different value functions across varying reservoir-capacity ratios. It illustrates the performance of:

- **Single Value Function**
- **Perfect Foresight**
- **Fixed-Spread Policy**
- **(7x3x3) Value Function**
- **(7x6x6) Value Function**

The y-axis represents the percentage from the lower bound, and the x-axis shows the reservoir-capacity ratio. The graph visually captures how each function varies with respect to the ratio.
# Approximation Quality

## Number of breakpoints for approximation

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</table>

Maximal relative differences between upper and lower bound
Convergence Speed

% from upper bound

no. of iterations

single value function
(7x3x3) value function
(7x6x6) value function
Balancing: Price Response vs Risk Measure

- w/o price response
- linear price response with $\beta_d^h = \beta_b^h > 0$
- linear price response with $\beta_d^h = 0, \beta_b^h > 0$
Conclusion

Model, Method & Results

- We solve a complex stochastic-dynamic decision process
  - Training a value function with ADP methods
  - Using a stochastic program as immediate profit function
- Applied the model to the PJM market of 2009
  - Diminishing returns to sophistication for value function approximation
  - ADP better than simple heuristics and deterministic approaches
  - Need risk measure or price response to obtain realistic bids on the RT market

Future Work

- Multiple (connected) storage facilities
- More careful estimation of price response function on the real time market
- Application to other markets (EEX, Nordpool)
References


Bidding Curve, 11h