Back to Basics: 
Basic Research Spillovers, Innovation Policy and Growth*

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Abstract

This paper introduces endogenous technical change through basic and applied research in a growth model. Basic research differs from applied research in two significant ways. First, significant advances in technological knowledge come through basic research rather than applied research. Second, these significant advances could potentially be applicable to multiple industries. Since these applications are not immediate, the innovating firm cannot exploit all the benefits of the basic innovations for production. We analyze the impact of this appropriability problem on firms’ basic research incentives in an endogenous growth framework with private firms and a public research sector. After characterizing the equilibrium, we estimate our model using micro level data on research expenditures and behavior by French firms. We quantitatively find that the competitive equilibrium features too little basic research and too much applied research investment. We show that uniform R&D subsidy to private firms does not improve welfare due to cross-subsidization of overinvested applied research. Optimal type-dependent policy dictates positive subsidy for private basic research and negative subsidy on applied research. We also find a strong complementarity between the property rights of the public researcher and optimal public funding of basic research. Our analysis highlights the need for crafting policies that directly target basic research.

Keywords: Innovation, basic research, applied research, research and development, government spending, endogenous growth, spillover.

JEL classification: O31, O38, O40, 043, 047, L78

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1 Introduction

Fostering economic growth is one of the primary objectives of economics and policymakers. Regardless of political persuasion, policymakers are often arguing for additional support for research and development. For instance, in a recent report by the US Congress Joint Economic Committee, it is argued that despite its value to society as a whole, basic research is underfunded by private firms precisely because it is performed with no specific commercial applications in mind. The level of federal funding for basic research is “worrisome” and it must be increased to overcome the underinvestment in basic research (JEC, 2010). However, the report also complains about the lack of available studies on the subject to measure this underinvestment and the lack of available data.\(^1\) Understanding the inefficiencies and designing appropriate industrial policy requires identifying the spillovers associated with innovation and research.

Countries allocate a significant part of their resources to researching new products and technologies to foster economic growth (see Figure 1). However, what is less known is the fact that almost half of that portion is allocated for basic research, while the other half goes to applied research (see Figure 2).\(^2\) Despite these sizable investments in both types of research activities, the distinctive roles played by basic and applied research in the growth process are still unknown and many related questions remain to be answered: What are the key roles of basic and applied research for productivity growth? What are the incentives of private firms to do basic research? How does public basic research contribute to innovation and productivity growth? How sizable are the spillovers from basic research? What are the potential inefficiencies in a competitive economy and what are the appropriate government policies to mitigate them and promote economic growth? Our goal in this paper is to shed light on these important questions which have long been at the heart of macroeconomic and industrial policy debates.

Basic research differs from applied research in two significant ways. First, significant advances in basic technological knowledge that forms the basis for the subsequent tangible applied innovation come through basic research. Second, these significant basic advances could potentially be applicable to many different industries and can be utilized by subsequent ap-

\(^1\)http://jec.senate.gov/public/?a=Files.Serve&File_id=29aac456-fce3-4d69-956f-4add06f11c1

\(^2\)Basic research is defined by the NSF as a “systematic study to gain more comprehensive knowledge or understanding of the subject under study without specific applications in mind. Although basic research may not have specific applications as its goal, it can be directed to fields of current or potential interest. This focus is often the case when performed by industry or mission-driven federal agencies”. Applied research is, on the other hand, defined as a “systematic study to gain knowledge or understanding to meet a specific, recognized need. In industry, applied research includes investigations to discover new scientific knowledge that has specific commercial objectives with respect to products, processes, or services”. http://www.nsf.gov/statistics/seind10/c4/c4s.htm#sb2
plied innovators within the same industry. Because of these potentially sizable spillovers both within and across industries, the consensus has been that there is an underinvestment in basic research -although the extent of the underinvestment has been unclear and hard to determine.\textsuperscript{3}

A satisfactory analysis for the study of the above listed questions requires a structural framework that models the incentives for different types of research investments by private firms. We strongly believe that the combination of a macro model with heterogenous firms and firm level data would greatly contribute to our understanding of the study of macro level policies and above listed questions. Our goal in this project is to take a first step towards developing this theoretical framework, to estimate it using micro-level data, and to discuss the effects of different research policies on productivity and growth. To the best of our knowledge, this would be the first study to model private investment into basic and applied research simultaneously as well as government investment into basic research in an endogenous growth framework, and the first that combines a quantitative analysis with micro-level evidence to identify sources of spillovers arising from basic research.

Our analysis proceeds in three steps. We first document some important empirical facts on basic research spillovers. Second, motivated by those empirical facts we propose a general equilibrium multi-industry framework with firms and an public research sector. Firms conduct both basic and applied research whereas the public sector mainly focuses on basic research.\textsuperscript{4}

\textsuperscript{3}Our paper is a contribution to the vast empirical literature on R&D spillovers (see for example, Griliches (1992), Jaffe, Trajtenberg and Henderson (1993), Audretsch and Feldman (1996), Anselin, Varga and Acs (1997), Bloom, Schankerman and Van Reenen (2011)).

\textsuperscript{4}Trajtenberg et al. (1992) show that academic research is more basic and less frequently patented.
spillovers for subsequent applied innovations. As the Ivory Tower perspective argues, in our model basic research by private firms will turn into consumer products faster than public research labs (for empirical evidence, see Trajtenberg et al. (1992), Rosenberg and Nelson (1994), Henderson, Jaffe and Trajtenberg (1998) and more recently Evans (2010) and Bikard (2011)). Applied research on the other hand, advances these new technologies further. To highlight the key economic forces, we will first consider a benchmark economy with tractable functional forms, solve it in closed form and discuss the main dynamics and inefficiencies. Next, we will introduce the general framework. After characterizing the equilibrium of the general model, we estimate the structural parameters. Finally, we use the estimated model to assess the extent of inefficiencies in basic and applied research and to study the implications of several important R&D policies.

The reduced for analysis of our paper contributes to the empirical innovation literature by introducing two new ways of identifying spillovers. First, we use variation in the level of basic research spending between firms in different numbers of industries to infer the magnitude of cross-industry spillovers. Second, we use heterogeneous citation patterns across public and private patents in order to identify within-industry spillovers.

Our model also constitutes a contribution to the literature on endogenous growth. Although the different characteristics of basic and applied research on the one hand and public and private research on the other hand have been widely recognized to be of first-order importance, these issues have received insufficient attention from the economic growth literature. In particular, Romer (1986, 1990), Lucas (1988), Aghion and Howitt (1992), Grossman and Helpman (1991) and Segerstrom, Anant and Dinopolous (1990) mainly considered a uniform type of (applied) research and ignored basic research investment in the economy. Few exceptions, such as Aghion and Howitt (2009), Morales, Gersbach et al (2008), Narin et al. 1997, Mansfield 1998, Martin 1998, Park (1998) have considered theoretical models where basic research is done publicly and ignore the private investment into it. In addition, we enrich the notion of basic research to include spillovers both within and across industries.

Quantitatively, our contribution is to consider various spillovers associated with basic research, model private firms’ incentives to invest in basic research and hence quantify the size of the inefficiencies in the decentralized economy. Our quantitative analysis suggests that the size of the inefficiency is around 8.7 percentage points in consumption equivalent terms. This

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5By fundamental innovation, we mean the major technological improvements that generate a much bigger contribution to the aggregate knowledge stock in a society relative to the rest of innovations, and also have long-lasting spillover effects on the quality of subsequent innovations within the same field.

6Bloom, Van Reenen and Schankerman (2011), XXX
raises an important question: By how much can policies undo this inefficiency? We analyze different innovation policies that are commonly considered by policymakers. The first policy of interest is a uniform research subsidy to private firms. This policy has been analyzed by various studies (Aghion and Howitt, pp 487), yet the findings have been mixed. These results come from models with a single type of research. Once the distinction between basic and applied research is introduced, the results can differ greatly. We show that in an economy with both types of research efforts, the major underinvestment is in basic research due to its sizable spillovers. In this environment subsidizing overall private research is less effective since this policy would *oversubsidize applied research*, which is already overinvested in due to competition. Therefore the welfare improvement from a uniform subsidy is limited unless the policymaker is able to discriminate between types of research projects at the firm level, which is considered to be quite impractical. Nevertheless, we consider a hypothetical type-dependent research subsidy, and find that the optimal policy is to subsidize basic research by 35% and to tax applied research by 11%.

Then we analyze the optimal allocation of funding to public research labs. Due to the Ivory Tower feature of public basic research, allocating more money to the academic sector without giving property rights to the researcher is not necessarily a good idea. Therefore, we mimic a policy exercise similar to the Bayh-Dole Act that was enacted in the US in 1980. We consider alternative scenarios in which the public researcher has no property rights, then 50% and 100% property rights. We find a complementarity between the level of property rights and the optimal allocation of resources to academic research. The optimal combination turns out to be granting full property rights to the academic researcher and allocating 1.6% of GDP to public research. This reduces the welfare gap to from 8.7 to 4.6.

The rest of the paper is organized as follows. Section 2 introduces some new empirical facts on basic research spillovers to motivate our modeling approach. Section 3 presents our theoretical framework and consists of two parts: In Section 3.1 we provide a benchmark version of the main model with more tractable functional forms to develop intuitions via closed form solutions and in Section 3.2 we describe the generalizations of the benchmark model which we will estimate. Section 4 describes our quantitative analysis including the welfare properties of the estimated general model. Section 5 provides a detailed discussion of the welfare impacts of various policies on the decentralized economy. Section 6 concludes. Appendix contains omitted proofs and derivations (A), the data description (B), further robustness checks on the stylized facts (C), and further details on within-industry spillover (D).
2 Empirical Facts

Despite the vast literatures in industrial organization and endogenous growth focusing on aggregate research investment, the literature on firms’ basic research decisions has been very thin and empirical studies are even rarer. Notable exceptions include Mansfield (1980, 1981), Griliches (1986) and Link (1981) which empirically document the positive contribution of basic research to firms’ productivity. Existing studies on basic research have been mainly theoretical and devoted their attention to academic/public research as the source of basic research and ignored the *private* side of it (Segerstrom, 1998; Aghion and Howitt, 1996, Morales, 2004). We believe that part of the reason for this outcome, as argued in one Congressional report (JEC 2010), has been the lack of firm-level data on distinct types of private research. Our empirical evidence also contributes to the literature on innovating firms (eg. Klette and Kortum (2004) Lentz and Mortensen (2008), Akcigit and Kerr (2010), Acemoglu and Cao (2010)), characterizing the innovation and firm dynamics of R&D conducting firms.

In this paper we use unique data on the French economy combining information not only on product markets and R&D characteristics of individual firms, but also on firm ownership status for the period 2000-2006. R&D information comes from an annual survey conducted by the French Ministry of Research that covers a large, representative cross-section of innovating French firms. Details regarding data sources are provided in section ??.

The next section presents the main empirical facts emerging from this data.

2.1 Basic versus Applied Research

First we document that private firms’ investment in basic research forms a non-negligible fraction of both total private research spending and total basic research spending.\(^7\) Table 1 reports official statistics from the French Ministry of Research on public investment in basic research and private investments into applied and basic research for the period 2000-2006. Private spending on basic research amounted to an average of 1 billion Euros per year as opposed to 8.3 billion on applied research for the period 2000-2006. During the same period, public expenditures on basic research represented an average of 7 billion Euros per year in France. This implies that more than 11% of private research is spent on basic research. More importantly, almost 15% of total basic research in the economy is undertaken by private

\(^7\)Public statistics define basic research expenditures following the Frascati Manual: “Investment into basic research is undertaken either for pure scientific interest or to bring a theoretical contribution to the resolution of technical problems.” Alternatively: “The objective of applied research is to identify the potential applications of results from fundamental research or to find new solutions to a precisely identified problem.”

6
### Table 1: Expenditures into Basic R&D

<table>
<thead>
<tr>
<th>Year</th>
<th>Private Basic</th>
<th>Private Applied</th>
<th>Public Basic</th>
<th>Public Applied</th>
<th>Private Basic</th>
<th>Private Applied</th>
<th>Public Basic</th>
<th>Public Applied</th>
</tr>
</thead>
<tbody>
<tr>
<td>2000</td>
<td>802</td>
<td>7005</td>
<td>6425</td>
<td>5 .11</td>
<td>.13</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2001</td>
<td>795</td>
<td>7748</td>
<td>6786</td>
<td>8 .1</td>
<td>.12</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2002</td>
<td>959</td>
<td>8899</td>
<td>7037</td>
<td>9 .11</td>
<td>.14</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2003</td>
<td>1092</td>
<td>8928</td>
<td>7133</td>
<td>10 .12</td>
<td>.15</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2004</td>
<td>1175</td>
<td>9482</td>
<td>7338</td>
<td>11 .12</td>
<td>.16</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2005</td>
<td>1227</td>
<td>9469</td>
<td>7331</td>
<td>12 .13</td>
<td>.17</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2006</td>
<td>1213</td>
<td>10278</td>
<td>7755</td>
<td>13 .12</td>
<td>.16</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>


The picture that emerges therefore hints at a significant involvement of the private sector in undertaking basic scientific research. Thus, ignoring the private incentives behind basic research might prevent economists and policymakers from designing more effective policies for productivity growth.

#### 2.2 Multi-Industry Distributions

Another stylized fact emerging from the data is the extent of multi-industry presence of firms. Figure 3 uses our micro-level data on French companies between 2000 and 2006 in order to plot their empirical distribution into multiple industries.

To measure multi-industry presence, we count the number of distinct SIC codes a firm is present in. Our data allows us to identify a firm’s links to different industries not only through product lines within the same firm but also through its majority ownership links. To avoid misclassification of related industries we consider as our benchmark case the number of distinct 1 digit SIC codes (10 industries).

On average firms are present in 2 distinct industries as defined by 1-digit SIC codes. Although nearly 44% of the firms are operating in only one industry, the remaining firms occupy a large spectrum of industries. Insights from Figure 3 are very similar when using more disaggregate SIC classifications (up to the 4 digit SIC level), or when changing the definition of an industry link.\(^9\)

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\(^8\) Similarly, Howitt (2003) using a NSF survey finds that around 22% of all basic research in the US during the period 1993-1997 was performed by private enterprises.

\(^9\) Figures available upon request.
2.3 Basic Research and Cross-industry Spillovers

Next we link our stylized fact on multi-industry presence to private incentives for basic research. More specifically, we test Nelson’s hypothesis that the main basic research investors would be those firms that have *fingers in many pies*. According to this argument, as the range of its products and industries gets more diversified, a firm’s incentive for investing into basic research relative to applied research should increase due to better appropriability of potential knowledge spillovers.

Figure 4 plots average basic research intensity against the total number of distinct 1 digit SIC codes in which the firm is present together with a simple linear fit of the data. Basic research intensity is defined as the ratio of total firm investment into basic research divided by total firm investment into applied research.

Figure 4 shows that a simple linear fit of the data suggests a positive and statistically significant relationship between the two variables. Table 2 provides further evidence about the relationship between multi-industry presence and basic research intensity. To take into account the corner solution at 0 we estimate the relation between a firm’s basic research intensity and its multi-industry presence using a Tobit model.

In all specifications basic research intensity is increasing in the number of industries. According to the benchmark estimation, presence in an additional industry increases a firm’s basic research intensity by 3 percentage points on average. In terms of magnitude this corresponds to a 50% increase in the average research intensity of a single industry firm. Note also that
**Note:** Linear plot on pooled data for the period 2000-2006. Basic Research Intensity is defined as the ratio of total firm investment into basic research divided by total firm investment into applied research. Number of 1 Digit SIC Industries is the number of distinct SIC codes a firm is present in.

**Table 2: Basic Research Intensity and Multi-Market Activity**

<table>
<thead>
<tr>
<th></th>
<th>1 Digit SIC</th>
<th>2 Digit SIC</th>
<th>3 Digit SIC</th>
<th>4 Digit SIC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Log # of Industries</td>
<td>0.032***</td>
<td>0.027***</td>
<td>0.024***</td>
<td>0.021***</td>
</tr>
<tr>
<td></td>
<td>(0.01)</td>
<td>(0.00)</td>
<td>(0.00)</td>
<td>(0.00)</td>
</tr>
<tr>
<td>Log Employment</td>
<td>0.003**</td>
<td>0.002</td>
<td>0.001</td>
<td>0.001</td>
</tr>
<tr>
<td></td>
<td>(0.00)</td>
<td>(0.00)</td>
<td>(0.00)</td>
<td>(0.00)</td>
</tr>
<tr>
<td>Year &amp; Organization FE</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
</tr>
<tr>
<td>N</td>
<td>13708</td>
<td>13708</td>
<td>13708</td>
<td>13708</td>
</tr>
</tbody>
</table>

Notes: Pooled data for the period 2000-2006. Estimates are obtained using Tobit models and relate to the marginal effect of the regressors at the sample mean. Robust standard errors clustered at the firm level in parentheses. See appendix for the definition of variables. One star denotes significance at the 10% level, two stars denote significance at the 5% level, and three stars denote significance at the 1% level.

the magnitude of the estimated coefficient on multi-industry presence is stronger for higher levels of SIC aggregation, in other words for non related activities, since it decreases from 3 percentage points at the 1 digit SIC level to 2.1 percentage points at the 4 digit SIC level. Table Y in the appendix provides a rich set of robustness checks in terms of control variables, alternative measures of multi-industry presence, instrumental variable approaches and different
2.4 Basic Research and Within-industry Spillovers

Basic research contributes to economic growth through its impact on subsequent innovations within the same industry. Applied research builds on the latest technological knowledge in the product line but the returns from building on the original break-through innovation vanishes as more and more firms exploit it.

To empirically capture these spillovers we turn to patent data. The idea is to pin down the age at which a patent derived from basic research cannot be distinguished from a patent derived from applied research in terms of its importance for follow-up innovations. Two empirical issues need to be addressed: (i) distinguish patents derived from basic and applied research and, (ii) capture the idea of successively less original contributions. We address the first point by distinguishing patents applied for by corporations from patents applied for by public institutions. We address the second point by computing a citation based measure of marginal contribution of citing patents over time.

To do so we use the NBER patent dataset covering the period 1974-2006. The analysis of our final dataset will focus exclusively on French patent depositors but the construction of the different variables uses information from the entire dataset. For each patent we first identify citing patents across time. The age of a patent is given by the difference between the grant year of the patent and the current year. For each of the citing patents we compute the cumulative 10 year-forward citations these citing patents receive. For each originally cited patent we are then able to compute across age the mean of the citing patents’ cumulative 10 year-forward citations. Our measure, Average Citations of Citing Patents, captures the marginal importance of each successive citing patent. Appendix D provides further explanations on the construction of this variable.

Figure 5 presents graphical evidence for French patents between 1975 and 1985. It plots Average Citations of Citing Patents for French public patents (blue line) and French private patents (red line).

The figure measures Average Citations of Citing Patents computing the 10 year forward citations of the citing patents. Patents citing private patents receive on average 1.6 citations within the first 10 years. The relative importance of patents citing private patents remains stable and slightly increasing through the age of the private patent. Patents citing public patents receive on average two citations within their first ten years. The importance of citing
Figure 5: Citation Patterns for French Public and Private Patents

Note: Panel plots Average Citations of Citing Patents for French public patents (blue line) and French private patents (red line) across patent age. Average Citations of Citing Patents is computed as the 10 year forward citations of the citing patents and is measured for patents granted in the period 1975-1985.

Table 3: Citation Differences for French Public and Private Patents

<table>
<thead>
<tr>
<th>Age</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Difference</td>
<td>.3**</td>
<td>.3**</td>
<td>.62***</td>
<td>.28**</td>
<td>.41**</td>
<td>.23</td>
<td>.71***</td>
<td>.08</td>
<td>.39</td>
<td>.14</td>
</tr>
<tr>
<td></td>
<td>(0.15)</td>
<td>(0.15)</td>
<td>(0.17)</td>
<td>(0.14)</td>
<td>(0.18)</td>
<td>(0.17)</td>
<td>(0.25)</td>
<td>(0.16)</td>
<td>(0.25)</td>
<td>(0.24)</td>
</tr>
</tbody>
</table>

Notes: Differences in citation patterns of 15383 patents granted by USPTO to French private (92%) and public (8%) depositors. The difference is computed in terms of Average Citations of Citing Patents across patent age. Average Citations of Citing Patents is computed as the 10 year forward citations of the citing patents and is measured for patents granted in the period 1975-1985. Two sample t-test with unequal variances. One star denotes significance at the 10% level, two stars denote significance at the 5% level, and three stars denote significance at the 1% level.

Patents is stable until the original patent is eight years old. At this moment we observe a significant drop of citations of citing patents from 2.4 to 1.7. This is when the difference between private and public in terms of citings’ citations becomes non-significant. Although public citations of citing patents slightly increase again after this drop, the difference remains smaller and statistically non-significant as indicated in Table 3. Results are similar when we relax the assumption that the variable is a normally distributed interval variable using the
Wilcoxon-Mann-Whitney test. Appendix D provides further robustness checks related to the computation of the citations variable and related to the public and private patent classification. Our results are consistent with previous stylized facts related to citations of private and academic patents. Henderson et al. (1998) as well as Trajtenberg et al. (1997) show that corporate patents relative to academic patents tend to be relatively less cited and less general in the subsequent technological fields that cite them. Evidence on European patent data is more scarce but Bacchiocchi and Motobbio (2006) shows that in France university patents and public research organizations are more highly cited during the first five years but then become similar in terms of citations.

3 Theoretical Analysis: Endogenous Growth with Basic and Applied Research

We are first willing to discuss our modelling choice before going into the details of it. Our theoretical framework will depart from standard endogenous growth models in a number of ways. In line with stylized fact 1, we will allow private firms to invest in both basic and applied research. Second, to capture stylized fact 2, firms will be able to operate in multiple industries. Third, our analysis relies on the appropriation of spillovers from basic research by multi-industry firms, hence there will be cross-industry spillovers from basic research. Fourth, to capture stylized fact 4, there will also be within-industry spillovers from basic research. Finally, we will also introduce public research sector which can be thought as the universities or publicly funded research labs. The key distinction between the private basic research and public basic research will be that the outcome of the former will turn immediately into a consumer product of the innovating firm, while the latter will contribute to the general pool of basic knowledge and will not turn into a consumer product until a firm utilizes that knowledge. This will induce a delay in the effect of public basic research as was argued in the Ivory Tower perspective of basic research. Hence, the social trade-off will be that while private firms are better in turning an abstract basic research into consumer products, they do not internalize all the spillovers associated with it. Hence, there will be room for interesting policy discussions, which we will provide after our quantitative analysis.

In our quantitative analysis, we will consider flexible functional forms in order to best match the data. However, this will come at the cost of tractability and ease of intuition. Therefore, in this section, we will first outline a simplified benchmark framework that highlights the key elements of the main model. After deriving the theoretical results and the main economic intuition, in the next section we will describe the generalizations of the benchmark model.
In the benchmark model, first we will introduce the static problem of goods production (section 3.1.1). Then we will introduce the dynamic innovation decisions of firms (section 3.1.2). Finally, in order to highlight the effects of the two types of spillovers (cross-industry and within-industry), we will consider two subeconomies that utilize only one type spillover at a time and discuss the implications of each (sections 3.1.3 and 3.1.4, respectively).

3.1 Benchmark Model

We will provide our benchmark analysis in two steps to transpire the static and dynamic features of our model. First, we will introduce the static environment, define the production technologies and firm’s static profit maximization problem and solve for the static equilibrium. Next, we will introduce the dynamic components of our model such as the household’s and firms’ intertemporal decisions and government’s policy tools through which it can affect the economy’s productivity growth.

3.1.1 Preferences and Production Technology

We consider a continuous time economy with a representative household.

**Household** The household is populated by a measure $M$ individuals each of which supply one unit of labor inelastically. The household consumes final good and maximizes the following lifetime utility

$$U_0 = \int_0^\infty \exp(-\delta t) \ln C(t) \, dt$$

where $C(t)$ is the consumption at time $t$ and $\delta$ is the discount rate. The household owns all the firms in the economy, which generates a risk-free flow return of $r$ in aggregate. The household also supplies labor in the economy through which it earns wage rate $w(t)$. Finally, the household pays a lump sum tax $T(t) \geq 0$ every instant. Thus the household’s intertemporal maximization is simply to maximize (1) subject to the following budget constraint,

$$C(t) + \dot{A}(t) \leq r(t) A(t) + Mw(t) - T(t)$$

where $A(t)$ is the asset holdings of the household. The Euler equation of the household maximization will pin down the steady state interest rate as

$$r^* = g^* + \rho$$

where $g^*$ denotes the steady state growth rate of the economy.
Production is divided into three major sectors: Downstream, midstream and upstream sectors. The upstream sector produces intermediate goods $y_{ij}$. These are used to produce industry aggregates $Y_i$ in the midstream sector. Finally, the downstream sector combines these industry aggregates into a final good $Z$.

**Downstream Sector** The household consumes the final good $Z(t)$ which is produced in the downstream sector by infinitely many competitive firms that combine inputs from $M$ different industries according to the constant elasticity of substitution production function

$$Z(t) = \left[ \sum_{i=1}^{M} \mu_i Y_i(t) \right]^{\frac{1}{\gamma}}$$

where $\mu_i \in (0, 1)$ and $\sum_{i=1}^{M} \mu_i = 1$. In this production function, $Y_i(t)$ is the aggregate output from industry $i \in \{1, \ldots, M\}$. The economy consists of $M \in \mathbb{Z}_+$ industries. In the context of firm-level data, each industry $i$ can be thought of as a different one-digit Standard Industrial Classification (SIC) code.\(^\text{11}\) We normalize the price of the final good $P(t)$ to 1 at every instant $t$ without any loss of generality. The maximization problem of a downstream firm can be expressed as

$$\max_{\{Y_i\}_{i=1}^{M}} \left\{ \left[ \sum_{i=1}^{M} \mu_i Y_i(t) \right]^{\frac{1}{\gamma}} - \sum_{i=1}^{M} P_i(t) Y_i(t) \right\}.$$  

Since there is no capital and all costs are in terms of labor units, the resource constraint of the economy is simply $C(t) \leq Z(t)$.

**Midstream Sector** Next we turn to the midstream production of the industry aggregates $Y_i(t)$. Each industry aggregate $Y_i(t)$ is produced competitively by a continuum of firms that combine inputs from a unit continuum of product lines. Let $y_{ijk}(t)$ denote the production of upstream good $j$ in industry $i$ by the $k^{th}$ inventor in that product line. Denote the index of the latest inventor by $K(t)$. Industry aggregate $i$ is produced according to the following Cobb-Douglas production function

$$\ln Y_i(t) = \int_0^{1} \ln \left( \sum_{k=1}^{K(t)} y_{ijk}(t) \right) dj.$$  

We will see that the equilibrium in the upstream sector will feature only one firm (latest inventor) actually producing in each product line. Thus we will simply denote the output in

\(^{11}\)Note that we introduce multi-industry structure in order to model cross-industry spillovers. To avoid any additional theoretical complications, we will focus on symmetric equilibrium in which industry aggregates achieve the same values.
product line $j$ by $y_{ij}$ and drop the variety index $k$. In this case, the maximization problem of a representative firm producing $Y_i$ can be written as

$$
\Pi_i(t) = \max_{\{y_{ij}\}_{j \in [0,1]}} \left\{ P_i(t) \exp \left( \int_0^1 \ln y_{ij}(t) \, dj \right) - \int_0^1 p_{ij}(t) y_{ij}(t) \, dj \right\}
$$

where $P_i(t)$ and $p_{ij}(t)$ are the prices of $Y_i$ and $y_{ij}$ at time $t$. For notational simplicity, time subscripts will henceforth be suppressed.

**Upstream Sector** In the upstream sector, the firm that has made the latest invention $K$ has the monopoly on the usage of that particular production technology. The upstream good is produced according to a linear production technology that takes only labor as an input

$$
y_{ijk} = q_{ijk} l_{ijk}
$$

where $q_{ijk} > 0$ is the labor productivity associated with technology $k$ and $l_{ijk}$ is amount production workers employed. Let us denote the wage rate in the economy by $w$ in terms of the final good. The specification in (5) implies that each product $y_{ijk}$ has a constant marginal cost of production $w/q_{ijk} > 0$. Therefore, in any product line $j$, the latest inventor will capture the submarket through limit pricing at the previous leader’s marginal cost $(w/q_{ij,K-1})$. All other firms in that submarket will optimally choose zero production. The aggregate variables in industry $i$ will be a function of an aggregate quality index, therefore we find it informative to define the following quality index

$$
\ln Q_i = \int_0^1 \ln q_{ij} \, dj.
$$

**Definition of a Firm** In this model, as in Klette and Kortum (2004), a firm is defined as a collection of product lines in which it is the lead producer. Firm $f$ owns $n_{if} \in \mathbb{Z}_+$ product lines in a given industry $i$. Therefore we will denote the portfolio of firm $f$ in industry $i$ by a multiset $^{12}$

$$
q_{if} = \{ q_{if}(1), q_{if}(2), \ldots, q_{if}(n) \}.
$$

Similarly firm $f$ can operate in multiple industries ($m_f \geq 1$). Let $I_f$ and $F_i$ denote the set of industries in which firm $f$ operates and set of firms that operate in industry $i$. A firm’s entire product portfolio is then given by

$$
q_f = (q_{1f}, q_{2f}, \ldots, q_{M_f}).
$$

---

12 A multiset is a generalization of a set which can contain more than one instances of the same member. For instance, given $j \neq j'$, a multiset $q_{if}$ can contain $q_{if}(j)$ and $q_{if}(j')$ regardless of whether $q_{if}(j) = q_{if}(j')$. 
Working with such a large and complex state space proves burdensome in practice. Later on, we will make assumptions that allow us to use a much simpler equivalent representation for a firm’s product portfolio.

A firm’s portfolio of products will expand through successful innovation. Likewise, it will lose product lines when other firms or potential entrants successfully innovate on one of its product lines (thus stealing it). After a successful innovation in a product line $j$, the quality will improve by a step size $s_{ij} \in \{\eta, \lambda\}$ with $\eta > \lambda > 0$,

$$q_{ijK+1} = (1 + s_{ij}) q_{ijK}.$$ 

We will describe the step size determinations later in the text. These two step sizes will determine two different mark-ups in the model. We will denote the current shares of product lines with a high mark-up ($\eta$) in industry $i$ by $\alpha_i$ and the remaining share $1 - \alpha_i$ will denote the fraction of low markup ($\lambda$) product lines.

**Static Equilibrium** In this economy, there are five variables relevant to producers that will be determined in the dynamic environment: the fraction of production workers $L^P$ in the labor force, the distribution of markups $\alpha_i$, the interest rate $r$, the growth rate $g$ and the quality index $Q_i$. Our focus will be on symmetric equilibrium in which all industries have the same industry aggregates $\alpha_i = \alpha$, $Q_i = Q$ and $Y_i = Y$ and all industries have the same weights $\mu_i = 1/M$. Thus we will drop the industry indicies when it causes no confusion. Before we go into the dynamics of the model, we first find it informative to solve for the static equilibrium given these equilibrium values $(L^P, \alpha, r, g, Q)$.

**Definition 1 (Static Equilibrium)** For any given set of variables $(L^P, \alpha, r, g, Q)$, a static equilibrium in this model is defined as a tuple $(y_j^*, p_j^*, \ell_j^*, w^*, Y^*, Z^*)$ such that $(y_j^*, p_j^*)$ solve monopolist maximization problem in (4), $\ell_j^*$ is consistent with the optimal output choice in (5), $w^*$ equates the labor demand to $L^P$ and $(Y^*, Z^*)$ determined in (2) and (3) and consistent with the monopolist optimal choices.

The following proposition summarizes the key variables in the static equilibrium.

**Proposition 1** For any given set of values $(L^P, \alpha, r, g, Q)$, the static equilibrium consist of

- firm’s optimal production and pricing decisions and profit in product line $j$

$$y_j^* = q_j (1 - \tilde{s}_j) \frac{L^P}{1 - \bar{s}}, \quad p_j^* = \frac{w^*}{(1 - \tilde{s}_j) q_j} \quad \text{and} \quad \pi_j^* = \tilde{s}_j \frac{Z^*}{M}$$  

(6)
monopolist’s optimal labor decision

\[ \ell_j^* = (1 - \tilde{s}_j) \frac{L^P}{1 - \tilde{s}} \]  \hspace{1cm} (7)

equilibrium wage rate

\[ w^* = (1 - \bar{\eta})^\alpha (1 - \bar{\lambda})^{1-\alpha} \left( \frac{Q}{M} \right) \]  \hspace{1cm} (8)

and final output

\[ Z^* = \left[ \frac{(1 - \bar{\eta})^\alpha (1 - \bar{\lambda})^{1-\alpha}}{\alpha (1 - \bar{\eta}) + (1 - \alpha)(1 - \bar{\lambda})} \right] \times (QL^P) \]  \hspace{1cm} (9)

where \( \tilde{s}_j \equiv \frac{s_j}{1+s_j} \) for \( s_j \in \{\eta, \lambda\} \) and \( \bar{s} = \alpha \bar{\eta} + (1 - \alpha)\bar{\lambda} \).

\[ \text{Proof.} \] See Appendix A. \[ \blacksquare \]

Couple of observations are in order. First, firms are charging a constant mark-up \( s_j \in \{\eta, \lambda\} \) over their marginal cost \( (w^* / q_j) \). Since there are two different mark-ups \( (\eta \text{ and } \lambda) \) in the economy, the distribution of the mark-ups \( \alpha \) plays a role in the equilibrium aggregate values. Second, the wage rate and the final output are proportional to the quality index \( Q \). Therefore the growth of the quality index will determine the aggregate growth in this economy. An important question is the efficiency property of the static equilibrium. The following lemma sheds light on this issue.

Lemma 1 For any given \( Q \) and \( L^P \), in the static model, the efficient level of output is \( Z^{SP} = QL^P \). This is achieved by allocating the same amount of workers to each product line, \( \ell_i = L^P \).

\[ \text{Proof.} \] The social planner’s problem can be expressed as \( \max_{\ell_j \in [0,1]} \exp \left( \int_0^1 \ln y_j dj \right) \) subject to \( L^P = \int_0^1 \ell_j dj \) and \( y_j = q_j \ell_j \). Substituting the last expression into the objective function, this becomes \( \max_{\ell_j \in [0,1]} Q \exp \left( \int_0^1 \ln \ell_j di \right) \) subject to \( L^P = \int_0^1 \ell_j di \). Due to concavity in each product line in the objective function, we find that \( \ell_j = L^P \). Substituting this into the objective function yields \( Z^{SP} = QL^P \). \[ \blacksquare \]

Note that in the static equilibrium, the labor allocation in (7) and output in (9) depart from their efficient levels \( L^P \) and \( QL^P/M \) due to heterogeneous markups. The efficient production plan allocates the same amount of workers to every product line. Our model features heterogeneous markups \( (\eta, \lambda) \), and hence monopolists put too much labor into low markup product lines, and vice versa. Note also that these static distortions vanish when markups are uniform, i.e., \( \alpha \in \{0, 1\} \).
3.1.2 Dynamic Environment

Technological Progress Through Innovation  In this economy, there are two sources of productivity growth. First, the government uses tax revenues to fund public research labs to produce basic innovations. Second, private firms invest in both basic and applied innovation with the goal of increasing their market share. In what follows, we are going to describe firms’ research technology and the distinction between basic and applied research. Then we will describe the public research technology.

Firms innovate by investing in two types of research: basic and applied. In accordance with Nelson’s (1959) description, significant advances in technological knowledge come through basic innovation in our model. Applied innovation builds on the existing basic innovations. Therefore there will be complementarity between the two types of innovation at the aggregate level.

Basic Research by Private Firms  Innovation through basic research introduces a new generation of fundamental technical knowledge. The utilization of these new fundamental ideas for production requires what we call industry specific working knowledge. Each firm has some set of industries $\mathcal{I}_f$ in which it has working knowledge, which we will describe later.

Each innovation by a firm builds on the state-of-the-art technology in that product line. Let $q_{jk}$ be the highest quality technology for producing $j$. When firm $f$ produces a basic innovation that has a direct application in industry $i \in \mathcal{I}_f$ and product line $j$, firm $f$ utilizes this basic knowledge for production and patents this new high-value technology. As a result, firm $f$ improves $q_{jk}$ by $\eta > 0$

$$q_{ijk} = (1 + \eta) \cdot q_{ijk}.$$  

(10)

and gains one product line that generates per-period profit of $\pi_j$ as in expression (50). Moreover basic research features two potential spillovers:  

- **within-industry spillover**: Each new basic innovation changes the evolution of the product line by introducing a radically new technology. The introduction of the new basic technology causes subsequent applied innovations to be larger until the latest basic technology becomes outdated through some random process. We refer to product lines just hit by basic innovation as hot product lines, as opposed to cold product lines, whose latest basic innovation has become outdated.

---

13Each of these effects is challenging to measure empirically. For within-industry spillovers, we look at patent citations. Specifically, we compare citation patterns between applied and basic innovations. For cross-industry spillovers, since this generates a greater incentive to undertake basic research for firms in many industries, we look at the correlation between multi-industry presence and basic research spending.
• **cross-industry spillover**: Each new basic innovation has the potential for spillovers into other industries. With some probability a basic innovation will generate an additional basic innovation some other industry. If the firm has working knowledge in this other industry, it can utilize the innovation for production. Otherwise, the new technology contributes to the pool of existing basic knowledge and will turn into a consumer products in the future by some other producer.

Each of these spillovers affects the incentives of private firms to invest in innovation. These incentives are the main drivers of productivity growth in this economy and lie at the heart of our analysis. First, we will describe each of these in detail. Then we will consider two distinct economies in which we analyze each spillover and the incentives generated by them separately.

**Applied Research by Private Firms and Within-industry Spillover**  
*Applied research* makes use of the *within-industry* spillover from basic research and builds on the existing latest basic technological knowledge in a product line. The contribution of each applied innovation to the technology stock is a function of how depreciated the latest basic technology is. If the latest basic technology is undepreciated (i.e. still hot), a successful applied innovation will benefit from it and improve the latest quality \( q_{ijK} \) of that product line by \( \eta \), as in (10). If the latest basic technology of the product line is depreciated (i.e. cold), a successful applied innovation will improve the latest quality only by \( \lambda \) so that

\[
q_{ijK+1} = (1 + \lambda) \cdot q_{ijK}.
\]  

(11)

We assume that a new basic technology depreciates at a Poisson rate \( \zeta > 0 \). As a result, \( 1/\zeta \) is a measure of the intensity of within-industry spillovers. We will denote the share of product lines in which that latest basic innovation is undepreciated by \( \alpha_i \in [0, 1] \).

Firms choose their flow rate of innovation and pay a labor cost that is increasing and convex in this rate. Basic and applied research levels are chosen separately, and there is no complementarity between them in terms of research costs. In terms of the innovation production function, we will follow the literature (See Klette and Kortum, 2004; Lentz and Mortensen, 2008, Acemoglu et.al., 2010). Firms undertake innovation by combining their existing, non-tradable intangible capital with researchers (hired at wage rate \( w \) as with the production workers) in a Cobb-Douglas production function. In our model, the intangible capital stock in a particular industry \( i \) is proxied by the number of product lines \( n_{ij} \) that a firm owns in that industry. The production function for applied and basic research then takes
the following form

\[ A_{if} \equiv n_{if}^{1/2} H_{A,if}^{1/2} a_{if}^{1/2} \quad \text{and} \quad B_{if} \equiv \xi^{1/2} n_{if}^{1/2} H_{B,if}^{1/2} a_{if}^{1/2} \quad (12) \]

where \( H_{A,if} \) and \( H_{B,if} \) denote the number of researchers that firm \( f \) needs to hire in order to generate the Poisson flow rates applied \((A_{if})\) and basic research \((B_{if})\).

The above specifications, which are standard in this class of models, capture the idea that a firm’s knowledge capital facilitates innovation.\(^{14}\) Let us define \( a_{if} \equiv A_{if}/n_{if} \) and \( b_{if} \equiv B_{if}/n_{if} \) as the applied and basic innovation intensities. As a result, we can summarize the cost of doing applied and basic research as,

\[ C_a (a_{if}, n_{if}) = w n_{if} \cdot \frac{1}{2} a_{if}^2 \quad \text{and} \quad C_b (b_{if}, n_{if}) = w n_{if} \cdot \frac{1}{2} b_{if}^2 \quad (13) \]

Notice that total cost is directly proportional to the number of product lines. Therefore, we can also think of innovation decisions as being made on a per product line basis.

Both applied and basic research are directed towards particular industries but undirected within those industries. In other words, once a firm chooses \( A_{if} \) and \( B_{if} \), the realization of innovations will take place on a random product within industry \( i \).

There is a measure of potential entrants in the economy. They innovate at flow rate \( a_0 \), paying a flow cost of \( \chi a_0 \) in terms of labor. Successful entrants randomly innovate in one product line and replace the existing incumbent. They also gain working knowledge in that particular industry.

While some firms are gaining new product lines, on the flip side, other firms are losing those product lines. Exit will take place once a firm loses all of its product lines to other incumbents or outside entrants. We will denote the endogenous poisson rate of creative destruction by \( \tau \).

\textbf{Cross-Industry Spillover from Basic Research} Basic research features an additional element of uncertainty arising from random spillovers into other industries. When a firm successfully innovates through basic research, the resulting new fundamental knowledge will be applied first by that firm to increment the productivity of a random product in the target industry.

The characteristic feature of basic research that we wish to capture is that it often has applications in many fields other than the one it was originally intended for (Nelson, 1959). Therefore, we will assume that when a basic innovation occurs, it applies with probability one to the target industry, and with probability \( p \in (0, 1) \), it generates an additional basic

\(^{14}\)It also simplifies the analysis by making the problem proportional to the number of product lines.
innovation in another industry determined by nature at random. Thus \( p \) is our measure of the intensity of cross-industry spillovers. Let \( 1_{i,i'} \) be an indicator function that takes a value of one if a basic innovation in industry \( i \) has an application in industry \( i' \) and zero otherwise. Then the unconditional probabilities satisfy

\[
\Pr[1_{i,i'} = 1] = \begin{cases} \frac{p}{M-1} & \text{if } i' \neq i \\ 1 & \text{if } i' = i \end{cases}
\] (14)

The spillover innovation in industry \( i' \) will be of step size \( \eta \) as well, but will not generate its own spillovers. This new innovation will be utilized by the same firm \( f \) if it has working knowledge in \( i' \). Otherwise the production potential of this innovation will be used by the next inventor in that product line.\(^{15}\)

When a firm generates basic knowledge, it can turn this into an immediate application only in the sectors in which it has working knowledge. Nelson (1959) observes that in order to capture the full return from new basic scientific knowledge in industries where a firm is not present but the knowledge could have an application, the innovating firm must first patent and then license or sell the innovation to other firms in those industries. However, the applications of significant scientific advances are often not immediate and firms can turn them into patentable applications mostly in their own industries due to their expertise in the field.

Let \( m_f \) denote the number of industries in which firm \( f \) has working knowledge. Then the probability of a utilized spillover for firm \( f \) is

\[
\rho_{m_f} = \frac{p(m_f - 1)}{M-1} \in [0, 1)
\]

Note that this structure highlights the well-known appropriability problem of basic research. There is a significant chance that the new basic knowledge will be relevant to multiple industries, but it is not always clear that a firm will be in a position to exploit all of these avenues of production and patenting. However, firms operating in more industries will have a greater probability of being able to directly utilize all facets of a basic innovation. As Nelson puts it, firms that have fingers in many pies have a higher probability of utilizing the results of basic research. A broad technological base increases the probability of benefiting from successful basic research.

As a result, if we summarize the state of a firm by \( n_\eta \) high markup product lines and \( n_\lambda \)

\(^{15}\)Firms investing into basic research face appropriability problems because applications from this type of research are often not immediate, and firms are only able to transform them into patentable applications in their own industries. The reader is referred to the discussion of Nelson’s hypothesis (1959) in the introduction.
low markup product lines, the law of motion for this state is given according to

\[
(n_\eta(t + \Delta t), n_\lambda(t + \Delta t)) = \begin{cases} 
(n_\eta(t) + 1, n_\lambda(t)) & \text{with probability} \quad (1 - \rho_m) nb_m \Delta t \\
(n_\eta(t) + 2, n_\lambda(t)) & \text{with probability} \quad \rho_m nb_m \Delta t \\
(n_\eta(t) + 1, n_\lambda(t)) & \text{with probability} \quad \alpha n a_m \Delta t \\
(n_\eta(t) + 1, n_\lambda(t) + 1) & \text{with probability} \quad (1 - \alpha) n a_m \Delta t \\
(n_\eta(t) - 1, n_\lambda(t)) & \text{with probability} \quad n_\eta \tau \Delta t \\
(n_\eta(t), n_\lambda(t) - 1) & \text{with probability} \quad n_\lambda \tau \Delta t \\
(n_\eta(t), n_\lambda(t)) & \text{with probability} \quad 1 - n (b_m + a_m + \tau) \Delta t
\end{cases}
\]

The intuition here is as follows. With probability \((1 - \rho_m) nb_m \Delta t\) a firm with \(n\) product lines will come up with a basic innovation that does not have a cross-industry spillover, which increases the number of hot product lines by one. With probability \(\rho_m nb_m \Delta t\) it will receive a basic innovation with a cross-industry spillover, and hence will gain two hot product lines. The firm will get a successful applied innovation that builds on a hot product line with probability \(\alpha n a_m \Delta t\) or on a cold product line with probability \((1 - \alpha) n a_m \Delta t\). The firm will lose a hot product line with probability \(n_\eta \tau \Delta t\) or a cold product line with probability \(n_\lambda \tau \Delta t\). Otherwise, the firm’s portfolio of products will not change.

**Public Basic Research** In our model, the academic sector will be the other source of basic knowledge creation. One of the main tasks of the public research labs in an economy is to produce the necessary basic scientific knowledge that will be part of the engine for subsequent applied innovations and growth. We assume that the public research sector consists of a measure \(U\) of research labs per industry. The government raises a total tax revenue of \(\hat{T}\) and allocates this across \(MU\) research labs equally. Each lab receives the same transfer \(\hat{T}_u = \hat{T}/(MU)\) from the government to finance its research.

We assume that each single public research lab generates a flow rate of \(u\) by hiring \(h_u\) researchers with the same basic research technology as a one product firm in (13), so that \(u = (2h_u)^{1/2}\). This specification implies that the government can affect the basic knowledge pool in the economy through the amount of funds \(\hat{T}_u\) allocated to the academic sector. The flow rate of basic innovation from the academic sector will satisfy

\[
u = \left(\frac{2\hat{T}_u}{w}\right)^{1/2}
\]

where \(u\) is the academic basic innovation flow per lab. In this economy, \(\hat{T}\) is a policy lever controlled by the policymaker. As with private firms, each basic innovation generated by the

\footnote{In reality, public research labs may have a different research technology than private labs. However, obtaining data on both the inputs and outputs of individual public labs is difficult. The separate estimation of public and private innovation production functions is left for future research.}
academic sector applies to industry $i$ and a random product line $j$ and makes that product line hot. However, this innovation by public labs will turn into output only upon a subsequent private applied innovation. In addition to $i$, the same basic knowledge will contribute to the basic knowledge pool in another industry $i' \neq i$ and line $j'$ with probability $p \in (0, 1)$. Note that the equilibrium fraction of hot product lines $\alpha$ will be determined by the aggregate rates of public ($u$) and private ($\bar{b}$) basic research as well as the cooldown rate ($\zeta$).

**Remark** It is important to note that we follow the empirical Ivory Tower aspect of basic research and assume that innovation done by public labs are turned into consumer products only upon subsequent innovation by private firms. The lag between the creation of publicly funded innovations and actual goods production is empirically shown in a large literature Trajtenberg et al. (1992), Rosenberg and Nelson (1994), Henderson, Jaffe and Trajtenberg (1998) and more recently Evans (2010) and Bikard (2011). This important issue is generally overlooked in the theoretical growth literature. Inclusion of this feature generates some new interesting dynamics, such as the importance of involvement of the private sector in basic research.

**Dynamic Equilibrium** To further simplify things, we assume that at each instant every firm receives a random draw regarding the number of industries in which they have working knowledge $m$. These values are i.i.d. and are drawn from a distribution $\mathcal{M}$ that has support \{1, ..., $M$\}. We can now solve for the *Symmetric Steady State Markov Perfect Equilibrium* (SSSMPE) of this economy where strategies are a function of the payoff relevant variable $m$.

**Definition 2 (Symmetric Steady State Markov Perfect Equilibrium)** Given the initial condition for the quality index $Q_0 > 0$, a SSSME is defined as optimal choices $(g_{ij}^*, p_{ij}^*, a_m^*, b_m^*)$ of a firm that has working knowledge in $m$ industries, entrants’ optimal choice $a_0^*$, aggregate creative destruction rate $\tau^*$, midstream and upstream producers’ optimal choices $Y_i^*$ and $Z^*$ taking the price $P_i^*$ as given, the normalized wage $\bar{w}^*$ that clears the labor market, the growth rate $g^*$ and the fraction of high markup product lines $\alpha^*$ that are consistent with the firms’ innovation decisions, the quality index $Q^*$ is consistent with the initial condition $Q_0$ and the equilibrium growth rate $g^*$ and the interest rate $r^*$ that satisfies the household’s maximization problem.

We first focus on the household’s problem. The household’s maximization in (1) will deliver the standard Euler equation

$$r^* = \delta + g^*.$$  
(16)
Denote the flow rate of innovations by outside entrants by \( a_0 \). An incumbent firm’s choice of innovation intensity will depend on the number of industries it has working knowledge in that period. Thus we will have one for each value of \( m \) and will denote them by \( b_m \) and \( a_m \) for basic and applied innovation, respectively. Recall that each basic innovation by an \( m \)-firm will generate a utilized spillover with probability \( \rho_m \), so the true arrival rate of basic innovations for such a firm is \( (1 + \rho_m)b_m \).

Since innovations occur randomly and uniformly over product lines, each product line will face replacement by a competitor at a common rate \( \tau \). We call this the aggregate rate of creative destruction, which is comprised of innovation flows from entrants and incumbents

\[
\tau = \mathbb{E}_m (1 + \rho_m)b_m + \mathbb{E}_m a_m + a_0.
\]

In the above equation, the expectation is taken using the distribution over firm working knowledge.

**Free Entry**  
Entrants in the economy chose the flow rate of an innovation \( a_0 \) by paying a linear flow labor cost of research with marginal cost \( \chi > 0 \) in terms of labor units. Let us denote the entrant’s value by \( V_0 \) and a firm’s value of a firm with \( n_\eta \) high and \( n_\lambda \) low markup product lines and a working knowledge in \( m \) industries by \( V_{m,n_\eta,n_\lambda} \). Then an entrant’s maximization problem is

\[
V_0 = \max_{a_0} \{ a_0 \Delta t \cdot \mathbb{E}_m [\alpha V_{m,1,0} + (1 - \alpha)V_{m,0,1}] - \chi \cdot a_0 \Delta t \}.
\] (17)

Note that the outcome of the research investment and the working knowledge \( m \) will be realized after \( \Delta t \), hence the entrant forms its expectation over \( m \) and the markup \( s \in \{\eta, \lambda\} \). With probability \( \alpha \), the entrant will land on a high markup product line \( \eta \) and with the remaining probability on a low markup product line \( \lambda \). Thus any equilibrium with a positive rate of entry must satisfy \( \mathbb{E}_m [\alpha V_{m,1,0} + (1 - \alpha)V_{m,0,1}] = \chi \). Next we focus on the value function maximization of an incumbent firm that is of type \( m \).

**Firm Value Function**  
As we have assumed complete symmetry across industries, the exact industry in which a particular product lies is not important to the firm. The only necessary state variables are the total number of products that the firm owns \( n \), which determines the cost structure, and the number of industries in which it has working knowledge \( m \), which determines the probability of spillover.
Since there is no heterogeneity in step sizes, the flow profit is the same across all product lines. The value function of a firm that has \( n \) products and working knowledge in \( m \) industries is given by

\[
V_{m,n_\eta,n_\lambda} = \max_{a_m,b_m} \left\{ \begin{array}{c} \Delta t n_\eta \pi_\eta - \Delta t n_\eta \pi_\eta - \Delta t n_\eta \pi_\eta - \Delta t n_\eta \pi_\eta - \Delta t n_\eta \pi_\eta - \Delta t n_\eta \pi_\eta - \Delta t n_\eta \pi_\eta - \Delta t n_\eta \pi_\eta \\
+ \Delta t \, nb_m \rho_m \mathbb{E}_m V_{m,n_\eta + 1, n_\lambda} + \Delta t \, nb_m (1 - \rho_m) \mathbb{E}_m V_{m,n_\eta + 1, n_\lambda} + \Delta t \, na_m \mathbb{E}_m [\alpha V_{m,n_\eta + 1, n_\lambda} + (1 - \alpha) V_{m,n_\eta + 1, n_\lambda}] + \Delta t \, \tau [n_\eta \mathbb{E}_m V_{m,n_\lambda - 1, n_\lambda} + n_\lambda \mathbb{E}_m V_{m,n_\lambda - 1, n_\lambda}] + (1 - \Delta t (nb_m + na_m + n \tau - g)) \mathbb{E}_m V_{m,n_\eta + 1, n_\lambda} \end{array} \right\}.
\]

The terms here are fairly intuitive. There are flow terms from production profit and research costs, as well as probabilistic terms from successful innovation, be it applied, basic without spillover, or basic with spillover. In addition, there is a term for the loss of product lines due to creative destruction. Importantly, these terms is proportional to \( n_\eta \) and \( n_\lambda \). This leads to the following proposition.

**Proposition 2** The value function in (18) is linear in the number of product lines such that

\[
\mathbb{E}_m V_{m,n_\eta,n_\lambda} = n_\eta \mathbb{E}_m V_{m,n_\eta} + n_\lambda \mathbb{E}_m V_{m,n_\lambda}.
\]

where

\[
\mathbb{E}_m V_{m,n_\eta} = \frac{(\pi_\eta^* - \pi_\lambda^*) (1 - \alpha)}{\delta + \tau^*} + w^* \chi, \quad \text{and} \quad \mathbb{E}_m V_{m,n_\lambda} = \frac{1}{\delta + \tau^*} \left[ \pi_\lambda^* + \frac{\xi \Phi [\mathbb{E}_m V_{m,n_\eta}]^2 + w^* \chi^2}{2w^*} \right].
\]

Moreover the optimal research decisions are simply

\[
a_m^* = \chi, \quad \text{and} \quad b_m^* = \xi (1 + \rho_m) \left[ \frac{(\pi_\eta^* - \pi_\lambda^*) (1 - \alpha^*)}{(\delta + \tau^*) w^*} + \chi \right].
\]

**Proof.** See Appendix A. ■

This proposition establishes our first result. The fact that technologies scale up linearly in the number of product lines allow us to express the expected value function in linear form. The research incentives are determined by the value of next innovation, which corresponds to the value of a new product line in this economy. These values crucially depend on the expected present discount value of future profit streams and the option value of , Basic research incentives scale up as firms operate in more industries. The intuition for this result is very transparent. As firms operate (have working knowledge) in more industries, the likelihood of utilizing a cross-industry spillover increases. As a result, as Nelson (1959) stated, firms that invest in
more basic research are the ones that have fingers in many pies, i.e., those that operate in more industries.

Now we continue with the solution of the benchmark economy. Let us define the average spillover multiplier

$$\Phi \equiv E_m \left[(1 + \rho_m)^2\right].$$

This can be taken as a parameter, since it summarizes all relevant information about the probability of spillover and the distribution of working knowledge. Note that the free-entry condition in (17) delivers the following equality for the expected values

$$\alpha^*E_mV_{m,\eta} + (1 - \alpha^*)E_mV_{m,\lambda} = w^*\chi. \quad (21)$$

Note that we have three equalities ((19) for \(\eta\) and \(\lambda\) and (21)) and three endogenous values \((E_mV_{m,\eta}, E_mV_{m,\lambda}, \text{and } \tau)\) to solve for. The following proposition summarizes the rest of the equilibrium.

**Proposition 3** Consider the above economy when \(\xi > 0\). For any given \(\alpha\), the equilibrium creative destruction is implicitly determined by

$$\sqrt{\left[\left(\delta + \tau^* - \frac{\chi}{2}\right)\chi - \frac{\alpha^*\pi^*_\eta + (1 - \alpha^*)\pi^*_\lambda}{w^*}\right]} \frac{2}{\Phi^*} = \frac{(\pi^*_\eta - \pi^*_\lambda)(1 - \alpha^*)}{w^*(\delta + \tau^*)} + \chi. \quad (22)$$

Moreover the equilibrium growth rate is

$$g^* = \left\{ \begin{array}{l} \frac{E_m(1 + \rho_m)b^*_m \ln (1 + \eta)}{+ (a^* + a^*_0)E_s \ln (1 + s)} \end{array} \right. \quad (23)$$

**Proof.** See Appendix A. ■

The next equilibrium variable to be determined is the share of hot product lines in this economy. Let us denote the average private basic research spending per product line by \(\bar{b} \equiv E_mb_m\). The share of hot product lines is found using the flow equation that equates the measure of product lines that turn from cold to hot \((1 - \alpha) \left(\delta + \bar{b}\right)\) to the measure of product lines that turn from hot to cold \(\alpha \zeta\). This equality delivers the equilibrium measure of hot product lines

$$\alpha^* = \frac{\sqrt{\zeta^* + \bar{b}^*}}{\zeta + \sqrt{\zeta^* + \bar{b}^*}} \quad (24)$$

Note that a government can increase the equilibrium measure of basic knowledge by hiring more researchers through taxation \(T\). This will be the key mechanism in our main model, through which government can affect growth indirectly.
Finally we focus on the labor allocations. Recall that $a^*_m$, $b^*_m$ and $\tau^*$ are characterized in terms of the parameters of the model. Hence we get

$$a^*_0 = \tau^* - E_m(1 + \rho_m)b^*_m - E_m a^*_m$$

(25)

and therefore the labor allocation is found as

$$L^{P*} = 1 - \left[ L^G + \frac{E a^2_m + E b^2_m}{2} + a^*_0 \right].$$

(26)

We conclude the section on equilibrium by summarizing the solution of a SSMPE equilibrium. It is composed of $(y^*_j, l_j, w^*, Z^*, a^*_m, b^*_m, a^*_0, \tau^*, \alpha^*, r^*, g^*, L^{P*}, Q^*)$ such that $y^*_j$, $l_j$, $w^*$, $Z^*$ are equal to (6), (7), (8) and (9), respectively; $a^*_m$ and $b^*_m$ are equal to (20), $\tau^*$ solves (22), $a^*_0$ and $\alpha^*$ are as in (25) and (24), growth rate is equal to (23), interest rate is expressed in the Euler equation (16), $L^{P*}$ is found as in (26) and $Q^*_t = Q_0 \exp(g^*t)$.

**Welfare** From the household’s utility specification, welfare can be expressed purely as a function of the initial level of production and the growth rate,$^{17}$

$$W = \frac{\ln Z_0}{\delta} + \frac{g}{\delta^2}$$

The social planner chooses per-product-line innovation rates of incumbents ($a$ and $b$) and entrants ($a_0$). The planner is assumed to having working knowledge in every industry, and therefore utilizes all spillovers. This effectively means we set $\Phi^{SP} = (1 + p)^2$. Note that due to concavity in the R&D technology, the chosen rates of applied innovation $a$ and basic innovation $b$ will be the same across product lines.

This is the first paper to introduce the two types of spillovers and public basic research into an endogenous growth framework. Therefore our aim is to transpire the impacts of each of these new elements on the equilibrium productivity process and growth. We will achieve this in our benchmark analysis by focusing on each one of the spillovers at a time. In what follows, we will first consider an CIS economy in which we abstract from public research spending and within-industry spillovers by setting the innovation step sizes equal and consider only cross-industry spillovers. Next we will turn to WIS economy, where we will focus on within-industry spillovers and allow for public basic research and abstract from cross-industry spillovers. The

---

$^{17}$To see this, $W = \int_0^\infty \exp(-\delta t) \ln(C_0 \exp(gt)) \, dt = \frac{\ln Z_0}{\delta} + \frac{g}{\delta^2}$.
following table summarizes the elements of each model.

<table>
<thead>
<tr>
<th></th>
<th>Within-industry Spillover</th>
<th>Cross-industry Spillover</th>
<th>Public Basic Research</th>
<th>Private Basic Research</th>
</tr>
</thead>
<tbody>
<tr>
<td>WIS Economy:</td>
<td>✓ → (η &gt; λ)</td>
<td>× → (p = 0)</td>
<td>✓ → (T, d &gt; 0)</td>
<td>× → (ξ = 0)</td>
</tr>
<tr>
<td>CIS Economy:</td>
<td>× → (η = λ)</td>
<td>✓ → (p &gt; 0)</td>
<td>× → (T, d = 0)</td>
<td>✓ → (ξ &gt; 0)</td>
</tr>
</tbody>
</table>

These subeconomies will not only transpire the main dynamics in closed form, they will also allow us to emphasize the direction of inefficiencies and set the stage for policy discussion. The reader should note that we consider these two subeconomies are for expositional purposes and the general model in Section 3.2 will contain all these elements simultaneously with also more general functional forms.

### 3.1.3 Cross-industry Spillovers (CIS Economy)

In this section, we wish to construct a model that highlights the dynamics of cross-industry spillovers. We first shut down the public side of the economy by assuming that \( \hat{T} = 0 \) and hence \( d = 0 \). Moreover, we eliminate meaningful within-industry spillovers by assuming that \( \eta = \lambda \). This way, we do not need to keep track of the fraction of hot and cold product lines in the economy. There will be applied research undertaken by outside entrants, as well as basic and applied research undertaken by incumbents.

**Proposition 4** *Consider the above CIS economy. Then*

- **the optimal research decisions are given by**
  \[
  b^*_m = (1 + \rho_m)\chi, \quad a^*_m = \chi, \quad \text{and} \quad a^*_0 = \tilde{\eta} \left[ \frac{1}{\chi} + \delta \right] - \frac{(1 + \Phi \xi)}{2} \chi - \delta,
  \]
- **the creative destruction rate**
  \[
  \tau^* = \left( \frac{\lambda}{1 + \lambda} \right) \left( \frac{1}{\chi} + \delta \right) + \frac{(1 + \Phi)\chi}{2} - \delta,
  \]
- **production labor fraction**
  \[
  L^P = \frac{1 + \chi\delta}{1 + \lambda}, \tag{27}
  \]
- **the growth rate is equal to**
  \[
  g^* = \left[ \left( \frac{\lambda}{1 + \lambda} \right) \left( \frac{1}{\chi} + \delta \right) + \frac{(1 + \Phi)\chi}{2} - \delta \right] \ln (1 + \lambda).
  \]

**Proof.** See Appendix A. ■

We now need to impose a parametric assumption to ensure that there is positive entry and that the fraction of workers engaged in production is less than one.
**Assumption 1** \( \left( \frac{\lambda}{1 + \lambda} \right) \left( \frac{\delta}{\chi} + \delta \right) > \frac{\chi(1 + \Phi \xi)}{2} + \delta \)

For any given discount rate \( \delta \) and spillover multiplier \( \Phi \), this assumption intuitively requires that the profits to be generated upon entry are large enough (high \( \lambda \)) and the entry costs \( \chi \) are low enough. This assumption directly assures that the rate of outside entry is strictly positive. In addition, it also implies that \( \chi \delta < \lambda \), which ensures that the fraction of labor devoted to goods production is strictly less than one. This condition also has the virtue of ensuring that a decrease in the cost of entry \( \chi \) and an increase in spillover multiplier \( \Phi \) result in an increase in the aggregate rate of creative destruction \( \tau \) and the growth rate \( g \).

The steps above lead us to the following conclusion.

**Theorem 1** Under Assumption 1, the SSSMPE of the CIS economy exists, is unique, and features positive entry.

**Proof.** See Appendix A. ■

**Theorem 2** The decentralized CIS economy differs from the constrained efficient economy in such a way that it underinvests in research and overproduces.

**Proof.** See Appendix A. ■

Of particular interest is how the efficient levels compare to those of the decentralized equilibrium. The size of the static inefficiency in the initial consumption can be expressed as

\[
C_0^{SP} - C_0^* = Q_0 \left( \frac{\chi \delta}{\ln(1 + \lambda)} - \frac{1 + \chi \delta}{1 + \lambda} \right) \approx Q_0 \left( \frac{\chi \delta - \lambda}{\lambda(1 + \lambda)} \right) < 0
\]

Note that the decentralized equilibrium allocates more workers to the production sector than the efficient level. This is due to the underinvestment in innovation. The size of the inefficiency decreases with higher discount rate. This is due to the fact that the household cares more about future consumption, so innovation plays a larger role in the economy.

When we compare the decentralized growth rate to the efficient level, we get

\[
g^{SP} - g^* = \ln(1 + \lambda) \left( \left( \frac{1}{1 + \lambda} \right) \left( \frac{1 + \delta}{\chi} \right) + \left( \frac{\Phi^{SP} - \Phi}{2} \right) \lambda \right) - \delta
\]

The dynamic inefficiency in the model is increasing in the gap between the spillover multipliers. This will be an important source of inefficiency in the main model because the spillover multiplier will be endogenously determined by the distribution of multi-industry presence of the firms.
3.1.4 Within-industry Spillovers (WIS Economy)

In this second economy, our aim is to highlight the mechanism through which within-industry spillovers function and how government can affect the economy through this mechanism. To highlight this feature, we will assume that basic research is undertaken only by the government, i.e. \( b_m = 0 \), whereas applied research is the domain of private firms. The government in this WIS economy can tax households at the lump-sum rate \( T \) and employ \( L^G \) researchers per industry to generate basic innovation at rate \( d = \sqrt{L^G} \).

We also shut down cross-industry spillover by setting \( p = 0 \), meaning \( \rho_m = 0 \) for all \( m \). However, we will distinguish between hot and cold product lines, denoting by \( \alpha \) the share of ‘hot’ product lines. Note that the static problem is identical to that described in section 3.1.1.

As described earlier, upon successful innovation in a particular product line, that product line becomes hot. So long as the product line remains hot, applied innovation within the same product line will have a bigger impact, generating a higher step size \( \eta > \lambda \). With flow probability \( \zeta \) basic technologies get outdated, meaning that hot product lines return to being cool product lines. Hence, we call \( \zeta \) the cool down rate. If an applied innovation by a private firm lands on a cool product line, the step size is the usual applied step size \( \lambda \). This specification captures the complementarities between basic and applied innovations. We will denote the value of hot and cold product lines by \( V_\eta \) and \( V_\lambda \), respectively.

**Proposition 5** Consider the above WIS economy. Then

- the optimal research decision is given by
  \[
  a^*_m = \chi \quad \text{and} \quad a^*_0 = \left[ \alpha^* \eta + (1 - \alpha^*) \bar{\lambda} \right] \left( \frac{1 - L^G}{\chi} + \delta \right) - \frac{\chi}{2} - \delta,
  \]
- the rate of private innovation is
  \[
  \tau = \left[ \alpha^* \eta + (1 - \alpha) \bar{\lambda} \right] \left( \frac{1 - L^G}{\chi} + \delta \right) + \frac{\chi}{2} - \delta,
  \]
  \[
  \tag{28}
  \]
- the fraction of production workers is simply
  \[
  L^P = (1 - L^G + \delta \chi) (1 - \bar{s}),
  \]
  \[
  \tag{29}
  \]
- the fraction of hot product lines is
  \[
  \alpha^* = \frac{\sqrt{L^G}}{\zeta + \sqrt{L^G}},
  \]
  \[
  \tag{30}
  \]
and the growth rate is

\[ g^* = \left( \alpha^* \left( \bar{\eta} - \bar{\lambda} \right) + \bar{\lambda} \left( \frac{1-L_G}{\bar{x}} + \delta \right) \right) \left[ \alpha^* \ln \left( \frac{1 + \eta}{1 + \lambda} \right) + \ln (1 + \lambda) \right] \] (31)

**Proof.** See Appendix A. ■

**Remark 1** Although the government does not directly contribute to growth, it increases the knowledge stock in the economy and allows firms to utilize the knowledge pool. This enhances growth (31).

This proposition highlights important effects of both basic research and public investments. First, public investment in basic research has a crowding out effect as can be seen in (29). As the government allocates more labor public research, this pushes up the wage rate through the labor market condition and hence lowers the private value of innovation. As a result, the aggregate rate of private innovation decreases, as seen in (28). Second, hiring more public researchers increases the fraction of hot product lines as in (30). This increases the value of private applied innovation through its complementarity with basic research. Hence we see a positive complementarity effect in (28). This expression highlights the economic incentives for the government to allocate public funds for basic research.

**Assumption 2** \( \left( \frac{\Lambda}{1+x} \right) \left( \frac{1-L_G}{\bar{x}} + \delta \right) > \frac{\bar{\eta}}{\bar{x}} + \delta \).

Next we state the main theorems of the WIS economy.

**Theorem 3** Under Assumption 3.1.4, the SSSMPE of the WIS economy exists, is unique, and features positive entry.

**Proof.** See Appendix A. ■

**Theorem 4** Allocating public funds for basic research is growth enhancing when \( L_G \) is zero. When \( L_G \) becomes sufficiently large, the crowding out effect dominates, leading to a decrease in the growth rate.

**Proof.** See Appendix A. ■

The welfare analysis can be undertaken as was illustrated in the CIS economy. In this setting, there will be static inefficiency arising from heterogeneity in step size. This will be a function of the fraction of hot product lines \( \alpha \). Still, the efficient production level is achieved by allocating an equal amount of labor to each product line. In the main model, we will quantitatively investigate the effects of government policy in great detail. Therefore, we postpone this discussion until later to save space.
3.2 General Model

We will now generalize the above baseline model in a number of dimensions. The previous sections served primarily as an exposition of the key mechanisms in the model. In taking the model to the data, we introduce more flexible functional forms for goods production and innovation technologies. By doing so we can both produce a realistic fit of the data and extract more information on the structural underpinnings of the environment. Below is a table documenting these changes.

<table>
<thead>
<tr>
<th>Name</th>
<th>Variable</th>
<th>Baseline Form</th>
<th>General Form</th>
</tr>
</thead>
<tbody>
<tr>
<td>Per-period Utility</td>
<td>$U$</td>
<td>$\log (C(t))$</td>
<td>$\frac{C(t)^{\sigma - 1} - 1}{\sigma - 1}$</td>
</tr>
<tr>
<td>Midstream Good</td>
<td>$Y_i$</td>
<td>$\exp \left[ \int_0^1 \ln (y_{ij}(t)) , dj \right]$</td>
<td>$\int_0^1 \frac{c_{ij} - 1}{d_{ij}} , dj$</td>
</tr>
<tr>
<td>Incumbent Applied Research</td>
<td>$A$</td>
<td>$2^{\frac{1}{2}} n^{\frac{1}{2}} H_a^{\frac{1}{2}}$</td>
<td>$\beta_a n^{1-\nu} H_a^\nu$</td>
</tr>
<tr>
<td>Basic Research</td>
<td>$B$</td>
<td>$\xi^{\frac{1}{2}} 2^{\frac{1}{2}} n^{\frac{1}{2}} H_b^{\frac{1}{2}}$</td>
<td>$\beta_b n^{1-\nu} (H_b - n\phi)^\nu$</td>
</tr>
<tr>
<td>Multi-industry Presence</td>
<td>$x$</td>
<td>Exogenous i.i.d</td>
<td>startup buy-outs</td>
</tr>
<tr>
<td>Outside Entrants</td>
<td>$a_0$</td>
<td>$h_0/\chi$</td>
<td>$\beta_a h_a^\nu$</td>
</tr>
<tr>
<td>Academic Research</td>
<td>$u$</td>
<td>$2^{\frac{1}{2}} h_u^{\frac{1}{2}}$</td>
<td>$\beta_u (h_u - \overline{\phi})^\nu$</td>
</tr>
</tbody>
</table>

In the case of the household’s problem, the midstream production technology, and applied innovation, the above table should be self explanatory. We provide a more thorough explanation for the cases of basic research, working knowledge expansion, outside entrants, and academic research below. We will then focus on the equilibrium of the model.

3.2.1 Generalizations

Basic Innovation Technology  We generalize the basic research technology as with the applied research technology, adding scale and curvature parameters. Additionally, we introduce a fixed cost of doing basic research. This will allow us to capture the fact that some firms simply do not undertake basic research. At each instant, a firm with $n$ product lines draws a fixed labor cost of doing basic research $n\phi \geq 0$, where $\phi$ is distributed according to the distribution $B(\cdot)$.

To generate new innovations, firms with $n$ product lines hire $H_b$ researchers to produce a flow rate of applied innovation $A$ according to the following production function

$$B = \beta_b n^{1-\nu} (H_b - n\phi)^\nu$$
where \( \nu \in (0, 1) \) and \( \beta_b > 0 \). This production function leads to the following cost function

\[
H_b(b, n) = nh_b(b) = \begin{cases} 
  n \left( \frac{b}{\beta_b} \right)^{\frac{1}{\nu}} + \phi & \text{if } b > 0, \\
  0 & \text{if } b = 0.
\end{cases}
\]

**Entry & Exit**  We move away from the linear entry cost of the baseline model and introduce curvature \( \nu \geq 1 \) to the cost function. The research technology for a single outside entrant is assumed to be the same as that of applied innovation for a firm with a single product line. Thus if an outside entrant hires \( h_0 \) researchers, it produces a flow probability of entry of

\[
a_0 = \beta_a h_0^\nu.
\]

As with applied innovation, this leads to a cost function of the form

\[
h_0(a_0) = \left( \frac{a_0}{\beta_a} \right)^{\frac{1}{\nu}}.
\]

There is a mass \( M_0 \) of outside entrants per industry. Varying this parameter will control the relative importance of outside entry in the economy. This will imply that creative destruction arising from new entrants will be equal to \( M_0 a_0 \).

In our model, there will be both endogenous and exogenous channels for firm exit. First, a firm that loses all of its product lines to other competitors will have a value of zero and thus will exit. Second, each firm has an exogenous death rate \( \kappa > 0 \). When this occurs, the firm sells all of its product lines to random firms at a “fire sale” price of \( \iota \). On the flip side, firms will receive a buyout option with a probability that is proportional to their number of products.

**Industry Expansion (Start-up Buy-outs)**  In the baseline model, we took the working knowledge of the firms (\( m \)) as exogenously given with an instantaneous iid draw. We now endogenize \( m \) by introducing buy-out options of new entrants. The economy features \( M_0 a_0 \) flow of entry at any instant. We will assume that \( \zeta \) fraction of the new entrants will meet randomly an incumbent firm which are measure \( F \). An incumbent will have a flow rate of a buyout option

\[
x = \zeta M_0 a_0 / F.
\]

Clearly this new company will be from a new industry with probability \( (1 - m/M) \) or from an industry that already exist in incumbent’s portfolio with probability \( m/M \). Our goal is to

\[18\]The exact value of this price will not play any role for the equilibrium determination.
keeps the M&A margin as tractable as possible and we will achieve this by assuming that the M&A price that the incumbent firm has to pay is equal to the full surplus of the new merger such that

\[ p_m = \beta_{m+1} + \mathbb{E}_{\tilde{q},s} (\tilde{q}_t + \Delta t + \tilde{s}) \quad \text{and} \quad p'_m = \beta_m + \mathbb{E}_{\tilde{q},s} (\tilde{q}_t + \Delta t + \tilde{s}). \]

In this expression \( p_m \) and \( p'_m \) stand for the prices paid to a startup from a new industry and from an existing industry, respectively.

**Public Research Sector** The public research sector employs the same technology as the private sector when doing basic research. That is, for each lab there is a fixed labor cost \( \bar{\phi} = \mathbb{E}[\phi] \) of operation and a variable cost in terms of researchers. Thus in the case that the lab is operating, it can use \( h_u \) units of labor to achieve a flow rate of innovation

\[ u = \beta_b \cdot (h_u - \bar{\phi})^\nu. \]

This can be thought of as assuming that each academic lab gets a separate fixed cost draw but chooses to operate regardless of its value. Let the mass of public research labs continue to be denoted by \( U \). As with outside entrants, this parameter will control the importance of academic research in the aggregate economy and will imply that the average flow rate of public innovation is \( U u \).

**Labor Market** We close the description of the generalized environment by writing explicitly the labor market clearing condition. The economy has a total labor supply of \( M \). In industry \( i \), \( L^P_i = \int_0^1 \ell_{ij} dj \) workers are employed in production, \( L^R_i = \int_0^1 \left( h_{aij} + h_{bij} \right) dj \) workers are employed in research by incumbents, \( L^0_i = M^0 \cdot (a_0/\beta_a)^{1/2} \) workers are employed by outside entrants, and \( L^U_i = Uh_u \) workers are employed in public research labs. Therefore, the labor market clearing condition can be summarized as

\[ M \geq \sum_{i=1}^M \left[ L^P_i + L^R_i + L^0_i + L^U_i \right] \quad (32) \]

**Quality Imitation** In this model, firm size is determined by the number of product lines and their productivities. In our current economy, aggregate variables such as the final output and wage rate are well-behaved, however a stationary productivity distribution cannot exist. (See Acemoglu and Cao, 2010 for further details). To achieve the latter with a minimal restriction, we follow Acemoglu and Cao (2010) and Luttmer (2007) and introduce “imitation” into our model.

Products whose productivity falls too low relative to the mean productivity in the economy \( Q \) benefit from the aggregate level of knowledge and are kept at a minimal level of productivity
where \( \hat{q} > 0 \). This imitation process continues to occur until that particular product line gets innovated upon and improved above \( \hat{q}Q \). Here the idea is that for the firms that fall too far below the average, imitation keeps a productivity level above a certain fraction of the average productivity in the economy. As a result, the productivity of levels are always above an endogenously determined threshold, \( q_{ij} \geq \hat{q}Q \).

This concludes the description of the model. The next section characterizes the equilibrium of this economy.

### 3.2.2 Equilibrium

In this generalized model, three variables affect the payoff of the firm: the number of product lines \( n \), the number of industries \( m \), and the relative quality \( \hat{q}_{ij} \equiv q_{ij}/Q_i \) of its product lines, which is the absolute quality in line \( j \) normalized by the average quality \( Q_i \) in industry \( i \). We will focus on equilibria that are symmetric across industries, meaning we can drop dependence on \( i \) in equilibrium variables. Thus, each incumbent firm is characterized by its state \( k \equiv (\hat{q}, n, m) \). Throughout, we focus on Symmetric Steady State Markov Perfect Equilibria (SSSMPE) where all allocations across the industries are the same and the strategies are only a function of the payoff relevant state variable \( k \). This will allow us to drop the industry and product line identities \((i, j)\). Let us denote the decision of a firm with state \( k \) by \( \xi_k \equiv \{p(\hat{q}), y(\hat{q}), A(n, m), B(n, m), E(m)\}_{\hat{q} \in \hat{q}} \). Let \( \Gamma(\hat{q}, n, m) \) denote share of product lines where the latest normalized productivity level is \( \hat{q} \), and owned by a firm that operates in \( m \) industries and \( n \) product lines so that

\[
\sum_{m=1}^{M} \sum_{n=1}^{\infty} \int_{\hat{q}}^{\infty} d\Gamma(\hat{q}, n, m) = 1 \text{ for all } t.
\]

Then a symmetric steady state allocation in this economy can be defined as follows.

Let \( \hat{T} \) be the policy. A **symmetric steady state allocation** for this economy is defined as a set of incumbent decisions \( \xi_k \), innovation decisions by outsiders \( a^0 \), public research lab innovation rate \( u \), the wage rate \( w \), and the distributions of product lines \( \Gamma(k) \).

A SSSMPE is a symmetric steady state allocation where \( \xi_k^* \) maximizes the net present discounted value of the incumbents, \( a^{0*} \) maximizes the net present discounted value of the entrants, academic basic research \( u^* \) is chosen according to the transfer \( \hat{T} \), downstream, midstream and upstream producers maximize their profits, and labor and goods market clear.

As in the definition, we will use “*” to denote the equilibrium values. Moreover our focus will be on steady state in which the equilibrium aggregate variables \((Z^*, Y^*, w^*, Q^*)\) grow at a constant rate \( g^* \).
Households and Production The standard maximization of the household leads to the usual Euler equations for CRRA preferences
\[
\frac{\dot{Z}}{Z} = g = \frac{r - \delta}{\sigma}
\] (33)

Throughout, we will characterize upstream firms’ production technologies by their relative quality \( \hat{q} = q/Q \), where \( Q \) is the mean quality aggregate in the economy
\[
Q = \left[ \int_0^1 q_j^{\epsilon - 1} dj \right]^{\frac{1}{\epsilon - 1}}.
\] (34)
This definition is analogous to that given in the baseline model but using a general elasticity. Note that this will satisfy \( \int_0^1 \hat{q}_j^{\epsilon - 1} dj = 1 \). Additionally, we will normalize growing variables by the per-industry production level \( Z/M \). Such variables are denoted with a “⇠”. With this, we present a proposition characterizing the symmetric equilibrium in the static production economy.

**Proposition 6** The profits and labor for a product line with relative quality \( \hat{q} \) are given by
\[
\bar{\pi}(\hat{q}) = \frac{\hat{q}^{\epsilon - 1}}{\epsilon} \quad \text{and} \quad \ell(\hat{q}) = \hat{q}^{\epsilon - 1} L^P.
\] (35)

Furthermore, the equilibrium wage is characterized by
\[
\bar{w} = \frac{1}{L^P} \left( \frac{\varepsilon - 1}{\varepsilon} \right).
\] (36)

**Proof.** See Appendix A. ■

This coincides with the efficient allocation. Thus, conditional on some allocation of production labor \( L^P \), there are no static distortions present in this model.

Value Functions and Innovation Rates So far we have solved the production decisions of the firms. In order to find the optimal research decisions, we must solve the value functions of the firms. Note that innovation in our model takes the form of business stealing in the sense that an innovating firm improves and captures a new product line that used to belong to some other firm in the economy. As a result, firms face a competitive threat from the rest of the firms, which will be summarized by the endogenous creative destruction rate \( \tau \).

Define a new step size as
\[
\hat{s} = \bar{q}s \quad \text{where} \quad s \in \{\eta, \lambda\}.
\]
Recall that the state of a firm is summarized by \( (\hat{q}, n, m) \). Each firm takes the the normalized wage rate \( \bar{w} \), the creative destruction rate \( \tau \), the interest rate \( r \), and the growth rate \( g \) as given.

As a result, the continuous time Hamilton-Jacobi-Bellman equation can be written as
The value function takes a similar form to that presented in the baseline model. There are instantaneous payoffs from profits and the cost of doing research. In addition, the continuation value depends upon whether the firm successfully innovated, lost a product, expanded, or exited, or was offered a buyout. In the case of the applied innovation, due to within-industry spillovers, the quality is incremented by $1 + \eta$ with probability $\alpha$ and $1 + \lambda$ otherwise. With basic innovation, there is some probability $\rho_m$ of receiving an additional innovation from the utilization of cross-industry spillovers. Note that the firm’s productivity portfolio depreciates due to growth in the quality aggregate.

We will show below that the value of a firm takes a very tractable form. First, we solve for the present production value of a successful innovation, then we use this to solve an auxiliary value function and find the optimal innovation rates. The following lemma gives the production value of a product line in closed-form.

**Lemma 2** The production value of a product line is $Z \beta(\hat{q})$, where the production value $\beta(\hat{q})$ is characterized by the following differential equation

$$(r + \tau + \kappa - g) \beta(\hat{q}) + \frac{\partial \beta(\hat{q})}{\partial \hat{q}} \hat{q} g - \frac{\hat{q}^{\varepsilon-1}}{\varepsilon} = 0$$

which has the following exact solution

$$\beta(\hat{q}) = \frac{[\hat{q}^{\varepsilon-1} - \hat{q}^{\varepsilon-1}] / \varepsilon}{r + \tau + \kappa + g (\varepsilon - 2)} + \frac{\hat{q}^{\varepsilon-1} / \varepsilon}{r + \tau + \kappa + g},$$

for $\hat{q} \geq \hat{q}_2$.

**Proof.** See Appendix A. ■
Note that the production value of a product line which is the discounted sum of future profit flow is increasing in the instantaneous profit $\hat{q}\varepsilon^{-1}/\varepsilon$ and decreasing in the effective discount rate. In addition to the interest rate, the effective discount rate contains the rate of creative destruction $\tau$ and the exogenous destruction rate $\kappa$ due to an increased frequency of product line losses and $g$ since the relative productivity of the firm $\hat{q} = q/Q$ deteriorates continuously.

Now we are ready to characterize firm innovation rates. Of prime importance will be the expected return from successful innovation. Define the expected production value of a newly innovated product with step size of $\hat{\lambda} = \hat{q}\lambda$ as

$$\beta_\eta = \mathbb{E}_\eta \beta(\hat{q} + \hat{\eta}) \quad \text{and} \quad \beta_\lambda = \mathbb{E}_\eta \beta(\hat{q} + \hat{\lambda}).$$

In the following proposition, the firm value is shown to be linear in the number of products (but not in the level of working knowledge $m$) and linearly separable between production value and the option value of research, which is a function of the firm’s level of working knowledge.

**Proposition 7** The the value function in (37) can be expressed as a sum of production value and an option value of being in $m$ industries $\beta_m$

$$V(\hat{q}, n, m) = \frac{Z}{M} \left[ \sum_{\hat{q} \in \hat{q}} \beta(\hat{q}) + n\beta_m \right]$$

where the option value is determined by

$$(r - g) \beta_m = \begin{cases} -\tilde{w} h(\alpha_m) + h_b(b_m) + 1_{(\phi < \Phi_m)} \phi \\ +a_m (\alpha \beta_\eta + (1 - \alpha)\beta_\lambda + \beta_m) \\ +b_m (1 + \rho_m) (\beta_\eta + \beta_m) \\ +x (\beta_{m+1} - \beta_m) - \tau \beta_m + \kappa \mathbb{E}_\eta \beta(\hat{q}) \end{cases}$$

Moreover, the optimal research and expansion decisions would be the solutions of the following equations

$$a_m = h^{-1}_a \left( \frac{\alpha \beta_\eta + (1 - \alpha)\beta_\lambda + \beta_m}{\tilde{w}} \right)$$

$$b_m = \begin{cases} h^{-1}_b \left( \frac{(1 + \rho_m) (\beta_\eta + \beta_m)}{\tilde{w}} \right) & \text{if } \phi < \Phi_m \\ 0 & \text{if } \phi \geq \Phi_m \end{cases}$$

$$\Phi_m = b_m (1 + \rho_m) (\beta_\eta + \beta_m) - \tilde{w} h_b(b_m)$$

**Proof.** See Appendix A. ■

A couple of key observations are in order. First, the value of a firm in (39) is determined by the interaction of three factors: the production value of a single product line ($\beta(\hat{q})$), the option
value \((\beta_m)\), and the firm scale \((n)\). Second, the option value associated with each product line reflects the fact that each additional product line expands the firm’s capacity for research (both basic and applied, \(a_m\) and \(b_m\)).

The first order conditions reflect the incentives associated with each type of research. In (41), the incentive for applied research comes from the interaction between the expected innovation size and the option value associated with gaining a new product line and the marginal cost of innovation. As is critical in our analysis, \(b_m\) in (41) reflects the fact that there may be additional gains from basic research through utilized spillover, the probability of which is increasing with number of industries \((\rho_m)\). Note that \(\phi_m\) in (41) indicates that firms will invest effort in expansion depending upon the additional option value generated from standing in an additional industry. Finally, the fixed cost structure will replicate the observed pattern on the extensive margin of basic research seen in the data. Firms that draw a high fixed cost \((\phi)\) will find it profitable not to invest in basic research.

The labor market condition requires the solution of the invariant distributions over \((n, m)\) and relative quality \(\hat{q}\), which are determined via some involved derivations. To save space, we describe these in detail in Appendix A.

4 Quantitative Analysis

We use the Generalized Method of Moments (GMM) to estimate our model. The next section provides details on the estimation method. Before we proceed, however, we need to specify the functional forms for the research cost functions and the distribution of the fixed cost of basic research \(F(h^b)\). We will assume that:

\[
    h_a(x) = \xi_1^a x^{\xi_2^a}, \quad h_b(x) = \xi_1^b x^{\xi_2^b} \quad \text{and} \quad h_e(x) = \xi_1^e x^{\xi_2^e}
\]

where \(\xi_1^a, \xi_1^b, \xi_1^e > 0\) and \(\xi_2^a, \xi_2^b, \xi_2^e > 1\). We will also assume that the fixed costs are drawn from a lognormal distribution with mean \(\mu\) and variance \(\sigma^2\). As a result, the set of parameters of the model is

\[
    \theta = \{\gamma, \delta, \varepsilon, \eta, \lambda, \rho, \zeta, \mu, \sigma, U, E, \xi_1^a, \xi_2^a, \xi_1^b, \xi_2^b, \xi_1^e, \xi_2^e\} \in \Theta.
\]

For the period that we consider, there was existing government support for R&D activities. Our dataset contains information on the publicly funded portion of private R&D. On average, 10% of private R&D was funded publicly. Therefore in our estimation, we introduce a uniform subsidy to the total R&D spending of the firm \(\psi = 0.10\), such that a firm of type \((m, n, h^b_f)\)
pays
\[(1 - \psi)nu^\ast (ha(a^\ast) + 1_{(h^a_n < h^b_n)}[h^b_n + h^b_f])\]
in net research costs. The government has a balanced budget every period, so that the sum of
total subsidies and academic funding must be equal to the tax revenues, that is
\[T = \hat{T} + \psi\hat{w}^\ast \sum_m \left( ha(a^\ast) + \mathcal{F}(h^b_n) \left[ h^b_n + \mathbb{E} \left( h^b_f \mid h^b_f < h^b_n \right) \right] \right) \hat{\Gamma}_m^\ast, \]
where \(T\) is a lump-sum tax on consumers.

4.1 Estimation Method

In our dataset, for each firm \(f\) and each time period \(t\), we have a vector of \(N\) observables
from the actual data \(y^A_{ft} = [y^A_{ft_1} \ldots y^A_{ft_N}]_{N \times 1}^{f}\) including the number of industries the firm is
present in, sales, profits, and labor costs. Let the entire dataset be denoted by \(y\).

We use GMM for the estimation. Define \(\Lambda^\ast(y)\) and \(\Lambda^\ast(\theta)\) to be, respectively, the vectors of
real data moments (generated from \(y\)) and equilibrium model moments (generated for some
vector of parameters \(\theta\)). Since certain moments require knowledge of the joint distribution over
the number of products and industries \((m, n)\) and the the portfolio of product qualities \(q\), we
simulate a large panel of firms to calculate \(\Lambda^\ast(\theta)\) to a high degree of accuracy.

Our proposed estimator minimizes a quadratic form of the difference between these two
vectors
\[\hat{\theta} = \arg\min_{\theta \in \Theta} [\Lambda(\theta) - \Lambda(y)] \cdot W \cdot [\Lambda(\theta) - \Lambda(y)]\]
where \(W\) is the weighting matrix. For microfounded moments, we use the optimal weighting
matrix given by the inverse of the covariance matrix of the moments. In the case of macroe-
conomic moments such as aggregate growth rates, we use a fixed proportional weighting with
no covariance terms.

This is asymptotically consistent under fairly general conditions, but not efficient\(^{19}\). The
standard error of our parameter estimates, given any weighting matrix \(W\) is:
\[\hat{Q} = \left( \hat{G}W^{-1}\hat{G} \right)^{-1} \cdot \hat{G}W^{-1}\hat{\Omega}W^{-1}\hat{G} \cdot \left( \hat{G}W^{-1}\hat{G} \right)^{-1}\]
where \(\hat{G} = \frac{\partial \Lambda(\theta)}{\partial \theta} \bigg|_{\theta = \hat{\theta}}\). We estimate the covariance matrix by bootstrapping the data at the
firm level. See Bloom (2008) and Lentz and Mortensen (2008) for further description and
usage. In our estimation, we use 26 moments, which are described next.

\(^{19}\)In the efficient case where \(W = \hat{\Omega}^{-1}\) this becomes simply \(\hat{Q} = \left( \hat{G}\hat{\Omega}\hat{G} \right)^{-1}\).
4.2 Target Moments and Identification

In this section we explain the moments that are used to identify our parameters. For convenience, define expressions for the per product line R&D employment

\[ H_a^m = h_a(a_m^*) \quad \text{and} \quad H_b^m(h^b) = \left[ h_b(b_m^*) + h^b \right] 1_{(h^b < h_m^b)} \]

for applied and basic research, respectively. Note that these are functions of \( m \), the number of industries a firm operates in, and the idiosyncratic fixed cost of basic research \( h^b \).

Below, expectations are assumed to be over the distribution of firm characteristics \((m, n, h^b, q)\). Note that here \( m \) denotes the number of industries in which a firm has one or more products, rather than the number of industries in which the firm has working knowledge. Since the latter in unobservable, we must compute the former to in order to match the data.

**Basic Research Intensity by Number of Industries** We define basic research intensity as the ratio of spending on basic research to spending on applied research. Since the effect of multi-industry presence on this quantity is of critical importance to our model, we have one moment for each \( m \in \{1, \ldots, M\} \). Given a set of parameters and an equilibrium of the model, this moment’s analytic value for a given \( m \) is

\[ \lambda(1-8) = E \left[ \frac{H_b^m(h^b)}{H_a^m} \right] \]

In our estimation, we use \( M = 10 \). However, in the data there are only a handful of firms with \( m > 8 \), so we have one moment for each \( m \in \{1, \ldots, 7\} \) and a final moment which is averaged over \( m \in \{8, 9, 10\} \). The way in which this moment increases with \( m \) identifies the cross-industry spillover parameter \( p \) in our model. Additionally, it provides us with some identification power for the basic research cost parameters \((\xi_1^b, \xi_2^b)\).

**Extensive Margin of Basic Research Investment by Number of Industries** We use the share of positive basic research spending by each \( m \) to identify the mean \( \mu \) and variance \( \sigma^2 \) of the fixed cost distribution basic research. This is simply the probability that the idiosyncratic fixed cost draw is less than the cutoff for a certain \( m \)

\[ \Lambda(9-16) = E \left[ 1_{(h^b < h_m^b)} \right] \]
Distribution of $m$  We track two moments relating to the distribution of $m$, the mean and mean squared. They are given by

$$\Lambda(17) = \mathbb{E}[m] \quad \text{and} \quad \Lambda(18) = \mathbb{E}[m^2]$$

These moments identify the merger probability parameter governing the rate of expansion $z$ as well as the mass of potential outside entrants ($E$). Together, these factors determine the equilibrium distribution of multi-industry presence.

Profitability  Firm profitability is defined as the ratio of profits to sales. For a given panel of firms, this moment is given by

$$\Lambda(19) = \frac{1}{\varepsilon} - \mathbb{E} \left[ \frac{n \bar{w} [H_m^a + H_m^b(h^b)]}{\sum_i \tilde{q}_i^{-1}} \right]$$

Notice that there is one fixed component from static production side that yields information on the value of $\varepsilon$ and another from dynamic R&D expenditures that yields information on R&D cost and step size parameters.

Exit Rate  As exit occurs when firms either receive the exogenous destruction shock or lose their last product, the predicted exit rate will be

$$\Lambda(20) = \kappa + \tau \cdot \sum_m \Gamma_{m,1}$$

However, for consistency, we simply use the value from the simulated firm sample. This moment serves primarily to determine the value of the rate of exogenous destruction $\kappa$, as well as the mass of outside entrants $E$, since the size of the pool of entrants affects the rate creative destruction and hence the exit rate of single-product firms.

Total Research Intensity  We have two moments to track levels of R&D: the ratio of total research spending to total employment spending. Since research spending is proportional to $n$, R&D expenditures per product will be the same across firms with the same $m$, while employment will be a function of the portfolio of product qualities. The R&D to employment ratio is given by

$$\Lambda(21) = \mathbb{E} \left[ \frac{n \bar{w} [H_m^a + H_m^b(h^b)]}{\left(\frac{\varepsilon}{\varepsilon - 1}\right) \sum_i \tilde{q}_i^{-1}} \right]$$

Conditional on innovation rates, this moment give us information on the research production function parameters.
**Private to Public Basic Research Ratio**  The rate of academic innovation \( d \) is estimated as a parameter. To determine this, we look at the ratio of private basic research spending to public basic research spending. The moment is given by

\[
\Lambda(22) = \frac{\mathbb{E}[H^b_m(h^b)]}{h_b(d) + h^b}
\]

This moment will allow us to identify the flow rate of academic innovation \( d \), as well as inform us as to the level of private basic research spending.

**Firm Growth**  We have a moment for employment growth amongst firms. This is calculated conditional on the firm not exiting, since we do not observe the last period’s growth rate for exiting firms. The moment is calculated by looking at the one-year growth rate of total employment by a firm. It is labeled \( \Lambda(23) \). The employment growth primarily informs on the rate of exogenous destruction \( \kappa \) and the R&D cost function parameters.

**Aggregate Growth**  The growth rate gives information on the effectiveness of research spending absent effects coming from the distribution of firm size and its relation to firm growth, particularly on innovation step sizes. This is moment \( \Lambda(24) \).

**Spillover Differential**  In order to quantify the spillovers associated with basic research, we turn to patent citation data. The model predicts that innovations that build off of previous basic research should have a larger step size on average. If we take citations as a proxy for step size, then patents that cite basic research should themselves have more citations.

This effect will diminish with the age of the patent due to product line cooldown. Thus the average time after which a public innovation is indistinguishable from a private innovation should be

\[
\Lambda(25) = \frac{1}{\zeta} \left( \frac{\tau_a}{\tau} \right)
\]

This yields direct information on the value of the cooldown rate \( \zeta \).

**Firm Age**  Firm age is highly correlated with firm size. We track the average age of firms for those above and below the median firm size. This yields information entry and exit patterns, as well as on the rate of creative destruction. Moment \( \Lambda(26) \) is the average age of firms below the median firm size, while moment \( \Lambda(27) \) is the average age for those firms above it.
4.3 Computer Algorithm Outline

An equilibrium of this model can be summarized by a vector of five variables \((\tau_a^*, \tau_b^*, \tau_s^*, \tilde{w}^*, g^*)\) satisfying five equations:

\[
\tau_a^* = \sum_m a_m^* \tilde{\Gamma}_m^* + \tilde{a}^* \quad \text{and} \quad \tau_b^* = \sum_m (1 + \rho_m) b_m^* \tilde{\Gamma}_m^*
\]

\[
\tau_s^* = \sum_m (p - \rho_m) b_m^* \tilde{\Gamma}_m^* + (1 + p) d \quad \text{and} \quad \frac{1}{M} = L_{prod}^* + L_{res}^*
\]

in addition to a consistency condition on the growth rate. Here, \(a_m^*, b_m^*, e_m^*, \text{ and } \tilde{a}^*\) are chosen in accordance with the optimality conditions put forth in Proposition 1, \(\tilde{\Gamma}^*\) satisfies the flow equations, and \(L_{prod}^*\) and \(L_{res}^*\) represent labor demanded for production and research. The first three equations impose consistency on the levels of applied and basic research and spillover. The final equation ensures labor market clearing.

Given a vector of candidate equilibrium variables, we can then evaluate the above equations by taking the following steps:

1. Calculate \(\Psi\) and the distribution over \(\hat{q}\) using \(\tau_a, \tau_b, \tau_s, \text{ and } g\).
2. Calculate \(g\) using, \(\tau_a, \tau_b, \text{ and the distribution over } \hat{q}\).
3. Now calculate \(\beta_\eta = \mathbb{E}[\beta(\hat{q} + \eta)]\) and \(\beta_\lambda = \mathbb{E}[\beta(\hat{q} + \lambda)]\).
4. Find \(a_m\) and \(b_m\) using first-order conditions.
5. Impose an upper bound on \(n\) and find the steady state \(\Gamma_{m,n}\) using iteration (or your preferred method, e.g. an eigensolver).
6. Plug these into the equations above.

We use a Powell-Hybrid equation solver to solve this set of equations for a given set of parameters. To minimize the GMM objective function, we perform a search over the joint parameter and equilibrium variable space, minimizing the objective function subject to the equilibrium constraints. For this we use a Nelder-Mead (simplex) algorithm and/or simulated annealing with multiple restarts.

4.4 Estimation Results of Baseline Economy

Table 4 reports the values of the estimated structural parameters and their asymptotic standard errors.
<table>
<thead>
<tr>
<th>Value</th>
<th>Std. Err.</th>
<th>Description</th>
<th>Symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.33578</td>
<td>0.02387</td>
<td>CRRA Utility Parameter</td>
<td>γ</td>
</tr>
<tr>
<td>4.49542</td>
<td>0.00219</td>
<td>Elasticity of Substitution</td>
<td>ε</td>
</tr>
<tr>
<td>0.12716</td>
<td>0.00068</td>
<td>Cross-industry Spillover</td>
<td>p</td>
</tr>
<tr>
<td>0.07355</td>
<td>0.00049</td>
<td>Basic Step Size</td>
<td>η</td>
</tr>
<tr>
<td>0.04619</td>
<td>0.00013</td>
<td>Applied Step Size</td>
<td>λ</td>
</tr>
<tr>
<td>0.54696</td>
<td>0.00054</td>
<td>Mass of Entrants</td>
<td>E</td>
</tr>
<tr>
<td>0.18393</td>
<td>0.01562</td>
<td>Mass of Academic Labs</td>
<td>U</td>
</tr>
<tr>
<td>1.43920</td>
<td>0.00039</td>
<td>Applied Cost Curvature</td>
<td>ξ_a</td>
</tr>
<tr>
<td>1.55524</td>
<td>0.00023</td>
<td>Basic Cost Curvature</td>
<td>ξ_b</td>
</tr>
<tr>
<td>1.82129</td>
<td>0.00142</td>
<td>Applied Cost Scale</td>
<td>ξ_a^i</td>
</tr>
<tr>
<td>8.72808</td>
<td>0.01976</td>
<td>Basic Cost Scale</td>
<td>ξ_b^i</td>
</tr>
<tr>
<td>0.00412</td>
<td>0.00002</td>
<td>Exogenous Exit Rate</td>
<td>κ</td>
</tr>
<tr>
<td>4.73215</td>
<td>0.00184</td>
<td>Basic Fixed Mean</td>
<td>μ</td>
</tr>
<tr>
<td>0.26624</td>
<td>0.00163</td>
<td>Basic Fixed Std. Dev.</td>
<td>σ</td>
</tr>
<tr>
<td>0.11831</td>
<td>0.00000</td>
<td>Product Cooldown Rate</td>
<td>ζ</td>
</tr>
</tbody>
</table>

### 4.4.1 Parameter Estimates

One of most important estimate of our model is the cross-industry spillover parameter $p = 0.13$ which measures the probability of having an additional immediate application of the basic innovation. This estimate affects the extent to which basic innovations contributes to cross-sectional growth. In equilibrium, firms operate in two industries on average. Therefore the firms have on average a 1/9 probability of internalizing a given horizontal spillover. Given the estimate value of $p$, the internalized spillover is 0.02 ($= (1/9) \times .18$). In other words with probability 0.16, each successful basic innovation will have an application that will not be utilized by the main innovator.

The estimated innovation size of basic research is $\eta = 7.4\%$ and the innovation size of each new applied innovation is $\lambda = 4.6\%$. As a result, a basic innovation contributes 1.6 times as much to aggregate growth as an applied innovation.

Additionally, each basic innovation has a within industry spillover. The cooldown rate of hot product lines is estimated to be $\zeta = 0.11$ which indicates that a basic innovation affects the subsequent innovations in the same product line for almost 10 years on average.

The elasticity of applied innovation counts with respect to the research dollars spent is estimated to be 0.69 ($= 1/\xi_a^2$) and similarly the elasticity of basic innovation with respect to the basic research investment is 0.64 ($= 1/\xi_b^2$). A single elasticity has been estimated for the US economy by several papers (Pakes and Griliches (1984), Hall et al. (1988) and Kortum (1992, 1993)) and the estimates are varying between 0.1 and 0.6. The fact that the elasticity is smaller than 1 suggests that there is more investment in duplicative research efforts as the
research investment increases (Kortum, 1993). Therefore, our estimate suggests the there is a smaller fraction of duplicative research efforts in France than in the US for a given proportional increase in research investment.

4.4.2 Goodness of Fit

Table 5 contains the values of moments from the actual data and our estimated model.

<table>
<thead>
<tr>
<th></th>
<th>Description</th>
<th>Model</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-8</td>
<td>Basic Research Extensive</td>
<td>See Figure 7</td>
<td></td>
</tr>
<tr>
<td>9-16</td>
<td>Basic Research Intensive</td>
<td>See Figure 6</td>
<td></td>
</tr>
<tr>
<td>17</td>
<td>Mean Industries</td>
<td>1.9518</td>
<td>2.2037</td>
</tr>
<tr>
<td>18</td>
<td>Mean Square Industries</td>
<td>5.2424</td>
<td>6.9756</td>
</tr>
<tr>
<td>19</td>
<td>Return on Sales</td>
<td>0.0349</td>
<td>0.0326</td>
</tr>
<tr>
<td>20</td>
<td>Exit Rate</td>
<td>0.0675</td>
<td>0.0919</td>
</tr>
<tr>
<td>21</td>
<td>R&amp;D/Labor</td>
<td>0.2413</td>
<td>0.2603</td>
</tr>
<tr>
<td>22</td>
<td>Private/Public Basic</td>
<td>0.1293</td>
<td>0.1157</td>
</tr>
<tr>
<td>23</td>
<td>Employment Growth</td>
<td>0.1876</td>
<td>0.1032</td>
</tr>
<tr>
<td>24</td>
<td>Aggregate Growth</td>
<td>0.0124</td>
<td>0.0150</td>
</tr>
<tr>
<td>25</td>
<td>Spillover Differential</td>
<td>8.6460</td>
<td>8.0000</td>
</tr>
<tr>
<td>26</td>
<td>Age, Small Firms</td>
<td>14.2440</td>
<td>14.9965</td>
</tr>
<tr>
<td>27</td>
<td>Age, Large Firms</td>
<td>24.5041</td>
<td>24.8733</td>
</tr>
</tbody>
</table>

The results indicate that the model generates firm and industry dynamics similar to those in the data. In line with stylized fact 1, a significant fraction of innovating firms invest in basic research. In particular, 23% of firms are investing in basic research, which was 27% in the data. We also capture the positive relationship between the extensive margin of basic research and multi-industry presence, as evidenced in rows 17-24 and Figure 1.

In addition, the ratio of private basic research investment to public basic research investment is 13%, while it is 12% in the data. This is partly driven by our assumption in the
model that academic labs operate regardless of their fixed cost draw, whereas private firms are optimizing in this dimension. In our model, the share of basic research in total private research investment by incumbents is 4%. This number is lower than the 10% figure listed in stylized fact 2, however, it is more in line with the firm level data as seen in Figure 2.

The positive correlation between multi-industry presence of a firm and its basic research intensity was one of the primary motivations for introducing multi-industry presence into our model. As explained previously in the text, multi-industry presence plays an important role in increasing basic research incentives, by allowing a greater potential to internalize the positive spillover from basic research. In our reduced form analysis, we found a significant and positive correlation between multi-industry presence and basic research intensity. This has been the key moment to identify the cross-industry spillover parameter. Our model successfully generates this positive correlation.

In the data, firms operate on average in 2.2 industries compared to 2.0 in our model. Furthermore, we find the mean squared in the model to be 5.2, compared to 7.0 observed in the data.

The table above reports some additional moments that are not captured by the stylized facts. For instance, the mean profitability is 3.5% in the data, yet our model predicts a value of 3.3%. The prime determinants of profitability are the step sizes for basic and applied innovation. However, these also affect the investment levels for both types of research, since this increases the return to successful innovation. Therefore, the step size parameters are set to compromise between hitting the profitability moment and the research investment and growth moments.

We are targeting additional moments regarding research investments. The first is the average ratios of total research investment to firm sales. The model overshoots this ratio (18.7% vs 11.3%) largely in order to hit the aggregate growth and return on sales. This also results in an overshooting of the average firm employment growth moment. The third moment that we are targeting is the ratio of private to public basic research investment.

All of these components of the economy determine the aggregate growth rate. Our model matches the observed growth rate closely. Our model economy grows at a rate of 1.3%, while the French economy grew at an average rate of 1.5% during the period studied (2000-2006).

4.4.3 Untargeted Moments

In this part, we discuss our model’s prediction about some of the moments that we did not directly target. In the data there is a positive correlation between basic research intensity and
firm size. This is captured in our model as well. We find a value of 0.31 for the correlation between basic research intensity and log employment.

In the data the correlation between profitability and basic research intensity is not significantly different from zero. The same implication emerges from our model. In the baseline model, the correlation between profitability and basic research intensity is only 0.03. This result emerges because basic research investment is determined through instantaneous idiosyncratic shocks which are history independent, whereas profitability is a direct function of the history of step size realizations.

Our model naturally generates a positive correlation between multi-industry presence and firm size, which is also empirically true in the data. This arises since both of these moments are strongly correlated with firm survival. In the model, we find a correlation of 0.66 between the log employment and multi-industry presence. In the data, this value is 0.76.

Another stylized fact in our data is that firm size distribution is highly skewed. This is a well known feature which is documented extensively in the literature. In our model, we capture this fact with a skewness of the firm size distribution of 2.28. This value is 3.07 in the data.

We will now focus on the social planner’s problem to quantify the underinvestment in our model economy. Then we will turn to various policies that could address this inefficiency.

### 4.4.4 Equilibrium Values of Endogenous Variables

The following table provides equilibrium values for some of the important endogenous variables in the model.

<table>
<thead>
<tr>
<th>Decentralized Economy: Endogenous Variables (in percentages)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\psi$</td>
</tr>
<tr>
<td>10</td>
</tr>
</tbody>
</table>

As the table indicates, much of the applied innovation in the economy is done by incumbents. Approximately 65% of applied innovations come from incumbent firms. In any given period, about 14% of incumbents produce an applied innovation. On the basic research side, around 64% of basic innovations come from the academic sector. However, one must keep in mind that basic innovations by the academic sector, while they will make a product line hot, do not have an immediate impact on quality. The arrival rate of basic innovation is significantly smaller than that of applied innovation. Applied innovations arrive 30 times more frequently than basic innovations. However, each basic innovation will have additional contributions to economic growth which we analyze in more detail below.
Overall, we find a fairly small role for cross-industry spillovers. Approximately 13% of basic innovations generate an additional basic innovation in another industry. As discussed earlier, only about 1 in 9 of these are internalized. Combined with the low rate of basic innovation overall, this does not produce a sizeable effect.

Within-industry spillovers appear to be much more important. The estimate for $\Psi^*$ captures the within-industry spillovers from basic research. Thanks to this spillover, 15% of applied innovations hit hot product lines and have a larger contribution $(\eta - \lambda)$ to growth than they otherwise would. The resulting improvements lead to an aggregate growth rate of 1.2% per annum for the economy as a whole, which we analyze in the next section.

**Growth Decomposition** In expression (23), we show that research efforts contribute to aggregate growth along several dimensions. The following table decomposes these effects according to their source and their immediate impact.

<table>
<thead>
<tr>
<th>Research Type</th>
<th>Direct Applied</th>
<th>Direct Basic</th>
<th>Spillover Basic</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(\bar{a}^* + \tilde{a}^*) \lambda$</td>
<td>$(\bar{d}^* + \tilde{d}^*) \eta$</td>
<td>$p \left( \bar{d}^* + \tilde{d}^* \right) \eta + (\bar{a}^* + \tilde{a}^<em>) \Psi^</em> (\eta - \lambda)$</td>
<td></td>
</tr>
<tr>
<td>$1.08% \ (87%)$</td>
<td>$0.03% \ (3%)$</td>
<td>$0.13% \ (10%)$</td>
<td></td>
</tr>
</tbody>
</table>

Overall 87% of total growth comes from the direct effect of applied innovations, while 3% of growth comes from the direct effect of basic innovation. In addition to the direct effect, basic research contributes to overall growth through cross-industry spillovers, which account for less than 1% of total growth, and through within industry spillovers, which account for 10% of total growth. In sum, roughly 10% percent of the contribution of basic innovation to growth comes through its two types of spillovers, which are not fully internalized by innovating firms.

<table>
<thead>
<tr>
<th>Institution</th>
<th>Entrants</th>
<th>Incumbent Applied</th>
<th>Incumbent Basic</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tilde{a}^* \left[ \Psi^* \eta + (1 - \Psi^*) \lambda \right]$</td>
<td>$\tilde{a}^* \left[ \Psi^* \eta + (1 - \Psi^*) \lambda \right]$</td>
<td>$(1 + p) \tilde{b}^* \eta$</td>
<td></td>
</tr>
<tr>
<td>$0.43% \ (34%)$</td>
<td>$0.78% \ (63%)$</td>
<td>$0.03% \ (3%)$</td>
<td></td>
</tr>
</tbody>
</table>

Our estimates indicate that entrants play an important direct role in overall growth. That being said, we also recognize that without direct data on entry rates, our estimate relies largely on inference from other aspects of the data, which are listed in section 4.4.2. To give an idea of size of the contribution, keeping the innovation rates of entrants and outsiders the same, we recompute $\Psi^*$ excluding the contribution from academic basic innovation and recalculate the implied growth contributions from the first two columns. In this case we find that growth falls to 1.20%. This implies that the dynamic effects of academic innovation
account for $14\% (= (1.40 - 1.20)/1.40)$ of total growth, which increases the overall contribution of the academic sector to 26%.

Finally, we conclude this section by providing a decomposition of the labor force utilization. This is informative because in our model, the only input for both production and innovation is labor. Therefore, it is important to observe the allocation of labor amongst its various possible uses in order to understand the mechanism behind the current results and the upcoming policy analysis.

<table>
<thead>
<tr>
<th>Labor Decomposition by Activity (in percentages)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Production</td>
</tr>
<tr>
<td>79.0</td>
</tr>
</tbody>
</table>

In our benchmark economy, 79% of labor is used for production, and 21% is employed for innovation activities. Amongst this 21% percent, roughly 22% of researchers are engaged in innovation activities with additional spillovers. The policies that will consider will not only govern the split of labor resources between production and research, but also the composition within research across different types of innovation, since uninternalized spillovers are one of the main sources of inefficiency.

4.4.5 Quantifying the Social Planner’s Optimum

In this section, we are going to provide the solution to the social planner’s problem described in section ???. Recall that we are considering a planner who controls the research labs of firms but not the firms’ production decisions. The following table summarizes these results:

<table>
<thead>
<tr>
<th>Social Planners Optimum (in percentages)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a^{SP}$</td>
</tr>
<tr>
<td>11</td>
</tr>
</tbody>
</table>

One striking feature of the solution to the social planner’s problem is that it actually features less labor devoted to research activities in the aggregate than the baseline model. Total labor allocated to research activities was 21% in the decentralized economy, while it is only 17% when set by the social planner.

Indeed, the dominant misallocation here not that between production and research, as is common in this class of models, but amongst the various types of research activities, in this case, applied and basic innovation. In the decentralized economy, only 2% of research labor is used to undertake basic research. Meanwhile, the social planner devotes 31% of research labor to basic research. In absolute terms, we go from 0.4% to 5.0% of total labor being devoted to basic research. This happens both on the intensive and extensive margin of basic research. In
fact, the planner finds it optimal to employ nearly all research labs, regardless of their fixed cost draw.

In the case of applied innovation, there is actually an overinvestment in the baseline economy. Total applied research (including entrants) is 17.2% in the baseline economy. This figure drops to 11.1% in the social planner’s solution. This is in spite of the fact that the fraction of hot product lines rises from 15% to 27%, meaning the average step size of an applied innovation rises by a quarter.

One thing to note is that for simplicity, we assume that the social planner can fully utilize academic innovations for productive purposes and choose whether or not to operate labs as a function of the fixed cost. As such, we can simply treat the basic research capacity of the economy as a mass of $1 + U$ labs. Similarly, we can treat the applied research capacity of the economy as a mass of $1 + E$ labs, where $E$ is the mass of potential entrants.

The net result of the above changes is that growth rises from to 1.3% from 1.2%. Overall, the decentralized economy’s welfare corresponds to an 92% consumption equivalent with respect to the social planner’s optimum. The following policy experiments will try to bridge this gap.

5 Policy Analysis

In this section, we analyze the impact of different types of research subsidies. Given our distinction between basic and applied research, it seems natural to propose different subsidy policies for different types of research spending. However, such a policy would generate a moral hazard problem since firms would have an incentive to misreport the type of research they undertake, which is almost impossible to verify for a policymaker. In the previous section, we provided the solution to the social planner’s problem as a benchmark. We next consider a hypothetical case where the policymaker can observe the type of research project and subsidize it accordingly.

Secondly, to imitate real-world research subsidies, we consider the case in which only uniform subsidies to all research types are implemented. We then consider varying the amount of funding going to academic research labs. Finally, we allow for both uniform subsidies and changes in academic research funding.

5.1 Type-Dependent (TD) Research Subsidy

Assume first that the policymaker can distinguish between different types of research efforts and accordingly provide differentiated subsidy rates. Let $\psi_a$ and $\psi_b$ denote the applied research and basic research subsidy rates respectively. The share of GDP allocated to academic research

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$(T/Z)$ is kept constant by the policymaker. Total spending by firm $f$ of type $(m, n, h^b)$ is then simply

$$n (1 - \psi_a) \tilde{w}^{TD} h_a(a_{mn}^{TD}) + n (1 - \psi_b) \tilde{w}^{TD}\mathbf{1}_{(h^b < h_m^{TD})} \left[ h_b(h_m^{TD}) + h^b \right]$$

The first-order conditions for the value function in (37) become:

$$\Psi^{TD} \beta^{TD}_\eta + (1 - \Psi^{TD}) \beta^{TD}_\lambda + \beta^{TD}_m = (1 - \psi_a) \tilde{w}^{TD} h'_a(a_{mn}^{TD})$$

$$1 = (1 - \psi_b) \tilde{w}^{TD} h'_b(h_m^{TD})$$

In these expressions, the left side indicate the expected return to research investment and the right side is the relevant marginal cost. Note that an increase in the subsidy rate ($\psi_a$ or $\psi_b$) reduces of research cost for the firm and leads to an increase in the research effort as a result.

The advantage of this policy is clearly that the policymaker can target a particular type of research to correct the underinvestment in it. The following table reports the optimal subsidy rates under this policy. As there are spillovers associated with basic research that are not internalized, one would expect that the optimal subsidy to basic research would be higher.

**Type-Dependent Research Subsidy (in percentages)**

<table>
<thead>
<tr>
<th>$\psi_a$</th>
<th>$\psi_b$</th>
<th>$a^{TD}$</th>
<th>$b^{TD}$</th>
<th>$d^{TD}$</th>
<th>$\tilde{b}^{TD}$</th>
<th>$L_{prod}^{TD}$</th>
<th>$L_{bas}^{TD}$</th>
<th>$L_{app}^{TD}$</th>
<th>$\Psi^{TD}$</th>
<th>$\psi^{TD}$</th>
<th>$g^{TD}$</th>
<th>$a^{TD}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-11</td>
<td>35</td>
<td>11.1</td>
<td>6.00</td>
<td>0.31</td>
<td>3.08</td>
<td>80.2</td>
<td>4.5</td>
<td>11.9</td>
<td>30.2</td>
<td>95.0</td>
<td>1.27</td>
<td>94.4</td>
</tr>
</tbody>
</table>

Our quantitative results confirm this expectation. Since the underinvestment is mainly in basic research, the optimal type-dependent subsidy dictates a much larger subsidy rate for it, namely $\psi_b = 35\%$ and $\psi_a = -11\%$. Here, we not only see a lower subsidy on applied innovation, but a tax.

The large value for the basic subsidy is straightforward to understand. There are spillovers associated with basic innovation that are not internalized by firms. By subsidizing this type of innovation, we can mitigate this effect. There are two partially offsetting effects that determine the optimal subsidy (tax) value on the applied side. Given the size of the overinvestment in applied innovation in the baseline economy, one would expect a tax to be necessary. However, it is not as large as might otherwise be due to the subsidization of basic research. The higher rate of basic innovation raises the fraction of hot product lines ($\Psi$) in the economy. Therefore, the mean step size for applied innovation rises, mitigating the overinvestment problem.

Note that the share of research labor does not change very much with these new policies. Almost all of the action is in taking a fixed set of researchers and dividing them appropriately between basic and applied research. Thus, in a real-world setting, we may not need to worry
as much about the supply of researchers, but more about what types of projects existing researchers are working on.

The equilibrium growth rate increases to 1.27%, nearly that of the social planner’s optimum. Additionally, initial consumption rises slightly as a result of the greater fraction of workers devoted to production. The consumption equivalent rises to 94.4% with respect to the social planner’s optimum, which is a 2.1 percentage point improvement over the baseline economy.

As discussed above, this policy is hard to implement in the real world due to the moral hazard problem. Therefore we focus on a policy providing a uniform subsidy across firms and research types.

5.2 Uniform Private (UP) Research Subsidy

With this policy, the government subsidizes a fraction \( \psi \) of each firm’s total research investment, keeping the share of funds allocated to academic research constant. This implies that the total spending by firm \( f \) is simply

\[
n(1 - \psi)\bar{\omega}^{UP} \left( h_a(a_{mn}) + 1_{(h^b<h^m)} \left( h_b \left( l^{UP}_{mn} \right) + h^b \right) \right).
\]

In that case the first-order conditions for the value function in (37) become

\[
\Psi^{UP} \beta^{UP}_n + (1 - \Psi^{UP}) \beta^{UP}_\lambda + \beta^{UP}_m = (1 - \psi) \bar{\omega}^{UP} h'_a(a_{mn}^{UP})
\]

\[
(1 + \rho_m)(\beta^{UP}_n + \beta^{UP}_m) = (1 - \psi) \bar{\omega}^{UP} h'_b(b_{mn}^{UP})
\]

Note that such a policy subsidizes not only basic research in (45), but also applied research in (44). The following table summarizes the results of the optimal uniform subsidy rate.

| Uniform Research Subsidy (in Percentages) |
|---|---|---|---|---|---|---|---|---|---|---|
| \( \psi \) | \( \bar{a}^{UP} \) | \( \bar{b}^{UP} \) | \( \bar{l}^{UP}_{pro} \) | \( \bar{l}^{UP}_{bas} \) | \( \bar{l}^{UP}_{app} \) | \( \Psi^{UP} \) | \( \sigma^{UP}_{mn} \) | \( \sigma^{UP}_{m} \) | \( g^{UP} \) | \( \alpha^{UP} \) |
| 18 | 15.0 | 8.20 | 0.30 | 0.69 | 77.5 | 0.87 | 18.4 | 17.2 | 92 | 1.34 | 92.4 |

Our analysis of the baseline economy and the planner’s economy documented a slight overinvestment in research overall and a large misallocation between the different types of research. A uniform subsidy is therefore ill suited to address these issues as it can not directly affect the allocation between research types, and any attempt to subsidize basic research will only worsen the overinvestment in applied research. Although the optimal type dependent basic subsidy is 35%, the optimal uniform subsidy is only 18%, due to cross-subsidization of applied research whose optimal level was -11%.

Under this policy, we are allocating a slightly larger fraction of the labor force to research relative to the baseline economy. Overall, the researcher share goes up to 23% from 21%. This
reduces the initial consumption to 92% of the baseline, yet increases the growth rate to 1.34%. The welfare gain from this policy is negligible, raising the consumption equivalent to 92.4%, a change of only 0.1 percentage points.

Although the underinvestment in basic research is sizable, the uniform policy partially makes up for this at the cost of worsening the overinvestment in applied research. There is an additional dynamic through the creative destruction channel. The increase in innovation rates (particularly applied) raises the overall rate of creative destruction. This reduces the incentives to innovate, leading to a partial canceling out of the initial effect. As a result, it is difficult to induce a sizable change in basic innovation rates using this type of policy.

The main lesson to be drawn from this is that a uniform research subsidy should take into account its negative welfare consequences through its oversubsidization of applied research. Finding a feasible method to differentiate basic and applied research is essential for better innovation policies.

5.3 Optimal Academic Fraction of GDP (AC)

In this section we will look for the optimal public funding level for academic research as a fraction of GDP \( \frac{T}{Z} \) keeping the baseline subsidies fixed. This is particularly important because, as the rate of academic innovation is a major factor in determining the share of hot product lines, which determines the effectiveness of applied innovation.

In the baseline economy, we assumed that academic innovations have no immediate effect on productivity in a particular product line. Instead, turn it into a hot product line, increasing the step size of future applied innovation. In the social planner’s problem, we made the more optimistic assumption that academic innovation affects productivity immediately, in addition to turning the product line hot.

When studying the optimal allocation of GDP to academic research, we look at both of the environments described above, in addition to an intermediate case where the probability of an innovation has immediate impact is 50%. The Bayh-Dole Act of 1980 attempted to enhance the applicability of academic innovations by allowing universities to retain ownership of inventions made using federal funds. As such, we refer to this as the Bayh-Dole factor throughout. This can be thought of as a composite of policy and the underlying structural features of academic innovation.

The following table summarizes the results of the optimal academic policy.

<table>
<thead>
<tr>
<th>Academic Funding when Bayh-Dole=0% (in percentages)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \psi )</td>
</tr>
<tr>
<td>-----------</td>
</tr>
<tr>
<td>10</td>
</tr>
</tbody>
</table>

54
Optimal academic funding levels are then uniformly below those seen in the baseline estimation, meaning there was previously an overinvestment. The baseline case featured academic funding of 3.3% of GDP, while optimal levels range from 0.3% to 1.6% depending on the value of the Bayh-Dole parameter. These optimal allocation bring with them large welfare gains, between 2 and 3 percentage points. Some of this is simply due to the increase in the Bayh-Dole factor, while the rest can be attributed to the optimal allocation of academic funding. The growth rate rises as well, attaining levels seen in the social planner’s optimum in the last case.

Our results highlight the special role of academic research in overall growth and show the complementarities present between public and private research. Allocating resources to academic research not only has a direct effect on growth, but an indirect effect by making private research more productive. However, one should also note that this particular policy alone cannot make up for the underinvestment in research on the part of the private sector. Therefore, the next policy experiment is of particular importance.

### 5.4 Optimal Feasible Policy: Uniform Subsidy and Academic Budget (AU)

Our final policy experiment combines both of the feasible policies that have been considered thus far individually. We will allow both the uniform subsidy rate and the academic funding rate to be chosen by the policymaker. The advantage of considering both types of policies is to introduce more freedom to control the incentives for both types of research in a largely separate way. In particular, $\psi$ and $\hat{T}/Z$ are going to be the choice variables in this exercise. The following table contains the results of this experiment.

<table>
<thead>
<tr>
<th>$\psi$</th>
<th>$\hat{T}/Z$</th>
<th>$\tilde{\alpha}^{UP}$</th>
<th>$\bar{\alpha}^{UP}$</th>
<th>$\bar{\alpha}^{UP}$</th>
<th>$\bar{a}^{UP}$</th>
<th>$\bar{b}^{UP}$</th>
<th>$L_{prod}^{UP}$</th>
<th>$\Psi^{UP}$</th>
<th>$\frac{C_{UP}}{C_{SP}}$</th>
<th>$g^{UP}$</th>
<th>$\alpha^{UP}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>16</td>
<td>1.5</td>
<td>14.9</td>
<td>8.15</td>
<td>0.18</td>
<td>0.63</td>
<td>79.5</td>
<td>12.3</td>
<td>94.1</td>
<td>1.37</td>
<td>95.4</td>
<td></td>
</tr>
</tbody>
</table>

By using the level of academic funding to reach the proper share of researchers, the policymaker is able to lower the uniform subsidy, thus reducing needless cross-subsidization of applied research. Under the current policy 21% of the labor force is allocated to research, roughly the same as in the baseline case. This time around, the composition of workers between applied and basic research is closer to the social optimum. By allocating 1.5% of GDP to academic
research and subsidizing private research at a rate of 16%, the policymaker can do even better than the hypothetical case of a type-dependent research subsidy. The growth rate goes up 1.37%. The resulting economy achieves a consumption equivalent of 95.4% with respect to the social planner’s optimum. Ultimately though, this is not a significant improvement over the academic funding scenarios seen in the previous section.

To summarize our findings, we first considered the most widely discussed policy, which is a uniform subsidy. Using this tool optimally yielded almost no improvement in welfare due to oversubsidization of applied research since the policy could not distinguish between the research types with different spillover and productivity implications. Considering a policy combination that governs both academic and private research could generate a significant improvement. The first main conclusion to be drawn for innovation policy is the importance of recognizing different types of innovations and the impact of policies on these types of research. The second is that it is important to take into account both the direct and indirect effects of academic research on productivity growth when considering growth and innovation policies.

6 Conclusion

In this paper, we distinguished between basic and applied research and identified spillovers associated with each. Our quantitative analysis highlighted the importance of this distinction. Indeed, in the competitive equilibrium, applied research is overinvested and basic research is underinvested. As a result, following a uniform research subsidy does not generate the expected welfare improvement due to inefficient cross-subsidization of applied research. An increase in the uniform subsidy improves the underinvestment in basic research by worsening the overinvestment in applied.

The key message of our paper is that standard R&D policies can accentuate the dynamic misallocation in the economy. Our findings point to the need for policies that target basic research more directly. One method of achieving this is by increasing the intellectual property rights granted to academic researchers. Alternatively, one can reward collaboration between universities and the private sector, which would encourage focusing on research that can more directly lead to tangible gains in production technologies.

Our paper took a first step in trying to quantify the inefficiencies regarding different types of research and innovation efforts. There are still important open questions awaiting further study. In particular, the effect of university licensing, collaboration opportunities between universities and the private sector are some examples. Unfortunately, the study of these questions suffers from a lack of data on the public sector’s research production function. An important
improvement would be to gather data on public sector innovation and use it to address the aforementioned questions and study the effects of relevant policies.

7 Bibliography


Appendix

A Theoretical Proofs

Proof of Proposition 1. For any given industry, the demand coming from the downstream sector for each midstream output is

\[ Y_i = \left[ \frac{\mu_i}{P_i} \right]^{\gamma} Z. \]  

(46)

Next we turn to the problem of the midstream sector producer. The optimization leads to the following demand equation

\[ y_{ij} = \frac{P_iY_i}{p_{ij}} \forall j \in [0, 1]. \]  

(47)

When there is a new innovation at time \( t \) of size \( s \in \{\eta, \lambda\} \), then the quality improves to \( q(t + \Delta t) \) according to \( q_{k+1} = (1 + s)q_k \). This implies that the marginal cost of the latest inventor \( K \) in product line \( ij \) will set the price equal to the marginal cost of the previous inventor \( K - 1 \),

\[ p_{ij} = \frac{w(1 + s_{ij})}{q_{ij}}. \]

We assume symmetry across industries, that is \( \mu_i = \mu = 1/M \). Therefore, we focus on symmetric equilibria and drop explicit dependence on industry \( i \). An immediate implication of the final good production technology is that \( Z = Y \). This, along with (46) implies that

\[ P_i = P = 1/M. \]  

(48)

We define the markup for a given step size \( s \) as \( \tilde{s} := \frac{s}{1+s} \) where \( s \in \{\eta, \lambda\} \). We also denote the share of high markup product lines (with step size \( \eta \)) by \( \alpha \) and the average markup by \( \bar{s} \equiv \alpha\eta + (1 - \alpha)\lambda \). Then the optimal quantity from (47) and (48) can be expressed as

\[ y_j = \frac{q_j(1 - \tilde{s}_j)}{w} Z \frac{Z}{M}. \]  

(49)

The resulting profit is simply given by

\[ \pi_j = \tilde{s}_j \frac{Z}{M}. \]  

(50)

Here we see that the markup is in fact the fraction of revenue \( (Z/M) \) that goes to profits. The remainder goes to labor. Equations (3) and (49) imply

\[ w = (1 - \tilde{\eta})^\alpha(1 - \tilde{\lambda})^{1-\alpha} \left( \frac{Q}{M} \right). \]  

(51)
where \( \ln Q = \int_{0}^{1} \ln q_j dj \) is the average quality index in the economy and \( \alpha \) is the share of product lines with step size \( \eta \) (high markup). This proves (8).

Next we satisfy the labor market clearing by equating labor supply to labor demand \( ML^p = M \int_{0}^{1} \ell_j dj \). From (49) we get \( \ell_j = \frac{(1-\xi)}{w} \frac{Z}{M} \). Using this expression together with the equilibrium wage rate in (51) we get (9). Finally, substituting (9) and (51) into (49) delivers (6). \( \blacksquare \)

**Proof of Propositions 2 and 3.** Consider (18). First order conditions deliver

\[
a_m = \frac{E_m \left[ \alpha V_{m,n_\eta+1,n_\lambda} + (1-\alpha)V_{m,n_\eta,n_\lambda} + 1 \right] - E_m V_{m,n_\eta,n_\lambda}}{w} \\
b_m = \frac{\xi \rho_m E_m V_{m,n_\eta+2,n_\lambda} + (1-\rho_m)E_m V_{m,n_\eta+1,n_\lambda} - E_m V_{m,n_\eta,n_\lambda}}{w}
\]

Conjecture \( E_m V_{m,n_\eta,n_\lambda} = n_\eta E_m V_{m,\eta} + n_\lambda E_m V_{m,\lambda} \). Then we have

\[
a_m = \frac{\alpha E_m V_{m,\eta} + (1-\alpha)E_m V_{m,\lambda}}{w} \\
b_m = \frac{\xi (1+\rho_m) E_m V_{m,\eta}}{w}
\]

Using the free entry condition we get

\[
\left[ \alpha E_m V_{m,\eta} + (1-\alpha)E_m V_{m,\lambda} \right] = w \chi \tag{52}
\]

Thus we have

\[
a_m = \frac{\chi}{w} \\
b_m = \frac{\xi (1+\rho_m) K}{w}
\]

where we defined \( K \equiv E_m V_{m,\eta} \). Substituting the optimal choices back into the value function, taking the expectations of both sides with respect to \( m \) and using the conjecture we get

\[
V_{m,n_\eta,n_\lambda} = \max_{a_m,b_m} \left\{ \Delta t n_\lambda \pi_\lambda + \Delta t n_\eta \pi_\eta + \Delta t n_\lambda w \cdot \frac{1}{2} a_m^2 + \Delta t n_\lambda w \cdot \frac{1}{8} b_m^2 \\
+ \Delta t n_m b_m \left\{ \rho_m E_m V_{m,n_\eta+2,n_\lambda} + (1-\rho_m)E_m V_{m,n_\eta+1,n_\lambda} \right\} \\
+ \Delta t n_m a_m \left\{ E_m \left[ \alpha V_{m,n_\eta+1,n_\lambda} + (1-\alpha)V_{m,n_\eta,n_\lambda} + 1 \right] \right\} \\
+ \Delta t \tau \left\{ n_\eta E_m V_{m,n_\eta-1,n_\lambda} + n_\lambda E_m V_{m,n_\eta \lambda-1} \right\} \\
(1+\Delta t g) E_m V_{m,n_\eta,n_\lambda} \right\}
\]

which reduces to

\[
(\delta + \tau) \left[ n_\eta E_m V_{m,\eta} + n_\lambda E_m V_{m,\lambda} \right] = n_\lambda \pi_\lambda + n_\eta \pi_\eta + n_\lambda w \cdot \frac{1}{2} \chi^2 + n \frac{\xi E_m (1+\rho_m)^2 K^2}{2w}.
\]

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Then we have

\[ (\delta + \tau) E_{m V_{m, \eta}} = \pi_\eta + \frac{\xi \Phi [E_{m V_{m, \eta}}]^2}{2w} + \frac{\alpha E_{m V_{m, \eta}} + (1 - \alpha) E_{m V_{m, \lambda}}}{2w} \]

\[ (\delta + \tau) E_{m V_{m, \lambda}} = \pi_\lambda + \frac{\xi \Phi [E_{m V_{m, \eta}}]^2}{2w} + \frac{\alpha E_{m V_{m, \eta}} + (1 - \alpha) E_{m V_{m, \lambda}}}{2w} \]

where \( \Phi \equiv E_{m} (1 + \rho_m)^2 \). Thus the two previous equations become

\[ (\delta + \tau) E_{m V_{m, \eta}} = \pi_\eta + \frac{\xi \Phi [E_{m V_{m, \eta}}]^2}{2w} + \frac{w \chi^2}{2}, \]

(53)

\[ (\delta + \tau) E_{m V_{m, \lambda}} = \pi_\lambda + \frac{\xi \Phi [E_{m V_{m, \eta}}]^2}{2w} + \frac{w \chi^2}{2}. \]

(54)

Moreover the optimal applied research effort is

\[ a_m = \chi. \]

(55)

Taking the difference get \( E_{m V_{m, \eta}} - E_{m V_{m, \lambda}} = \frac{\pi_\eta - \pi_\lambda}{\delta + \tau} \). Solving this expression together with the free entry condition (52) we get

\[ E_{m V_{m, \eta}} = \frac{(\pi_\eta - \pi_\lambda)(1 - \alpha)}{\delta + \tau} + w \chi \]

(56)

Thus

\[ b_m = \xi (1 + \rho_m) \left[ \frac{(\pi_\eta - \pi_\lambda)(1 - \alpha)}{(\delta + \tau) w} + \chi \right]. \]

(57)

Finally multiplying (53) and (54) by \( \alpha \) and \( 1 - \alpha \) respectively and summing together we get

\[ (\delta + \tau) w \chi = \alpha \pi_\eta + (1 - \alpha) \pi_\lambda + \frac{\xi \Phi [E_{m V_{m, \eta}}]^2}{2w} + \frac{w \chi^2}{2} \]

Now, we solve for \( E_{m V_{m, \eta}} \) and equate to (56) to get

\[ \sqrt{\left( \frac{\delta + \tau - \frac{\chi}{2}}{w} \right)} \chi \left( \alpha \pi_\eta + (1 - \alpha) \pi_\lambda \right) \frac{2}{\Phi \xi} = \frac{(\pi_\eta - \pi_\lambda)(1 - \alpha)}{w (\delta + \tau)} + \chi. \]

Finally the growth rate is found as follows. Consider the change in quality index

\[ \ln Q(t + \Delta t) = \int_0^1 \left\{ (a + a_0) \Delta t \alpha \ln [(1 + \eta) q_j(t)] + (a + a_0) \Delta t (1 - \alpha) \ln [(1 + \lambda) q_j(t)] + E_m (1 + \rho_m) b_m \Delta t \ln [(1 + \eta) q_j(t)] + (1 - a \Delta t - a_0 \Delta t - E_m (1 + \rho_m) b_m \Delta t) \ln q_j(t) \right\} dj \]

Then we have

\[ g = \lim_{\Delta t \to 0} \frac{\ln Q(t + \Delta t) - \ln Q(t)}{\Delta t} \]

\[ = (a + a_0) (1 - \alpha) \ln (1 + \lambda) + [(a + a_0) \alpha + E_m (1 + \rho_m) b_m] \ln (1 + \eta) \]
Therefore

\[ g^* = \ln (1 + \eta) \left( a^* + a_0^* \right) a^* + \mathbb{E}_m (1 + \rho_m) b_m^0 \ (1 + \lambda) \left( a^* + a_0^* \right) (1 - a^*) \]  

(58)

**Proof of Proposition 4.** We set \( \eta = \lambda \) and \( d = 0 \). Then in (20) \( \pi_\eta = \pi_\lambda \) which delivers

\[ b_m = \xi (1 + \rho_m) \chi \text{ and } a_m = \chi. \]

Similarly in (22) we get

\[ \tau = \frac{\pi}{\lambda w} + \frac{1 + \Phi \xi}{2} \chi - \delta. \]

From the static problem we have the following expressions

\[
\begin{align*}
\pi_j &= \tilde{s}_j \frac{Z}{M} \\
w &= (1 - \tilde{\eta})^\alpha (1 - \tilde{\lambda})^{1 - \alpha} \left( \frac{Q}{M} \right) \\
Z^* &= \left[ \frac{(1 - \tilde{\eta})^\alpha (1 - \tilde{\lambda})^{1 - \alpha}}{\alpha (1 - \tilde{\eta}) + (1 - \alpha) (1 - \tilde{\lambda})} \right] QL^P
\end{align*}
\]

From here we get

\[ \frac{\pi}{w} = \frac{\tilde{\eta}}{1 - \tilde{\eta}} L^P \]

Now we can find the entry rate using (55) and (57) as

\[ a_0 = \frac{\tau - a - \mathbb{E}_m b_m (1 + \rho_m)}{1 - \tilde{\eta}} = \frac{\tilde{\eta} L^P}{\chi (1 - \tilde{\eta})} - \frac{(1 + \Phi \xi) \chi}{2} - \delta \]

Now we can use the labor market clearing condition

\[
\begin{align*}
L^P &= 1 - L^E - L^I \\
&= 1 - \chi a_0 - \frac{1}{2} a^2 - \frac{1}{2 \xi} \mathbb{E}_m b_m^2 \\
&= (1 + \delta \chi) (1 - \tilde{\eta})
\end{align*}
\]

Thus

\[ a_0 = \tilde{\eta} \left[ \frac{1}{\chi} + \delta \right] - \frac{(1 + \Phi \xi) \chi}{2} - \delta \]

and

\[ \tau = \left( \frac{1}{\chi} + \delta \right) \tilde{\eta} + \frac{(1 + \Phi \xi) \chi}{2} - \delta. \]
The growth rate in (58) is
\[
g^* = (a^* + a_0^* + E_m (1 + \rho_m) b_m^*) \ln (1 + \eta)
= \left[ \left( \frac{1}{\chi} + \delta \right) \tilde{\eta} + \frac{(1 + \Phi \xi) \chi}{2} - \delta \right] \ln (1 + \eta)
\]

\[\blacksquare\]

**Proof of Proposition 5.** We know set \( p = 0 \) and \( \xi = 0 \). Hence we have \( \Phi = 1 \) and \( b_m = 0 \). Then from (22) we get
\[
\tau = \frac{\alpha \pi_\eta + (1 - \alpha) \pi_\lambda}{\chi w} + \frac{\chi}{2} - \delta.
\]
Recall that
\[
\pi_\eta = \tilde{\eta} \frac{Z}{M} \text{ and } \pi_\lambda = \tilde{\lambda} \frac{Z}{M}
\]
\[
w = (1 - \tilde{\eta})^\alpha (1 - \tilde{\lambda})^{1 - \alpha} \left( \frac{Q}{M} \right)
\]
\[
Z^* = \left[ \frac{(1 - \tilde{\eta})^\alpha (1 - \tilde{\lambda})^{1 - \alpha}}{\alpha (1 - \tilde{\eta}) + (1 - \alpha)(1 - \tilde{\lambda})} \right] QLP
\]
Thus
\[
\frac{\alpha \pi_\eta + (1 - \alpha) \pi_\lambda}{\chi w} = \left[ \frac{\alpha \tilde{\eta} + (1 - \alpha) \tilde{\lambda}}{1 - [\alpha \tilde{\eta} + (1 - \alpha) \tilde{\lambda}]} \right] \frac{LP}{\chi}
\]
Next the entry rate is simply
\[
a_0^* = \tau - a_m
= \frac{\alpha \pi_\eta + (1 - \alpha) \pi_\lambda}{\chi w} - \frac{\chi}{2} - \delta.
\]
Next from the labor market
\[
LP = 1 - L^G - L^E - L^I
= 1 - L^G - \left[ \frac{\alpha \tilde{\eta} + (1 - \alpha) \tilde{\lambda}}{1 - [\alpha \tilde{\eta} + (1 - \alpha) \tilde{\lambda}]} \right] LP + \delta \chi
\]
Thus we have
\[
LP = \left[ 1 + \delta \chi - L^G \right] \left( 1 - [\alpha \tilde{\eta} + (1 - \alpha) \tilde{\lambda}] \right)
\]
Thus
\[
a_0^* = \left[ \alpha \tilde{\eta} + (1 - \alpha) \tilde{\lambda} \right] \left( \frac{1 - L^G}{\chi} + \delta \right) - \frac{\chi}{2} - \delta
\]
and
\[ \tau^* = \left[ \alpha \tilde{\eta} + (1 - \alpha) \tilde{\lambda} \right] \left( \frac{1 - L^G}{\chi} + \delta \right) + \frac{\chi}{\alpha} - \delta \]

Finally the growth rate is
\[ g^* = (a^* + a_0^*) \ln (1 + \eta)^{\alpha^*} (1 + \lambda)^{(1 - \alpha^*)} \times \left( \left[ \alpha \left( \tilde{\eta} - \tilde{\lambda} \right) + \tilde{\lambda} \right] \left( \frac{1 - L^G}{\chi} + \delta \right) + \frac{\chi}{\alpha} - \delta \right) \left[ \alpha^* \ln \left( \frac{1 + \eta}{1 + \lambda} \right) + \ln (1 + \lambda) \right] \]

Proof of proposition 6. As the downstream production technology is unchanged in the generalized model and we continue to impose symmetry across the industries, we reach the same conclusions regarding prices and output at the industry level, namely that
\[ P_i = P = \frac{1}{M} \quad \text{and} \quad Y_i = Y = Z. \] (59)

Henceforth, we can drop the industry index \( i \). The perfectly competitive firm that produces midstream good \( Y_i \) takes equilibrium prices \( P \) and \( p_j \) as given while maximizing its profit
\[ \max_{y_j} \left\{ P \left[ \int_0^1 y_j \frac{\epsilon - 1}{\epsilon} dj \right] - \int_0^1 p_j y_j dj \right\}. \]

This maximization leads to the following inverse demand for upstream good \( j \)
\[ p_j = P \left( \frac{Y}{y_j} \right)^{\frac{1}{\epsilon}}. \]

Monopolist \( j \) has normalized productivity \( \hat{q}_j \). The firm takes the demand function for its product as given and solves the following maximization problem
\[ \pi_j = \max_{y_j} \left\{ PY^{\frac{1}{\epsilon}} y_j^{\frac{\epsilon - 1}{\epsilon}} - \left( \frac{w}{Q} \right) \hat{q}_j^{-1} y_j \right\}. \]

This delivers the following optimal quantity
\[ y_j = \left[ \frac{1}{M} \left( \frac{\epsilon - 1}{\epsilon} \right) \left( \frac{Q}{w} \right) \hat{q}_j \right]^{\frac{\epsilon}{\epsilon - 1}} Z \]

Plugging this into the production function for midstream goods, we find a relationship between wage \( w \) and aggregated quality \( Q \)
\[ w = \frac{1}{M} \left( \frac{\epsilon - 1}{\epsilon} \right) Q \] (60)

With this, we can greatly simplify the expression of the firm quantity and price choices
\[ y_j = \hat{q}_j^* Z \quad \text{and} \quad p_j = \frac{1}{M \hat{q}_j} \]

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Denote variables normalized by $Z/M$ with a “∼”. Then the normalized profit and labor are given by

$$
\tilde{\pi}_j = \frac{\hat{\pi}_j - 1}{\varepsilon} \quad \text{and} \quad l_j = \frac{\hat{\pi}_j - 1}{\tilde{w}} \left( \frac{\varepsilon - 1}{\varepsilon} \right).
$$

(61)

where $\tilde{w}$ is the normalized wage. As a result, the profit share and labor share are given by

$$
\tilde{\Pi} = \frac{1}{\varepsilon} \quad \text{and} \quad \tilde{w}L^P = \frac{\varepsilon - 1}{\varepsilon}.
$$

(62)

This implies that the labor allocated to each product line satisfies

$$
\ell_j = \hat{q}_j^{\varepsilon - 1}L^P
$$

Proof of Lemma 2. We write the value function as

$$
V_t (\hat{q}_t) = \frac{1}{\varepsilon} \hat{q}_t^{-1} \Delta t Z_t + (1 - r \Delta t) (1 - \tau \Delta t - \kappa \Delta t) V_{t+\Delta t} (\hat{q}_{t+\Delta t})
$$

Conjecture $V_t (\hat{q}_t) = \beta (\hat{q}_t) \frac{Z_t}{M}$:

$$
\beta (\hat{q}_t) Z_t = \frac{1}{\varepsilon} \hat{q}_t^{-1} \Delta t Z_t + (1 - r \Delta t) (1 - \tau \Delta t - \kappa \Delta t) \beta (\hat{q}_{t+\Delta t}) Z_{t+\Delta t}
$$

$$
= \frac{1}{\varepsilon} \hat{q}_t^{-1} \Delta t Z_t + (1 - r \Delta t) (1 - \tau \Delta t - \kappa \Delta t) \beta (\hat{q}_{t+\Delta t}) (1 + g \Delta t) Z_t
$$

which implies

$$
\beta (\hat{q}_t) = \frac{1}{\varepsilon} \hat{q}_t^{-1} \Delta t + (1 - r \Delta t - \tau \Delta t - \kappa \Delta t + g \Delta t) \beta (\hat{q}_{t+\Delta t})
$$

First subtract $\beta (\hat{q}_{t+\Delta t})$ from both sides and divide both sides by $\Delta t$ and take the limit to get

$$
\lim_{\Delta t \to 0} \frac{[\beta (\hat{q}_t) - \beta (\hat{q}_{t+\Delta t})]}{\Delta \hat{q}} = \lim_{\Delta t \to 0} \frac{1}{\varepsilon} \hat{q}_t^{-1} - (r + \tau + \kappa - g) \beta (\hat{q}_{t+\Delta t})
$$

which simply implies

$$
(r + \tau + \kappa - g) \beta (\hat{q}_t) + \hat{q}_t \frac{\partial \beta}{\partial \hat{q}} = \frac{1}{\varepsilon} \hat{q}_t^{-1}.
$$

We rewrite the normalized production value as

$$
\xi_1 \hat{q}_t^{-1} \beta (\hat{q}_t) + \frac{\partial \beta}{\partial \hat{q}} = \xi_2 \hat{q}_t^{\varepsilon - 2}
$$

(63)

where

$$
\xi_1 \equiv (r + \tau + \kappa) / g - 1 \quad \text{and} \quad \xi_2 \equiv \frac{1}{\varepsilon g}.
$$

Let us drop the indices for notational convenience. Recall that for a differential equation of the form

$$
f(x) y(x) + y'(x) = g(x), \quad (64)
$$

We rewrite the normalized production value as

$$
\xi_1 \hat{q}_t^{-1} \beta (\hat{q}_t) + \frac{\partial \beta}{\partial \hat{q}} = \xi_2 \hat{q}_t^{\varepsilon - 2}
$$

(63)
the general solution is
\[ y(x) = e^{-F(x)} \left( \int e^{F(x)} g(x) \, dx + C \right) \] (65)
where \( F(x) = \int f(x) \, dx \) and \( C \) is the constant of integration. We first define the following equalities
\[ f(\hat{q}) \equiv \xi_1 \hat{q}^{-1}, \quad y(\hat{q}) \equiv \beta(\hat{q}) \quad \text{and} \quad g(\hat{q}) \equiv \xi_2 \hat{q}^{\epsilon - 2}. \]

Then (63) – (65) imply \( F(\hat{q}) = \ln \hat{q}^{\xi_1} \) and
\[ \beta(\hat{q}_t) = \hat{q}^{-\xi_1} \left( \int \xi_2 \hat{q}^{\xi_1 + \epsilon - 2} \, d\hat{q} + C \right) = \frac{\xi_2 \hat{q}^{\epsilon - 1}}{\xi_1 + \epsilon - 1} + C \hat{q}^{\xi_1}. \]

Imposing the boundary condition \( \beta(\hat{q}) = \frac{\hat{q}^{\xi - 1/\epsilon}}{r + \tau + \kappa + g} \) we find
\[ C = \hat{q}^{\xi_1} \left[ \frac{\hat{q}^{\xi - 1/\epsilon}}{r + \tau + \kappa + g} - \frac{\xi_2 \hat{q}^{\epsilon - 1}}{\xi_1 + \epsilon - 1} \right] \]
Therefore substituting back the constant term and the subindices we get
\[ \beta(\hat{q}) = \frac{[\hat{q}^{\epsilon - 1} - \hat{q}^{\xi - 1}]}{\epsilon} + \frac{\hat{q}^{\xi - 1/\epsilon}}{r + \tau + \kappa + g} \]

\[ \text{Proof of Proposition 7.} \quad \text{Conjecture} \quad V_t(\hat{q}) = Z_t \left[ \sum_{\hat{q} \in \hat{q}} \beta(\hat{q}_t) + n \beta_m \right] \]
\[ Z_t \left[ \sum_{\hat{q} \in \hat{q}} \beta(\hat{q}_t) + n \beta_m \right] = \max_{a, b} \left( 1 - r \Delta t \right) \]
\[ \left\{ \begin{array}{l}
\sum_{\hat{q} \in \hat{q}} \frac{1}{\xi} \hat{q}^{\xi - 1} Z_t \Delta t - n M w_1 \Delta t \left[ h_a(a) + h_b(b) + 1_{(b > 0)} \phi \right] \\
+ n a \Delta t Z_t \left[ \sum_{\hat{q} \in \hat{q}} \beta(\hat{q}_t + \Delta t) + \mathbb{E}_{\hat{q}, s} \beta(\hat{q}_t + \Delta t + \hat{q}) + (n + 1) \beta_m \right] \\
+ n b (1 + \rho_m) \Delta t Z_t \Delta t \left[ \sum_{\hat{q} \in \hat{q}} \beta(\hat{q}_t + \Delta t) + \mathbb{E}_{\hat{q}, \alpha} \beta(\hat{q}_t + \Delta t + \alpha) + (n + 1) \beta_m \right] \\
+ n \kappa \Delta t Z_t \Delta t \left[ \sum_{\hat{q} \in \hat{q}} \beta(\hat{q}_t + \Delta t) + \mathbb{E}_{\hat{q}, \beta} (\hat{q}_t + \Delta t + \beta) + (n + 1) \beta_m \right] \\
+ \tau \Delta t Z_t \Delta t \left[ \sum_{\hat{q} \in \hat{q}} \beta(\hat{q}_t + \Delta t) - \beta(\hat{q}_t) + (n - 1) \beta_m \right]
\end{array} \right\} \]
\[ = \max_{a, b} \left( 1 - r \Delta t \right) \]
\[ \left\{ \begin{array}{l}
+ x \Delta t Z_t \left[ \sum_{\hat{q} \in \hat{q}} \beta(\hat{q}_t + \Delta t + \hat{q}) + n \beta_{m + 1} \right] \\
+ \frac{M - m}{M} \left\{ \mathbb{E}_{\hat{q}, \gamma} \beta(\hat{q}_t + \Delta t + \gamma) + \beta_{m + 1} - p_m \right\} \\
+ \frac{m}{M} \left\{ \mathbb{E}_{\hat{q}, \kappa} \beta(\hat{q}_t + \Delta t + \kappa) + \beta_m - p_m \right\}
\end{array} \right\} \\
+ \left( 1 - n a \Delta t - n b (1 + \rho_m) \Delta t - (n + 1) \kappa \Delta t - n \tau \Delta t - x \Delta t \right) Z_t \Delta t \left[ \sum_{\hat{q} \in \hat{q}} \beta(\hat{q}_t + \Delta t) + n \beta_m \right] \]
Recalling the definitions of $\beta_\eta$ and $\beta_\lambda$, some simple algebra yields

$$Z_t \left[ \sum_{q \in \mathcal{Q}} \beta(\hat{q}_t) + n\beta_m \right] = \max_{a,b} \left\{ \begin{array}{l} \sum_{q \in \mathcal{Q}} \frac{1}{\varepsilon} \hat{q}^t - 1 Z_t \Delta t - nM w_t \Delta t \left[ h_a(a) + h_b(b) + 1_{(b>0)} \phi \right] \\ \alpha \beta_\eta + (1 - \alpha) \beta_\lambda + \beta_m \\ + nb (1 + \rho_m) \Delta t Z_{t+\Delta t} \left[ \beta_\eta + \beta_m \right] \\ + n \kappa \Delta t Z_{t+\Delta t} \mathbb{E}_q \beta (\hat{q}_{t+\Delta t}) - n \tau \Delta t Z_{t+\Delta t} \beta_m \\ + n \left( \beta_{m+1} - \beta_m \right) \\ + x \Delta t Z_{t+\Delta t} \left[ \frac{M - m}{M} \mathbb{E}_q \beta (\hat{q}_{t+\Delta t} + \hat{s}) \right] \\ + \frac{m}{M} \mathbb{E}_q \beta (\hat{q}_{t+\Delta t} + \hat{s}) \right\} \\ + (1 - r \Delta t - \kappa \Delta t - \tau \Delta t) Z_{t+\Delta t} \sum_{q \in \mathcal{Q}} \beta(\hat{q}_{t+\Delta t}) \\ + (1 - r \Delta t) Z_{t+\Delta t} n \beta_m \right\} \right\}$$

Note that we assume

$$p_m = \beta_{m+1} + \mathbb{E}_q \beta (\hat{q}_{t+\Delta t} + \hat{s})$$

$$p'_m = \beta_m + \mathbb{E}_q \beta (\hat{q}_{t+\Delta t} + \hat{s})$$

Next collect the terms that are direct function if $\hat{q}$. We get

$$Z_t \sum_{q \in \mathcal{Q}} \beta(\hat{q}_t) = \sum_{q \in \mathcal{Q}} \frac{1}{\varepsilon} \hat{q}^{t-1} Z_t \Delta t + (1 - r \Delta t - \kappa \Delta t - \tau \Delta t) Z_{t+\Delta t} \sum_{q \in \mathcal{Q}} \beta(\hat{q}_{t+\Delta t})$$

In steady state we have $Z_{t+\Delta t} \approx (1 + g \Delta t) Z_t$

$$\lim_{\Delta t \to 0} \frac{\beta(\hat{q}_t) - \beta(\hat{q}_{t+\Delta t})}{\Delta t} = \lim_{\Delta t \to 0} \frac{1}{\varepsilon} \hat{q}^{t-1} - (r + \kappa + \tau - g) \beta(\hat{q}_{t+\Delta t})$$

which gives us

$$\beta(\hat{q}_t) \hat{q}_t g = \frac{1}{\varepsilon} \hat{q}^{t-1} - (r + \kappa + \tau - g) \beta(\hat{q}_t)$$

We know from the previous lemma that the solution is simply $\beta(\hat{q}_t) = \frac{\hat{q}^{t-1/\varepsilon}}{r + r + \kappa + \gamma (\varepsilon - 2)}$. Then the remaining terms in the value function are

$$Z_t n \beta_m = \max_{a,b} \left\{ \begin{array}{l} - nM w_t \Delta t \left[ h_a(a) + h_b(b) + 1_{(b>0)} \phi \right] \\ \alpha \beta_\eta + (1 - \alpha) \beta_\lambda + \beta_m \\ + nb (1 + \rho_m) \Delta t Z_{t+\Delta t} \left[ \beta_\eta + \beta_m \right] \\ + n \kappa \Delta t Z_{t+\Delta t} \mathbb{E}_q \beta (\hat{q}_{t+\Delta t}) \\ - n \tau \Delta t Z_{t+\Delta t} \beta_m \\ + n \tau \Delta t Z_{t+\Delta t} \left( \beta_{m+1} - \beta_m \right) \\ + (1 - r \Delta t) Z_{t+\Delta t} n \beta_m \right\} \right\}$$
First rewrite $Z_{t+\Delta t} = (1 + g\Delta t) Z_t$ which just replaces the last line by $(1 + g\Delta t - r\Delta t) Z_t n \beta_m$. Then subtract the last line from the lefthand side and divide both sides by $nZ_t \Delta t$ and take the limit $\Delta t \to 0$ to get

$$(r - g) \beta_m = \max_{a,b} \left\{ \begin{array}{c} -\bar{w} \left[ h_a(a) + h_b(b) + 1_{(b>0)} \phi \right] \\ + a [\alpha_n + (1 - \alpha) \beta_m + \beta_m] \\ + b (1 + \rho_m) [\beta_n + \beta_m] \\ + x (\beta_{m+1} - \beta_m) - \tau \beta_m + \kappa \mathbb{E}_q \beta(\bar{q}) \end{array} \right\}$$

Finally the first order conditions follow from the value function. This completes the proof. ■
B Data & Data Organization

Data

Empirical investigation on the relationship between R&D investment and multi-market activity of a firm requires reliable and extensive information not only on product markets and on R&D characteristics of individual firms, but also on firm ownership status. The latter allows us to identify the product markets to which the firm is linked via its business group. We obtain this information from three different data-sets.

R&D Information  Information about R&D investment comes from the annual R&D Survey conducted by the French Ministry of Research. The R&D survey comes in annual waves of cross-sectional data, where the same firms are not necessarily sampled year after year (Mairesse and Mohnen, 2010). The survey covers a representative sample of French firms of more than 20 employees investing into R&D. However firms with less than .8 Million Euros of R&D investment fill out a shorter and simplified survey. The survey includes extensive information about the financing of R&D. It not only breaks down R&D investment according to the source of the funds, but also provides its allocation to different types of R&D. More specifically all firms are asked to report their R&D investment into basic and applied research.

Multi-Market Activity  The identification of business group structures is based on a yearly survey by INSEE called “Enquete Liasons Financieres” (LIFI). It covers all economic activities but restricts its attention to firms which either employ more than 500 employees, or generate more than 60 Million Euros of revenues, or hold more than 1.2 Million Euros of traded shares. However since 1998 the survey is crossed with information from Bureau Van Dijk and thus covers almost the whole economy. The LIFI survey contains information which makes it a unique data set to study the relationship between multi-market activity and investment into basic research. Besides providing information on direct financial links between firms, it also accounts for indirect stakes and cross-ownerships when identifying the head of the group. This is important as it allows to precisely reconstruct the group structure even in the presence of pyramids. This feature allows us to obtain a reliable account of the structure of business groups in the French economy and, as a consequence, reliable measures of our key variable, the multi-market presence of business groups.

Since each firm itself can be active in several markets, we cross the data set with an extensive yearly survey by the Ministry of Industry (“Enquete Annuelle des Entreprises”). The survey is filled by French firms with more than 20 workers and contains information not only on the
different markets in which a firm operates but also information on market dedicated sales for each segment. The data covers the vast majority of French firms and spans over the period 2000-2006.

**Balance Sheet Information** We use the firm- and industry-level data sets based on accounting data extracted again extracted from the EAE files. The data also includes unique firm identifiers allowing us to match it to the R&D and LIFI data.

**Data Organization**

We first identify the ownership status of each firm in the economy and the head of the group to which the firm is affiliated. Indeed, our data source (LIFI) defines a group as a set of firms controlled, directly or indirectly, by the same entity (the head of the group). We rely on a formal definition of control, requiring that a firm holds directly or through cross-ownership at least 50 percent of the voting rights in another firm’s general assembly. We do not expect this to be a major source of bias in our sample as most French firms are private and ownership concentration is strong even among listed firms.\(^{20}\) Firms which do not conform to this definition are classified as stand alone firms.

We then match the ownership information to our balance sheet data and to our survey on lines of business within firms. We drop firms which appear in the ownership data but for which we cannot find balance sheet information. We also delete as outliers firm-year observations whose ROA falls outside a multiple of five of the interquartile range, which report 0 employment or which have negative sales. Based on our two sources of information we identify the main line of business from the balance sheets and the different segments of the firm from the survey on lines of business. For computational convenience we create a new firm-group identifier that allows us to aggregate at the same time business groups, business groups with multidivisional firms, exclusively multidivisional firms and true stand alone firms.

We then define four measures of multi-market activity. The first measure counts each market in which the firm-group is present either via its ownership links or its multidivisional structure. The second measure counts each market in which the firm-group is present with at least 9 employees via its ownership links or its multidivisional structure. The third measure counts

\(^{20}\)In their overview of ownership structures and voting power in France, Bloch and Kremp (1999) show that ownership concentration is pervasive: for non-listed companies with more than 500 employees the main shareholder’s ownership stake is 88%. The degree of ownership concentration is slightly lower for listed companies but still above 50 percent in most cases.
each market in which the firm-group is present exclusively via its ownership links. The final measure counts each market in which the firm-group is present exclusively via its ownership links and excluding financial activities.

We then use our patent data to compute measures of the number of technology classes in which a firm is present. We define as the number of technology classes a firm is present as the cumulative distinct patent classes granted to the firm between 1993 and \( t \). If a firm has made no application during this period its number of technology classes are 0.\(^{21}\) Alternatively we use patent applications instead of received patents on theoretical and practical grounds. First because patent applications constitute a direct measure of where the firm believes it is technologically active. Second because of the relatively short time-span for which we are able to establish a link between the patent data and the firm identifier.

We then define firm characteristics from balance sheet data. There are three possible organizational types and comparison issues might arise. Taking the firm as the economic unit of interest has the advantage of simplicity since information is directly available in the balance sheets. However this method has the disadvantage of not being comparable across organizational types. Indeed most information for multidivisional firms is aggregated across lines of segment whereas firms belonging to business groups have market specific information. Similar to existing studies by the Ministry of Research (Dhont-Peltrault and Pfister E., 2009), we decided to aggregate the information to the economic unit at the highest level of control. The firm level for stand alone and multidivisional firms, the business group level for firms affiliated through majority ownership.\(^{22}\)

In a final step we match the firms’ balance sheet and patent information to information contained in the R&D Survey. We focus on firms for which we have R&D information. Again we aggregate at the highest level of control. As before one has to be cautious in aggregating on the basis of variables which might be prone to double-counting. When constructing information on the basic R&D intensity of a firm this is not the case as we are focusing exclusively on “internal” research expenditures. Therefore if a member of the group contracts out research with another member of the group then one will be counted as “external” research expenditures and the other one as “internal” expenditures. To correct for outliers in the dependent variable we drop firm-year observations whose basic research intensity, conditional

\(^{21}\) Alternatively we assume that the firm is present in one technological market. Results remain qualitatively similar.

\(^{22}\) In addition to the economic rationale for constructing the data at the highest level of control there is also a legal argument. Indeed most public administrations and tribunals define eligibility of firms to subsidy programs with respect to the business groups to which they belong.
on positive basic research, falls outside a multiple of five of the interquartile range. In addition we exclude firm-year observations whose total R&D to sales ratio falls outside a multiple of five of the interquartile range. 23

Variable List

All variables are organized and computed according to the method set out in the previous section. To summarize, we decided to aggregate the information to the economic unit at the highest level of control. The firm level for stand alone and multidivisional firms, the business group level for firms affiliated through majority ownership. In the remainder of the document we will, for the sake of notational convenience, refer generically to firms.

- **Basic Research Intensity**: total basic research by firm $i$ in year $t$ divided by total applied research of firm $i$ in year $t$. The formulation of the survey questions related to the type of research undertaken is directly derived from the definitions provided by the Frascati Manual;

- **# of Industries**: sum of all distinct SIC codes within firm $i$ in year $t$ irrespective of organizational form (business group or multidivisional structure). Industries are successively defined at the 4,3,2 and 1 digit SIC levels;

- **# of Industries - Weighted Sum**: weighted sum of all distinct bilateral 1 digit SIC links within firm $i$ in year $t$. Weights are computed on the basis of the empirical frequency of each bilateral SIC link in each year $t$;

- **# of Patent Classes Applied**: sum of cumulated distinct patent-class applications within firm $i$ in year $t$. Cumulated patent-class applications are computed for the period leading from 1993 to year $t$. Patent classes are successively defined at the 5,4,3,2 and 1 digit levels (EPO Classification);

- **# of Patent Classes Granted**: sum of cumulated distinct patent-class grants within firm $i$ in year $t$. Cumulated patent-class grants are computed for the period leading from 1993 to year $t$. Patent classes are successively defined at the 5,4,3,2 and 1 digit levels (EPO Classification);

23 Alternatively we exclude firm-year observations whose basic to applied R&D ratios falls above the 99th percentile of the distribution. Remains are qualitatively similar.
• **Financial Int.**: binary indicator equal to 1 if firm $i$ in year $t$ is present in a financial industry, 0 otherwise;

• **Foreign HQ**: binary indicator equal to 1 if the headquarter of firm $i$ in year $t$ is located outside France, 0 otherwise;

• **Market Share**: weighted average of total sales of firm $i$, year $t$ in industry $k$ divided by total industry sales year $t$. Weights are computed on the basis of the industry share of employment within firm $i$ in year $t$;

• **Outsourcing to Univ.**: binary indicator equal to 1 if firm $i$ in year $t$ has outsourced R&D to French universities, 0 otherwise;

• **Profitability - ROA**: weighted average of EBIDTA divided by total fixed assets of all subsidiaries within firm $i$ in year $t$. Weights are computed on the basis of the subsidiaries share of employment within firm $i$ in year $t$;

• **Profitability - ROS**: weighted average of EBIDTA divided by total sales of all subsidiaries within firm $i$ in year $t$. Weights are computed on the basis of the subsidiaries share of employment within firm $i$ in year $t$;

• **Public R&D Funds**: binary indicator equal to 1 if firm $i$ in year $t$ has received French public funds, 0 otherwise;

• **Research Area**: weighted average of the share of respectively biotech / software / environment research in research expenditures in firm $i$ year $t$. Weights are computed on the basis of the subsidiaries' share of total R&D within firm $i$ in year $t$;

• **Total Employment**: total employment of firm $i$ in year $t$;

• **IV - State Present in 1986**: binary indicator equal to 1 if the French state had a non-zero equity stake in firm $i$ in 1986;

• **IV - SOE in 1986**: binary indicator equal to 1 if the French state had a controlling equity stake in firm $i$ in 1986;
C Robustness Checks on Reduced Form Results

Appendix C provides further robustness checks on the correlation between basic research incentives of a firm and its multi-industry presence. Our baseline specification is related to the number of distinct 1 digit SIC activities in which a firm operates but extends to finer SIC classifications.

Confounding Factors Columns (1) and (2) check robustness of the results with respect to confounding factors. Column (1) estimates the model only allowing for year and organization fixed effects, whereas column (2) includes a set of potential confounding factors. Results in column (1) suggest that presence in an additional industry, not accounting for other variables such as size, is associated on average to 1.4 percentage points higher basic research intensity of firms. In column (2) the set of regressors includes controls for size, profitability and headquarter localization. The impact of multi-industry presence is slightly lower but remains statistically significant.\textsuperscript{24} Estimates on the localization of headquarters are also statistically significant at the 5\% level. Total employment and profitability on the other hand are not.

Measures of Multi-Industry Presence Columns (3) and (4) provide alternative measures for the multi-industry presence of firms. Column (3) defines multi-industry presence on the basis of a firms technological spectrum. Technological spectrum is proxied through the cumulated patent class applications of firm i in the period 1993 to year t. The coefficient is very similar in magnitude and precision to the one obtained using distinct 1 digit SIC industries. Column (4) measures multi-industry presence as a weighted sum of all distinct bilateral 1 digit SIC links within firm i in year t considering only distinct legal entities linked by majority ownership. Weights are computed on the basis of the empirical frequency of each bilateral SIC link in each year t. Intuitively, if a given bilateral industry link is rare then industries are more likely to be very different. Multi-industry presence is still positively related to basic research intensity, the different point estimate being linked to the different support of the weighted industry variable.

Causality and Instrumental Variables Columns (5) and (6) address the potential concern of reverse causality, i.e. basic research leading to larger economic scope of firms. We exploit historical ownership structures that affected a firms’ multi-industry presence as instrumental

\textsuperscript{24}Further checks on control variables included market shares, R&D subsidies, collaborations with universities, the presence of financial intermediaries / state in the capital of the firm, industry fixed effects and the use of a mean patent scaling method.
variables. The two instruments are defined as *State Ownership 1985-1987* and *State Owned between 1985-1987*.

The rationale behind our identification strategy is as follows. In 1981 Francois Mitterrand was elected President of the Republic and implemented a vast nationalization programme across industries. Even before that period the tradition of French state intervention resulted in a significant fraction of the economy being under state control. Consistent with Colbertist policies the State also modified the economic scope of its firms by merging unrelated firms into large conglomerates of national champions. In 1987 however, Jacques Chirac was elected Prime Minister on a liberal platform and this marked the beginning of privatizations which continued into the 90’s. The embedded exclusion restriction therefore requires that state control in the 80’s is associated today to a greater basic research intensity of firms only because of politically motivated mergers. The implicit assumption is that when these firms became private they adjusted their research spending from the social to the private optimum, but did not adjust their multi-industry presence. First stage estimates show that state ownership in the 80’s is associated on average to 1.2 more industry links for firms between 2000 and 2006. The associated F-test is well above the critical levels related to weak instruments tables. The instrumented LATE coefficient related to multi-industry presence in the second stage is nearly twice as large in magnitude with respect to the non-instrumented coefficients.

**Estimation** Columns (7) and (8) use alternative estimation methods for the baseline model with covariates. Column (7) presents estimates of the Heckman selection model, whereas column (8) presents estimates from a negative binomial model. In both cases estimates suggest a positive and statistically significant relation between basic research intensity and multi-industry presence.
D Construction of Within-Industry Spillover

Figure 8 provides a graphical intuition for the computation of the citation information.

**Figure 8: Computing the Cooling Down Rate**

Patent A is granted in 1978. In 1981, when patent A is 3 years old, it receives citations from both patent B and patent C which applied in 1981. Patent B in the following 10 years was cited by patents V, W and X, whereas patent C was only cited by patents Y and Z. The average citation of citing patents for patent A at age 3 is therefore 2.5. The timing of the computation implies that we need to be cautious with respect to possible truncation. We therefore compute our measure for patents between 1975 and 1985. This implies that, inclusive of the 10 year forward lag, we can observe without truncation all our patents until the age of 10.
**Robustness Checks**  Figure 9 provides robustness checks for the estimates on the cooling down rate of patents originating from basic and applied research. The left panel of the figure measures *Average Citations of Citing Patents* computing the 5 year forward citations of the citing patents and is measured for patents granted in the period 1975-1985. The right panel re-classifies university patents which were defined as private depositors. In both cases results are unchanged with a citation difference between public and private patents that becomes statistically non-significant at year 8. Indeed in France most of the academic patents are accounted for in the “public” category. French universities generally manage their patents through public research institutions to which academics are typically affiliated, one example being the CNRS.

**Figure 9: Citation Patterns for French Public and Private Patents**

![Citation Patterns for French Public and Private Patents](image)

<table>
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<th>Age</th>
<th>1</th>
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<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
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<th>8</th>
<th>9</th>
<th>10</th>
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<td>.16**</td>
<td>.28***</td>
<td>.16**</td>
<td>.22**</td>
<td>.15**</td>
<td>.33***</td>
<td>.08</td>
<td>.18</td>
<td>.15</td>
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<td>(0.07)</td>
<td>(0.08)</td>
<td>(0.06)</td>
<td>(0.07)</td>
<td>(0.07)</td>
<td>(0.11)</td>
<td>(0.08)</td>
<td>(0.11)</td>
<td>(0.12)</td>
<td></td>
</tr>
<tr>
<td>10 Yr Forward Citations</td>
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<td>.3**</td>
<td>.62***</td>
<td>.28**</td>
<td>.42**</td>
<td>.23</td>
<td>.71***</td>
<td>.08</td>
<td>.39</td>
<td>.15</td>
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<td>(0.15)</td>
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<td>(0.14)</td>
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<td>(0.16)</td>
<td>(0.25)</td>
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</table>

*Note:* The figures separately plot *Average Citations of Citing Patents* for French public patents (blue line) and French private patents (red line) across patent age. The left panel computes *Average Citations of Citing Patents* computing the 5 year forward citations of the citing patents and is measured for patents granted in the period 1975-1985. The bottom panel computes *Average Citations of Citing Patents* computing the 5 year forward citations of the citing patents and re-classifying university patents as public patents. The table reports differences in citation patterns using two sample t-test with unequal variances. One star denotes significance at the 10% level, two stars denote significance at the 5% level, and three stars denote significance at the 1% level.
<table>
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<tr>
<td></td>
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<tr>
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<td>-0.048**</td>
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Notes: Columns 1 and 2 re-estimate the Tobit model with different sets of regressor. Columns 3 and 4 modify the measure of a firm's multi-industry presence. Column 3 uses patent applications of French firms to the European patent office (1993-2003) to count the number of distinct technological fields in which they are present (1 digit IPC classification). Column 4 weights each bilateral industry link of a firm by the empirical frequency of this link in the French economy, thus giving more weight to less related industries. Columns 5 and 6 re-estimate the model by instrumenting contemporary multi-industry presence by historical ownership structures. More specifically we exploit the nationalization wave of the Mitterrand era that preceded the privatization of the 90’s. The idea is that state ownership effectively increased the scope of a firm’s economic activities. Column 5 uses state participation in the capital of a firm in 1986 as an instrument. Column 6 uses state ownership of a company in 1986 as an instrument. Both instruments accurately predict an increased multi-industry presence nowadays. Column 7 and 8 estimate the relationship between multi-industry presence and basic research intensity by using a Heckman model and a negative binomial model. Tobit estimates relate to the marginal effect of the regressors with respect to the uncensored variable mean, and are evaluated at the sample mean of covariates (except for categorical variables evaluated for firm which are present in 1 industry, non-foreign owned, in 2002). Robust standard errors clustered at the firm level in parentheses. See Appendix B for the definition of variables.