Margin Regulation and Volatility

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Margin Regulation and Volatility*

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Abstract

In this paper we examine the quantitative effects of margin regulation on volatility in asset markets. We consider a general equilibrium infinite-horizon economy with heterogeneous agents and collateral constraints. There are two assets in the economy which can be used as collateral for short-term loans. For the first asset the margin requirement is exogenously regulated while the margin requirement for the second asset is determined endogenously. In our calibrated economy, the presence of collateral constraints leads to strong excess volatility. Thus, a regulation of margin requirements may have stabilizing effects. However, in line with the empirical evidence on margin regulation in U.S. stock markets, we show that changes in the regulation of one class of assets may have only small effects on these assets’ return volatility if investors have access to another (unregulated) class of collateralizable assets to take up leverage. In contrast, a countercyclical margin regulation of all asset classes in the economy has a very strong dampening effect on asset return volatility.

Keywords: collateral constraints, general equilibrium, heterogeneous agents, margin requirements, Regulation T.

JEL Classification Codes: D53, G01, G12, G18.

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1 Introduction

The stock market bubble of 1927–1929 and the subsequent “great crash” of 1929 were accompanied by an extraordinary growth and subsequent contraction of trading on margin, see White (1990). The crash and the following great depression led the United States Congress to pass the Securities Exchange Act of 1934 which granted the Federal Reserve Board (FRB) the power to set initial margin requirements on national exchanges. The introduction of this law had three major purposes: the reduction of “excessive” credit in securities transactions, the protection of buyers from too much leverage, and the reduction of stock market volatility (see, e.g., Kupiec (1998)). Under the mandate of this law, the FRB established Regulation T to set minimum equity positions on partially loan-financed transactions of exchange-traded securities. From 1947 until 1974, the FRB frequently changed initial margin requirements to manage the volatility in stock markets. During this time the FRB viewed margin requirements as an important policy tool.1 The introduction and frequent adjustments of the margin ratio provided a natural experiment that was analyzed by a sizable empirical literature. The vast majority of these studies, however, did not find substantial evidence that regulating margin requirements in stock markets had an economically significant impact on market volatility. Fortune (2001) concludes after a comprehensive review of the literature that it “does provide some evidence that margin requirements affect stock price performance, but the evidence is mixed and it is not clear that the statistical significance found translates to an economically significant case for an active margin policy.”

Eighty years after the great crash, in the aftermath of the financial crisis of 2007–2009, it has again been argued that excessively low margin requirements or haircuts,2 this time in repo and securities lending markets, caused the build-up of leverage before the crisis and contributed to procyclicality in financial markets (see, e.g the Committee on Global Financial Stability, CGFS (2010)). The CGFS concluded that regulators and competent authorities should consider the introduction of “minimum constant through-the-cycle margins and haircuts, with a possible countercyclical add-on” (CGFS (2010), p.9). In light of the recent interest in margin regulation, a better understanding of the economic mechanism underlying margin regulation is necessary. For this purpose we revisit Regulation T, the most thoroughly studied margin regulation policy. We provide a model-based explanation for the inconclusive findings of the empirical literature on Regulation T. In addition, we explore how to design a successful regulation of margin requirements. For both purposes we analyze the effects of margin regulation on asset return volatility within a calibrated infinite-horizon asset-pricing model with heterogeneous agents. We first show that, in line with the evidence on Regulation T, changes in the regulation of one class of collateralizable assets may have only small effects on the assets’ return volatility if investors have access to another

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1 For example, in a U.S. Senate testimony in 1955, FRB chairman William McChesney Martin summarized the FRB’s view on margin policy as follows (as quoted in Moore (1966)): “The task of the Board, as I see it, is to formulate regulations with two principal objectives. One is to permit adequate access to credit facilities for securities markets to perform the basic economic functions. The other is to prevent the use of stock market credit from becoming excessive. The latter helps to minimize the danger of pyramiding credit in a rising market and also reduces the danger of forced sales of securities from undermargined accounts in a falling market.”

2 For what we call margin requirement, the term haircut is often used in the current policy debate.
(unregulated) class of assets to enter leveraged positions. We also demonstrate that a common countercyclical margin regulation of all asset classes in the economy has a very strong dampening effect on asset return volatility.

We develop a general equilibrium asset-pricing model with collateral constraints that allows us to assess the quantitative impact of margin regulation. In this economy, agents can default on a short position at any time without any utility penalties or loss of reputation. Financial securities are therefore only traded if the promised payments associated with selling these securities are backed by collateral. The margin requirement dictates how much agents can borrow using risky assets as collateral. In contrast to other papers considering such constraints (see, e.g. Kubler and Schmedders (2003), Cao (2011), Brumm and Grill (2013), and Brumm et al. (2013)), we analyze a setting that allows for two different ways to determine margin requirements. In our first rule the margin requirements are determined in equilibrium by market forces: they are endogenously set to the lowest possible value that still ensures no default in the subsequent period. This specification is a stochastic version of the collateral requirements in Kiyotaki and Moore (1997). In addition to market-determined margin requirements, we also consider regulated margin requirements which are set by a (not further modeled) regulating agency. The regulator requires debtors to hold a certain minimum amount of equity relative to the value of the loan-financed securities they hold.

To generate collateralized borrowing in equilibrium we assume that there are two types of agents that differ in risk aversion. To isolate the effect of heterogeneous risk aversion, we assume that the agents have identical elasticities of substitution (IES) and identical time discount factors. We represent these preferences by Epstein-Zin utility. The agent with the low risk aversion (“agent 1”) is the natural buyer of risky assets and takes up leverage to finance these investments. The agent with the high risk aversion (“agent 2”) has a strong desire to insure against bad shocks and is thus a natural buyer of risk-free bonds. When the economy is hit by a negative shock, the collateral constraint forces the leveraged agent 1 to reduce consumption and to sell risky assets to the risk-averse agent. This triggers a substantial change in the wealth distribution, which in turn affects asset prices.

We start our analysis of regulated margins by considering an economy with two long-lived assets where margin requirements are exogenously regulated for one long-lived asset (representing stocks) while the margin requirement for a second asset (representing housing and corporate bonds) is endogenous. We examine two forms of margin regulation: constant margin requirements and countercyclical margin requirements. For constant margins, the same margin requirement applies over the whole business cycle. For countercyclical margin regulation, minimum margin requirements apply over the whole business cycle and the regulator imposes additional margins (sometimes referred to as “macroprudential add-on”) in boom times. Regarding constant margin requirements on stocks, higher margins do not imply significantly different return volatilities. The reason for this result is that an increase in the margin requirement has two opposing effects: First, the regulated asset becomes less attractive as collateral. This implies that it is sold more frequently after bad shocks when agent 1 must de-leverage. As a result the price of the asset must fall to induce agent 2 to buy it. Second, higher stock margins decrease the agents’ ability to...
leverage. Therefore the amount of leverage decreases in equilibrium, leading to less de-leveraging after bad shocks. While the first effect increases the asset’s volatility, the second effect reduces it. In equilibrium, these two effects approximately offset each other and thus the return volatility of the regulated asset barely changes. We also show that for the asset with unregulated margins, the first effect leads to a reduction of its volatility since this asset becomes relatively more attractive as collateral. Consequently, for this asset, the two effects work in the same direction and therefore reduce its excess volatility. Our model thus predicts strong spillover effects from the margin regulation of one asset class on the return volatility of other assets.

Countercyclical margin regulation of the stock market has a slightly stronger impact on asset price volatility than constant regulation. In good times, this regulation dampens the build-up of leverage in the same way as with time-constant margins. However, the withdrawal of the macroprudential add-on in bad times decreases the de-leveraging pressure induced by binding collateral constraints. For this reason, volatility can be lowered through countercyclical margin regulation, yet the quantitative impact can hardly be interpreted as economically significant. To sum up, changes in the regulation of a class of collateralizable assets may have only small effects on the assets’ return volatility if investors have access to another (unregulated) class of collateralizable assets to leverage their positions. This result is in line with a popular argument mentioned by Fortune (2001), which echoes the sentiment of Moore (1966) that investors may substitute between margin loans and other debt: “If an investor views margin debt as a close substitute for other forms of debt, changes in margin requirements will shift the type of debt used to finance stock purchases without changing the investors total debt. The investors leverage will be unchanged but altered in form. The risks faced, and the risk exposure of creditors, will be unchanged. Little will be changed but the name of the paper.”

In the final step of our analysis, we consider a setting where all asset markets are subject to margin regulation. We document that the effects of countercyclical margin regulation can significantly reduce stock market volatility if this kind of regulation is applied to all collateralizable assets in the economy. In such a setting, agents are prohibited from excessively leveraging in unregulated markets thereby lowering aggregate asset price volatility. For this reason, the above described dampening effect on volatility of countercyclical margin regulation has an undiminished impact. Therefore, setting countercyclical margins in all markets is a powerful tool to considerably reduce asset market volatility.

While our model is designed for the analysis of stock market margin regulation (like Regulation T), we believe that our theoretical findings may also be relevant for the current debate on the regulation of margin requirements in repo and securities lending markets: In November 2012 and in August 2013, the Financial Stability Board (FSB) launched public consultations on a policy framework for addressing risks in securities lending and repo markets (see FSB (2012) and FSB (2013)). It explicitly includes a policy proposal to introduce minimum haircuts on collateral for securities financing transactions: “Such a framework would be intended to set a floor on the cost of secured borrowing against risky asset in order to limit the build-up of excessive leverage” (FSB (2012), p.12). Relating our findings to the current discussion, our analysis would imply that such a framework should allow regulators to set countercyclical margins. Moreover, our findings also
suggest that such a framework should have a broad scope as to maximize the quantitative impact on financial markets.

**Literature**

There is a large literature that examines the effects of financial constraints on asset prices. In this literature review we focus on a subset of this literature that formalizes the general idea that borrowing against collateral may increase asset price volatility and lower excess returns. Prominent early papers include Geanakoplos (1997) and Aiyagari and Gertler (1999). In these models, the market price may deviate substantially from the corresponding price in frictionless markets. Brunnermeier and Pedersen (2009) develop a model where an adverse feedback loop between margins and prices may arise. In their model, risk-neutral speculators trade on margin and margin requirements are determined by a value-at-risk constraint. Garleanu and Pedersen (2011) analyze how margin requirements affect first moments of asset prices. In contrast to the present study, these papers do not consider calibrated models and do not investigate quantitative implications of margin regulation.

Our economic model shares some features with the models by Coen-Pirani (2005), Rytchkov (2013) and Chabakauri (2013). The results in our paper, however, are in stark contrast to the findings of Coen-Pirani and Rytchkov. Both authors examine a setup similar to ours, yet they make crucial simplifying assumptions to solve the model which lead to results that are opposite to the ones obtained in this paper. Coen-Pirani (2005) also considers a discrete time Lucas style model with Epstein-Zin agents that differ in risk-aversion but have identical IES. By further assuming that the common IES is equal to one and that all income stems from dividend payments, he can show analytically that collateral constraints have no effect on stock return volatility. We find that this result changes dramatically for economies in which labor income finances a large part of aggregate consumption. In such an economy, collateral constraints substantially increase return volatility even if the common IES is equal to one. Rytchkov (2013) considers a continuous-time model where two agents maximize expected utility and differ in risk-aversion. As in Coen-Pirani (2005), all consumption stems from dividend payments of the tree. Collateral requirements force the less risk-averse agent to hold less of the stock than he would otherwise and typically lead to a reduction of stock-return volatility. Chabakauri (2013) examines general asset pricing implications of collateral constraints in a model with two stocks. In the context of a continuous-time model with CRRA preferences, he finds a positive relation between the amount of leverage and the conditional stock return correlations and volatilities.

The recent financial crisis has led researchers to suggest anew that central banks should regulate margin requirements. Ashcraft et al. (2010) proposes to use margins as a second monetary policy tool, whereas Geanakoplos (2009) suggest to regulate leverage. There is also an emerging literature which considers the regulation of financial intermediaries’ capital or funding constraints (see e.g. Adrian and Boyarchenko (2012) and He and Krishnamurthy (2013)). In this literature, intermediaries are explicitly modeled and regulators have the power to alleviate or tighten the financial constraints faced by the regulated intermediaries. In contrast, we consider market-wide types of regulations which affect non-regulated and regulated entities alike.
The remainder of this paper is organized as follows. We introduce the model in Section 2. Section 3 presents the calibration of our model. In Section 4 we show how collateralized borrowing may lead to substantial excess return volatility. Section 5 examines the effects of margin regulation; it contains the main results of the paper. In Section 6 we discuss the implications of the numerical results for margin policy, past and present. Section 7 concludes. In the Appendix we provide some sensitivity analysis.

2 The Model

We examine an infinite-horizon exchange economy with two infinitely-lived heterogeneous agents, two long-lived assets and margin requirements for short-term borrowing.

2.1 The Physical Economy

Time is indexed by $t = 0, 1, 2, \ldots$. A time-homogeneous Markov chain of exogenous shocks $(s_t)$ takes values in the finite set $S = \{1, \ldots, S\}$. The $S \times S$ Markov transition matrix is denoted by $\pi$. We represent the evolution of time and shocks in the economy by a countably infinite event tree $\Sigma$. The root node of the tree represents the initial shock $s_0$. Each node of the tree, $\sigma \in \Sigma$, describes a finite history of shocks $\sigma = s^t = (s_0, s_1, \ldots, s_t)$ and is also called date-event. We use the symbols $\sigma$ and $s^t$ interchangeably. To indicate that $s^{t'}$ is a successor of $s^t$ (or $s^t$ itself) we write $s^{t'} \succeq s^t$. We use the notation $s^{-1}$ to refer to the initial conditions of the economy prior to $t = 0$.

At each date-event $\sigma \in \Sigma$, there is a single perishable consumption good. The economy is populated by $H = 2$ agents, $h \in H = \{1, 2\}$. Agent $h$ receives an individual endowment in the consumption good, $e^h(\sigma) > 0$, at each node. In addition, at $t = 0$ the agent owns shares in long-lived assets (“Lucas trees”). There are $A = 2$ different such assets, $a \in A = \{1, 2\}$. At the beginning of period 0, each agent $h$ owns initial holdings $\theta^h_a(s^{-1}) \geq 0$ of asset $a$. We normalize aggregate holdings in each long-lived asset, that is, $\sum_{h \in H} \theta^h_a(s^{-1}) = 1$ for all $a \in A$. At date-event $\sigma$, we denote agent $h$’s (end-of-period) holding of asset $a$ by $\theta^h_a(\sigma)$ and the entire portfolio of asset holdings by the $A$-vector $\theta^h(\sigma)$.

The long-lived assets pay positive dividends $d_a(\sigma)$ in units of the consumption good at all date-events. We denote aggregate endowments in the economy by $\bar{e}(\sigma) = \sum_{h \in H} e^h(\sigma) + \sum_{a \in A} d_a(\sigma)$.

Agent $h$ has preferences over consumption streams $c^h = (c^h(s^t))_{s^t \in \Sigma}$ representable by the following recursive utility function, see Epstein and Zin (1989),

$$U^h(c^h, s^t) = \left\{ \left[ e^h(s^t) \right]^{\rho^h} + \beta \left[ \sum_{s_{t+1}} \pi(s_{t+1}|s_t) \left( U^h(c^h, s^{t+1}) \right)^{\alpha^h} \right]^{\frac{\rho^h}{\alpha^h}} \right\}^{\frac{1}{1-\rho^h}},$$

where $\frac{1}{1-\rho^h}$ is the intertemporal elasticity of substitution (IES) and $1 - \alpha^h$ is the relative risk aversion of the agent.
2.2 Financial Markets and Collateral

At each date-event, agents can engage in security trading. Agent $h$ can buy $\theta^h_a(\sigma) \geq 0$ shares of asset $a$ at node $\sigma$ for a price $q_a(\sigma)$. Agents cannot assume short positions of the long-lived assets. Therefore, the agents make no promises of future payments when they trade shares of physical assets and thus there is no possibility of default when it comes to such positions.

In addition to the physical assets, there are $J = 2$ one-period financial securities, $j \in J = \{1, 2\}$, available for trade. We denote agent $h$’s (end-of-period) portfolio of financial securities at date-event $\sigma$ by the vector $\phi^h(\sigma) \in \mathbb{R}^2$ and denote the price of security $j$ at this date-event by $p_j(\sigma)$. These two assets are one-period bonds in zero-net supply; their face value is one unit of the consumption good in the subsequent period. Whenever an agent assumes a short position in a financial security $j$, $\phi^h_j(\sigma) < 0$, she promises a payment in the next period. Such promises must be backed by collateral.

At each node $s^t$, we pair the first (second) one-period bond with the first (second) long-lived asset. If an agent borrows by short-selling a bond, $\phi^h_j(s^t) < 0$, then she is required to hold a sufficient amount of collateral in the corresponding long-lived asset $a = j$. The difference between the value of the collateral holding in the long-lived asset $a = j$, $q_j(s^t)\theta^h_j(s^t) > 0$, and the current value of the loan, $-p_j(s^t)\phi^h_j(s^t)$, is the amount of capital the agent put up to obtain the loan. A margin requirement $m_j(s^t)$ enforces a lower bound on the ratio of this capital to the value of the collateral,

$$m_j(s^t) \leq \frac{q_j(s^t)\theta^h_j(s^t) + p_j(s^t)\phi^h_j(s^t)}{q_j(s^t)\theta^h_j(s^t)},$$

whenever $\phi^h_j(s^t) < 0$ and $q_j(s^t)\theta^h_j(s^t) > 0$. If the agent holds no collateral, $\theta^h_j(s^t) = 0$, then she cannot borrow and so she faces the constraint $\phi^h_j(s^t) \geq 0$. We can combine these two cases in a single constraint,

$$m_j(s^t) \left( q_j(s^t)\theta^h_j(s^t) \right) \leq q_j(s^t)\theta^h_j(s^t) + p_j(s^t)\phi^h_j(s^t).$$

Using language from financial markets, we use the term ‘margin’ requirement throughout the remainder of the paper. Inequality (1) provides the definition of the term ‘margin’ according to Regulation T of the Federal Reserve Board. However, there does not appear to be a unified definition of this term. For example, in CGFS (2010) the term $m_j(s^t)$ is called a ‘haircut’ and instead a lower bound on the capital-to-loan ratio

$$\frac{q_j(s^t)\theta^h_j(s^t) + p_j(s^t)\phi^h_j(s^t)}{-p_j(s^t)\phi^h_j(s^t)}$$

is called a ‘margin requirement.’ Here, we use the definition and terminology according to Regulation T. It should be noted that, contrary to the unbounded capital-to-loan ratio, the capital-to-value ratio is bounded above by one for $\phi^h_j(s^t) \leq 0$ and $\theta^h_j(s^t) > 0$.

Following Geanakoplos and Zame (2002), we assume that an agent can default on her earlier promises without declaring personal bankruptcy. In this case the agent does not incur any penalties but loses the collateral she had to put up. Since there are no penalties for default, an agent who sold security $j$ at date-event $s^t$ defaults on her promise at a successor node $s^{t+1}$ whenever
the initial promise exceeds the current value of the collateral, that is, whenever
\[-\phi_j^h(s^t) > \theta^h(s^t) (q_j(s^{t+1}) + d_j(s^{t+1})) .\]

In this paper, we impose sufficiently large margin requirements so that no default occurs in equilibrium. We examine two different rules for the determination of such margin requirements. The first rule sets market-determined margin requirements while the second rule assumes exogenously regulated margin requirements.

2.2.1 Market-Determined Margin Requirements

Our first rule for margin requirements follows Geanakoplos (1997) and Geanakoplos and Zame (2002) who suggest a simple and tractable way to endogenize margin requirements. They assume that, in principle, financial securities with any margin requirement could be traded in equilibrium. Only the scarcity of available collateral leads to equilibrium trade in only a small number of such securities. In our economy, for each asset \(a\) only a single bond \(j = a\) collateralized by the asset is available for trade; this bond’s margin requirement \(m_j(s^t)\) is set to the lowest possible value that still ensures no default in the subsequent period,

\[m_j(s^t) = 1 - \frac{p_j(s^t) \cdot \min_{s^t+1} \{ q_j(s^{t+1}) + d_j(s^{t+1}) \}}{q_j(s^t)} .\]

Substituting this margin requirement into Inequality (1) we obtain

\[-\phi_j^h(s^t) > \theta^h(s^t) \min_{s^t+1} \{ q_j(s^{t+1}) + d_j(s^{t+1}) \} .\]

This market-determined margin requirement makes the bond risk-free. A short-seller of this bond will never default on her promise. This restriction is a stochastic version of the collateral requirement in Kiyotaki and Moore (1997).

2.2.2 Regulated Margin Requirements

The second rule for setting margin requirements relies on regulated capital-to-value ratios. A (not further modeled) regulating agency now requires debtors to hold a certain minimal amount of capital relative to the value of the collateral they hold. Put differently, the regulator imposes a floor on margin requirements so that for regulated assets the traded margin requirement \(m_j(s^t)\) is always larger of this minimal level and the market-determined margin level. If the margin requirement is one, \(m_j(s) = 1\), then the asset cannot be used as collateral. Note that even if \(m_j(s) < 1\) is constant across shocks and time, the resulting capital necessary to obtain a loan will depend on the endogenous equilibrium asset prices in the economy and thus will fluctuate across states and time periods.

2.3 Financial Markets Equilibrium with Collateral

We are now in the position to formally define the notion of a financial markets equilibrium. To simplify the statement of the definition, we assume that for both assets \(a \in A\) margin requirements are market-determined. Below, we explain how the definition differs when the margin requirement for some asset is exogenously regulated. We denote equilibrium values of a variable \(x\) by \(\bar{x}\).
Definition 1 A financial markets equilibrium for an economy with initial shock \( s_0 \) and initial asset holdings \( (\theta^h(s^{-1}))_{h \in H} \) is a collection of agents’ portfolio holdings and consumption allocations as well as security prices and collateral requirements for all one-period financial securities \( j \in J \),

\[
\left( \left( \hat{\theta}^h(\sigma), \hat{\phi}^h(\sigma), \hat{c}^h(\sigma) \right)_{h \in H}; (\bar{q}_a(\sigma))_{a \in A}, (\bar{p}_j(\sigma))_{j \in J}; (\bar{m}_j(\sigma))_{j \in J} \right)_{\sigma \in \Sigma},
\]

satisfying the following conditions:

(1) Markets clear:

\[
\sum_{h \in H} \hat{\theta}^h(\sigma) = 1 \quad \text{and} \quad \sum_{h \in H} \hat{\phi}^h(\sigma) = 0 \quad \text{for all } \sigma \in \Sigma.
\]

(2) For each agent \( h \), the choices \( (\hat{\theta}^h(\sigma), \hat{\phi}^h(\sigma), \hat{c}^h(\sigma)) \) solve the agent’s utility maximization problem,

\[
\max_{\theta \geq 0, \phi \geq 0} U_h(c) \quad \text{s.t.} \quad \text{for all } s^t \in \Sigma,
\]

\[
c(s^t) = c^h(s^t) + \sum_{j \in J} \phi_j(s^{-1}) + \theta^h(s^{-1}) \cdot (\bar{q}(s^t) + d(s^t))
\]

\[
- \theta^h(s^t) \cdot \bar{q}(s^t) - \phi^h(s^t) \cdot \bar{p}(s^t)
\]

\[
\bar{m}_j(s^t) \bar{q}_j(s^t) \theta^h_j(s^t) \leq \bar{q}_j(s^t) \theta^h_j(s^t) + \bar{p}_j(s^t) \phi^h_j(s^t) \quad \text{for all } j \in J.
\]

(3) For all \( s^t \), for each \( j \in J \), the margin requirement satisfies

\[
\bar{m}_j(s^t) = 1 - \frac{\bar{p}_j(s^t) \cdot \min_{a \in A} \{ \bar{q}_j(s^{t+1}) + d_j(s^{t+1}) \}}{\bar{q}_j(s^t)}.
\]

Note that the third condition relies on the innocuous assumption that the equilibrium prices of the long-lived assets are strictly positive. For an economy with regulated margin requirements for a bond \( j \), the third condition is replaced by the maximum between the floor set by the regulating agency and the market determined margin requirement.

For an interpretation of the results it is useful to understand the recursive formulation of the model. The natural endogenous state space of this economy consists of all agents’ beginning-of-period financial wealth as a fraction of total financial wealth (i.e. value of the assets cum dividends) in the economy. That is, we keep track of the current shock \( s_t \) and of agents’ wealth shares

\[
\omega^h(s^t) = \frac{\sum_{j \in J} \phi^h_j(s^{-1}) + \theta^h(s^{-1}) \cdot (\bar{q}(s^t) + d(s^t))}{\sum_{a \in A} (\bar{q}_a(s^t) + d_a(s^t))}.
\]

We compute prices, portfolios and individual consumptions as a function of the exogenous shock and the distribution of financial wealth. In our calibration we assume that shocks are i.i.d. and that these shocks only affect the aggregate growth rate. In this case, policy and pricing functions (normalized by aggregate consumption) are independent of the exogenous shock, and thus depend on the wealth distribution only.
We emphasize that the endogenous state variable in our model is time-stationary despite the heterogeneity of agents’ levels of risk aversion. Both agents “survive” in the long run since the collateral and short-sale constraints prohibit the agents from assuming more and more debt over time. Also note that in our numerical examples, both the market-determined and the regulated margin requirements are large enough to ensure that there is no default in equilibrium. Therefore, both one-period bonds are risk-free and thus we can treat them as a single bond and report statistics on a single interest rate.

3 The Baseline Economy

In this section we describe the details of the specification of our baseline economy. We calibrate the model to yearly U.S. data.

3.1 Growth Rates

The aggregate endowment at date-event \( s_t \) grows at the stochastic rate \( g(s_{t+1}) \) which only depends on the new shock \( s_{t+1} \in S \). So, for all date-events \( s^{t+1} \in \Sigma \),

\[
\frac{\bar{e}(s^{t+1})}{\bar{e}(s^t)} = g(s_{t+1}).
\]

There are \( S = 6 \) exogenous shocks. We declare the first three of them, \( s = 1, 2, 3 \), to be “disasters”. We calibrate the disaster shocks to match the first three moments of the continuous distribution of consumption disasters estimated by Barro and Jin (2011) who use data from Barro and Ursúa (2008). Also following Barro and Jin, we choose transition probabilities such that the six exogenous shocks are i.i.d. The non-disaster shocks, \( s = 4, 5, 6 \), are then calibrated such that the overall average growth rate is 2 percent and such that their standard deviation matches “normal” business cycle fluctuations with a standard deviation of about 2 percent. We sometimes find it convenient to call shock \( s = 4 \) a “recession” since it represents a (moderate) decrease in aggregate endowments of 3.2 percent. Table I provides the resulting growth rates and probability distribution for the six exogenous shocks of the economy. The disaster shocks play an important role in generating the endogenous dynamics of asset prices that we discuss in this paper. However, the sensitivity analysis in Appendix A.2 shows that the qualitative effects are robust to assuming much less severe disaster shocks.

<table>
<thead>
<tr>
<th>Shock s</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>( g(s) )</td>
<td>0.565</td>
<td>0.717</td>
<td>0.867</td>
<td>0.968</td>
<td>1.028</td>
<td>1.088</td>
</tr>
<tr>
<td>( \pi(s) )</td>
<td>0.005</td>
<td>0.005</td>
<td>0.024</td>
<td>0.0533</td>
<td>0.8594</td>
<td>0.0533</td>
</tr>
</tbody>
</table>

Table I: Growth rates and probabilities of exogenous shocks

Growth rates \( g(s) \) and their probabilities \( \pi(s) \) as a function of the shock \( s \in \{1, 2, \ldots, 6\} \).
3.2 Dividends

In our economic model, the long-lived assets ("Lucas trees") are claims to aggregate capital income that can be traded without transaction cost. The assets can also be used as collateral for borrowing on margin. In the U.S. economy three big asset classes roughly satisfy these conditions: Stocks, corporate bonds, and housing. We model the two long-lived assets in our economy to resemble these assets. As our analysis focuses on the margin eligibility of the two assets, we simply assume that the dividend streams have stochastic characteristics that are identical to those of aggregate consumption. Formally, for each long-lived asset $a$, we set $d_a(s^t) = \delta_a \epsilon(s^t)$, where $\delta_a$ measures the magnitude of the asset dividends.

To determine the size of the dividend streams we follow Chien and Lustig (2010) and use Table 1.2 of the National Income and Product Accounts (NIPA). We use annual data starting from 1947 (the year when Regulation T was first used to tighten margins for borrowing on stocks) until 2010 and report (unweighted) arithmetic averages below. Also following Chien and Lustig (2010), we define collateralizable income as the sum of ‘rental income of persons with capital consumption adjustment’, ‘net dividends’ and ‘net interest’. Between 1947 and 2010, the average share of this narrowly-defined collateralizable income was about 11 percent, thus we set $\sum_a \delta_a = 0.11$.\footnote{This definition of collateralizable income does not include proprietary income which constitutes a large share of income (about 10 percent, on average, between 1947 and 2010). However, it is difficult to assess what portion of this income is derived from assets that can be easily traded and collateralized. Up until the early 1980s, a significant share of this income was farm income, but nowadays it is almost entirely non-farm income. It includes income from partnerships such as law firms or investment banks, which is neither tradable nor collateralizable.}

We divide the total amount of tradable assets into two parts and model them as two long-lived assets that differ in how their margins are determined. The first asset represents the stock market. We model the margins of this asset to be regulated, since the Board of Governors of the Federal Reserve System establishes initial margin requirements for stocks under Regulation T. In NIPA data, the average share of dividend income for the time period 1947–2010 is about 3.3 percent.\footnote{During the time period 1947–1974 when there were frequent changes in the margin requirement, the share of the narrowly defined collateralizable income was about 8.5 percent on average. Net dividends constituted on average 33% of this income.} This fraction is smaller than the values typically assumed in the literature; values range from 4 to 5 percent, see, e.g., Heaton and Lucas (1996), since this number does not include retained earnings. To strike a compromise between these numbers we set $\delta_1 = 4\%$.

In order to simplify the analysis, we aggregate net interest and net rental income into the dividends of a second long-lived asset representing corporate bonds and housing. Since margins on (non-convertible) corporate bonds and mortgage-related securities as well as down payment requirements for housing have been largely unregulated, we assume margins on the second long-lived asset are determined endogenously.\footnote{Also, interest rates on margin loans against stocks exceed mortgage rates. From 2011 until the writing of this paper in 2013 margin rates of the discount broker Charles Schwab & Co. ranged from 8.5% for margin loans below $25,000 to 6% for loans above $2,500,000. See \url{http://www.schwab.com/public/schwab/investing/accounts_products/investment/margin_accounts} (accessed on April 24, 2013) By comparison, standard mortgage rates for 30-year fixed mortgage loans were below 3.5% in the U.S. in April 2013.} According to NIPA data, rental income constituted, on
average, about 2.3 percent and net interest about 4.9 percent of total income for the time period 1947–2010. In NIPA data, rental income includes the imputed rental income of owner-occupants of nonfarm dwellings. This figure is net of mortgage payments which are included in the category interest payments. Net interest also includes net interest paid by private businesses, but does not include interest paid by the government. We thus set $\delta_2 = 7\%$.

### 3.3 Endowment Shares

Recall that there are $H = 2$ types of agents in the economy. Each agent $h$ receives a fixed share of aggregate endowments as individual endowments, that is, $e^h(s^t) = \eta^h \bar{e}(s^t)$. We abstract from idiosyncratic income shocks because it is difficult to disentangle idiosyncratic and aggregate shocks for a model with two types of agents. Note that the agents’ endowments and the dividends of the two long-lived assets are collinear since all of them are fixed fractions of the aggregate endowment.

Below we specify the agents’ utility functions so that the first type, $h = 1$, is much less risk-averse than the second. As a result, agent 1 holds the two risky long-lived assets most of the time. This fact guides our choice of endowment shares $\eta^1$ and $\eta^2$. Since our objective in this paper is to analyze margin regulation, we ideally would like the endowment share $\eta^1$ of agent 1 to correspond to the labor income share of investors with a margin account. To the best of our knowledge, data on that rather specific share is unavailable. Data is available on the fraction of agents in the U.S. population that holds substantial amounts of stocks outside of retirement accounts. For example, Vissing-Jørgensen and Attanasio (2003) claim that about 20 percent of the U.S. population holds stocks. However, many of these households have only small stock investments, see Poterba et al. (1995). In addition, ideally we would like to match the labor income share of the stock-owning households instead of the simple population share of such households. As a compromise, we assume in our baseline economy that agent 1 receives 10 percent of all individual endowments, and agent 2 receives the remaining 90 percent. Since we set $\sum_a \delta_a = 0.11$, we have $\eta^1 = 0.089$ and $\eta^2 = 0.801$. This assumption implies that, along our simulations of the baseline economy, agent 1 has an average wealth share of about two-thirds. Guvenen (2009) reports that in the U.S. stockholders own slightly more than 80 percent of all net worth (including housing). Ideally, we would like to calibrate the model so that the average wealth share of agent 1 matches the average share of wealth of people owning a margin account. Unfortunately, such data appears to be unavailable.

### 3.4 Utility Parameters

The choice of an appropriate value for the IES is rather difficult. On the one hand, several studies that rely on micro-data find values of about 0.2 to 0.8; see, for example, Attanasio and Weber (1993). On the other hand, Vissing-Jørgensen and Attanasio (2003) use data on stock owners only and conclude that the IES for such investors is likely to be above one. Barro (2009) finds that for a successful calibration of a representative-agent asset-pricing model the IES needs to be larger than one. In our baseline economy, both agents have identical IES of 2, that is, $\rho^1 = \rho^2 = 1/2$.

Agent 1 has a risk aversion of $1/2$ while agent 2’s risk aversion is 7. Recall the weights for the two agents in the baseline economy, $\eta^1 = 0.089$ and $\eta^2 = 0.801$. The majority of the population
is therefore quite risk-averse, while 10 percent of households have low risk aversion. Recall that this number is chosen to match observed stock-market participation as we have discussed above. Finally, we set $\beta^h = 0.942$ for both $h = 1, 2$, because it matches an annual risk-free rate of 1% in an economy with a regulated margin of 60% on stocks, which is close to the average Regulation T margin requirement during the period from 1940 to 1974 (see Figure VI).

4 Collateral Constraints and Excess Volatility

The objective of this paper is to analyze the effects of margin regulation in general equilibrium and to compare the model predictions to empirical findings. Before we study the effects of regulation, we first describe how collateralized borrowing leads to substantial excess return volatility. This feature of the model is essential for our analysis as only in such an environment the regulation of margin requirements may have quantitatively significant effects. To understand these effects, a thorough understanding of the mechanisms at work in our model is necessary. For this purpose, we first consider an economy in which stocks are not margin-eligible (regulated margin requirement of 100 percent), whereas other assets are not regulated, that is, their margins are endogenously determined by market forces. We denote this baseline economy by $CC$: Collateral Constraints.

For an evaluation of the quantitative effects of borrowing on margin in the economy $CC$, we benchmark our results against an economy $NB$: No Borrowing in which agents cannot borrow, that is, there are no bonds. Due to the absence of borrowing in the benchmark economy $NB$, the agents cannot hold leveraged portfolios. Table II reports simulation statistics for each of the two economies. Throughout the paper we measure volatility by the average standard deviation (STD) of returns over a long horizon. We also report average excess returns (ER). While our paper does not focus on an analysis of this ER, we report it to ensure that our calibration delivers reasonable values. Recall that in our calibration, agents of type 1 are much less risk averse than type 2 agents. In the benchmark model $NB$, the less risk-averse agent 1 holds both assets most of the time. A bad shock to the economy leads to shifts in the wealth distribution and a decrease of asset prices. However, these effects are small. Asset prices are determined almost always by the Euler equations of agent 1, and so their volatility is quite low (5.3%). As financial frictions do not play a role in this economy, the exogenous shocks to the economy are the sole origin of volatility.

| Table II: Moments of asset returns with marginable and non-marginable assets |
|-------------------|--------|--------|---------|---------|---------|
|                    | STD    | ER     | agg STD | agg ER  | agg STD in $NB$ |
| Non-marginable asset ($\delta_1 = 0.04$) | 8.5%   | 6.8%   | 7.4%    | 5.0%    | 5.3%    |
| Marginable asset ($\delta_2 = 0.07$)     | 7.1%   | 4.4%   |         |         |         |

STD = standard deviation, ER = excess return, agg = aggregated over both long-lived assets, $NB$ = benchmark model without borrowing.

Note that both economies $NB$ and $CC$ are time-stationary: both agents survive in the long-run. In $NB$, agents cannot borrow. In $CC$, the margin and short-sale constraints prohibit the agents from taking on increasingly large debt positions.
Table II shows that the aggregate standard deviation (agg STD) substantially increases when we compare the benchmark model *NB: No Borrowing* to our baseline model, *CC: Collateral Constraints*. Specifically, the aggregate standard deviation of returns is 7.4 percent in the baseline economy *CC*, but only 5.3 percent for the benchmark model *NB*. The *excess return volatility* in the collateral-constrained economy, defined as the difference between the aggregate return volatilities in the two models *CC* and *NB*, is 7.4%-5.3% = 2.1%; in relative terms, the aggregate return volatility in the baseline model *CC* is almost forty percent larger than in the benchmark model *NB*. Moreover, we observe that the two long-lived assets in the collateral-constrained economy exhibit substantially different returns despite their collinear dividends. The marginable asset has a lower return volatility as well as a considerably lower average excess return than the non-marginable asset. To understand this difference in return volatilities, we need to have a closer look at the economic mechanisms at work in our model. For this purpose we now analyze simulation paths as well as policy functions.

Figure I displays the time series of six key variables in a simulation for a time window of 200 periods. Recall that we consider a stochastic growth economy. Therefore, we report normalized asset prices, that is, equilibrium asset prices divided by aggregate consumption. The first graph in Figure I shows the normalized price of the marginable asset. The second graph displays agent 1’s holding of the marginable asset. The next two graphs show the price and agent 1’s holding of the non-marginable asset, respectively. The last two graphs show agent 1’s bond position and share of financial wealth, respectively. In the displayed sample, shock $s = 3$ occurs in periods 71 and 155, while shock 2 occurs in period 168, and the worst disaster shock 1 hits the economy in period 50.

In addition to the simulated time series in Figure I, we also display equilibrium policy and price functions in Figure II. These equilibrium functions are all functions of agent 1’s wealth share, $m_1$, which is the endogenous state variable in our model. The first row of Figure II shows the normalized price functions of the marginable and the non-marginable asset, respectively, as well as the risk-free rate function. The second row shows agent 1’s holdings of the marginable asset, the non-marginable asset, and the risk-free bond.

When a bad shock occurs, both the current dividend and the expected net present value of all future dividends decrease. As a result, asset prices drop, but in the absence of further effects, the normalized prices should remain the same as we consider i.i.d. shocks to the growth rate. Figure I, however, indicates that additional effects occur in our baseline economy *CC* because we observe declines in the conditional price for the two long-lived assets. First, note that agent 1 is always leveraged since her bond position is always negative, as the simulation path in Figure I and more generally the bond function in Figure II indicate. When a bad shock happens, her beginning-of-period financial wealth falls relative to the financial wealth of agent 2 due to the price declines in the long-lived assets. This effect is the strongest when the worst disaster shock 1 occurs. The fact that collateral is scarce in our economy now implies that these changes in the wealth distribution strongly affect equilibrium portfolios and prices. In “normal times” agent 1 has a wealth share of about two-thirds and holds both long-lived assets. After a bad shock, her financial wealth drops and she has to sell some of these assets. In equilibrium, therefore, the
Figure I: Simulation Path of the Baseline Model $CC$

Normalized prices of the two long-lived assets, agent 1’s holdings of the two long-lived assets and the bond, and her wealth share for a snapshot from a simulation.

price has to be sufficiently low to induce the much more risk-averse agent 2 to buy a (substantial) portion of the assets.

In addition to the described within-period effect, there is also a dynamic effect at work. As agent 1 is poorer today due to the bad shock, she will also be poorer tomorrow implying that asset prices tomorrow are depressed as well. This effect further reduces the price that agent 2 is willing to pay for the assets today. Clearly, this dynamic effect is present not only for one but for several periods ahead, which is displayed in Figure I by the slow recovery of the normalized prices of the assets after bad shocks. Figure I shows that the total impact of the two described effects is very strong for shock $s = 1$ but also large for shock 2. Recall once more that the depicted asset prices are normalized prices, so the drop of the actual asset prices is much larger than displayed in the two figures. In disaster shock 1, agent 1’s wealth share falls below 0.4 and she is forced to sell the entire non-marginable asset; as a result, this asset’s normalized price drops by almost 25 percent while the actual price drops by approximately 55 percent. She is also forced to sell part of the marginable asset. In shock 2, she sells much less than half of her total asset holdings but the price effect is still substantial. Even in shock 3, the price effect is still clearly visible, although
Equilibrium prices and portfolios as a function of the endogenous state variable, $\omega^1$, the wealth share of agent 1. First row: normalized prices of marginable asset, non-marginable asset, resp., and risk-free rate. Second row: holdings of agent 1 in the marginable asset, non-marginable asset, and risk-free bond, resp.

agent 1 has to sell only very little.

The graphs in the two figures also illustrate important differences in the price and return dynamics of the two assets. First, the volatility for the non-marginable asset is larger than for the marginable one. Second, agent 1 holds the marginable asset in almost all periods but frequently sells the asset that is not marginable. When faced with financial difficulties, that is a declining wealth share after a bad shock, agent 1 holds on to the marginable asset as long as possible, because this asset allows her to hold a short position in the bond. So, after suffering a reduction in financial wealth, agent 1 first sells the non-marginable asset. In fact, as her wealth share decreases, agent 1 sells a portion of the marginable asset only after she has sold the entire non-marginable asset. In our sample path, this happens only after the worst disaster shock hits in period 50. Whenever agent 1 sells a portion of a long-lived asset to agent 2, the price of that asset must fall. So, one key factor contributing to the different volatility levels of the two assets is that the non-marginable asset is traded much more often and in larger quantities than the marginable one.
Table III reports agent 1’s average portfolio positions as well as the asset trading volume along long simulations. The numbers in the table confirm the visual impression from Figure I.

Table III: Average holdings and trading volume in long simulations

<table>
<thead>
<tr>
<th>$\theta_1^j$</th>
<th>$\theta_2^j$</th>
<th>$\phi_2^j$</th>
<th>$\Delta \theta_1^j$</th>
<th>$\Delta \theta_2^j$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.942</td>
<td>0.997</td>
<td>-1.11</td>
<td>0.030</td>
<td>0.003</td>
</tr>
</tbody>
</table>

Average holdings of agent 1 and asset trading volume. $\theta_1^j =$ agent 1’s average holding of asset $j$, $\phi_2^j =$ agent 1’s average holding of the risk-free bond, $\Delta \theta_j^j =$ average absolute per-period trading volume of asset $j$.

On average, agent 1 holds almost the entire marginable asset (asset 2). She only briefly reduces her position in response to the worst disaster shock 1. The average trading volume of the marginable asset is tiny (0.003). On the contrary, the average trading volume of the non-marginable asset is about ten times larger (0.030). Recall that in “normal” times, agent 1 holds both long-lived assets and is short in the risk-free bond. When she becomes poorer after a bad shock, the prices of both assets fall. But as she first sells the non-marginable asset, its trading volume is larger and its price falls much faster than the price of the marginable asset. This effect strongly contributes to the difference in the return volatilities of the two assets.

The preceding discussion demonstrates that the collateral feature of the marginable asset is very valuable to agent 1. For an assessment of this collateral value, we adopt the definition of Brumm et al. (2013) of the (relative) collateral premium at a node $s^t$ as

$$CP(s^t) = \frac{q_2(s^t) - q_1(s^t) \frac{d_2(s^t)}{d_1(s^t)}}{q_1(s^t) + q_2(s^t)}.$$ 

Note that if the two infinitely-lived assets had identical margin requirements, then their equilibrium prices would satisfy $q_2(s^t) = \frac{d_2(s^t)}{d_1(s^t)}q_1(s^t)$ for all $s^t$. We can, therefore, interpret the collateral premium, $CP(s^t)$, as the premium (as a fraction of the entire asset market) investors are paying for the additional collateralizability of the second infinitely-lived asset due to its lower margin requirement. Clearly, the collateral premium varies over time as the economy experiences different shocks. In this paper, we always report average collateral premia over repeated long simulations of the general equilibrium model. In the baseline economy, the collateral premium is 34.6%. That is, on average, the price of the marginable asset exceeds the price of an asset with identical cash flows and a margin requirement of 100 percent by more than one-third of the value of the entire asset market. Simply put, the price of the marginable asset contains a very substantial premium for its collateralizability. The relative collateral premium is so high since we consider the extreme case of an economy with a constant margin requirement on stocks (the non-marginable asset) of 100 percent. In the next section, we consider different margin requirements and, in particular, assess how changes in margin regulation affect asset return volatility in the economy.
5 Regulation of Margin Requirements

We now examine an economy in which a regulating agency sets the exogenous margin requirements. We first consider an economy in which the regulating agency imposes a margin restriction on the first long-lived asset but does not regulate the second long-lived asset. As in the previous section, margin requirements for the second asset are endogenously determined in equilibrium. We thus refer to these two assets\(^7\) as the “regulated” and the “unregulated” asset, respectively. We compare the effects of a uniform margin policy to those of a countercyclical policy. Subsequently, we examine an economy in which both long-lived assets are regulated.

5.1 Regulating the Stock Market

Figure III displays the volatility of both assets’ returns as a function of the margin requirement \(m_1\) for the regulated asset. Note that the case of a constant margin requirement of 100% on the right vertical axis corresponds to the baseline economy in the previous section. Most interestingly, over the entire range of values for the regulated margin requirement, the volatility of the regulated asset is rather flat. It initially increases slightly from 8.4 percent to 8.8 percent and then decreases slightly to about 8.5 percent. Thus, changes in the margin requirement of the regulated market have a non-monotone and rather small effect on its own volatility. However, this result should not be interpreted as to mean that margin requirements do not lead to excess volatility in asset markets. In fact they do, as we documented in the previous section. The point is rather that changing a uniform bound on margin requirements in the range of fifty to one-hundred percent on only some assets has little effect on their volatility.

In light of the economic mechanism discussed in the previous section, we can provide an explanation for the observation that margin regulation on the stock market has only little effect on its volatility. An increase in the margin requirement for stocks has two effects. As the margin requirement increases, the regulated asset becomes less attractive as collateral and at the same time the agents’ ability to leverage decreases. These two effects influence (the much less risk-averse) agent 1’s portfolio decisions after a bad shock occurs. First, when agent 1 must de-leverage her position, she always sells the regulated asset first, as it is the worse type of collateral due to its higher margins. Initially this effect leads to an increase in the return volatility of the regulated asset. However, the second effect of higher margins, a reduced ability to leverage, makes de-leveraging episodes less severe. This second effect decreases the return volatility of all assets. In our model specification, the two effects roughly offset each other and therefore a change in the margin requirement has almost no observable effect on the volatility of the regulated asset.

To provide evidence for the above explanation of Figure III, Table IV reports agent 1’s average portfolio positions as well as the asset trading volume and the collateral premium of the unregulated asset along long simulations for four different margin levels. The two aforementioned effects also roughly offset each other with respect to both the average positions and the trading

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\(^7\)Recall that both the market-determined and the regulated margin requirements are always so large that there is no default in equilibrium. Therefore, both one-period bonds are risk-free and thus we can treat them as a single bond and report statistics on a single risk-free bond.
Figure III: Volatilities as a Function of the Margin Requirement on the Regulate Asset

Standard deviations of asset returns as a function of the constant margin requirement $m_1$ on the regulated asset.

Contrary to the mild effect of changes in the margin requirement $m_1$ on the regulated asset, we observe a strong spillover effect on the unregulated asset. A tightening of the margin requirements on the regulated asset has two effects on the unregulated asset. First, the collateral value of the unregulated asset increases relative to the regulated asset as shown in the last column of Table IV. Second, de-leveraging episodes become less severe. Both of these effects act in the same direction and so the less risk-averse agent 1 holds, on average, more units of the unregulated asset and the trading volume of that asset decreases. Table IV reports that for $m_1 = 0.9$ the average trading volume of the regulated asset is less than half as large as for a margin level of $m_1 = 0.6$. The two effects also influence the return volatility of the unregulated asset. The dashed line in Figure III shows that the return volatility of the unregulated asset declines monotonically in the margin level of the other, regulated, asset.

The main message of this analysis is clear. A tightening of (constant) margin requirements on a regulated asset market may have almost no effect on the asset’s return volatility if agents have access to another asset that is not subject to margin regulation. In fact, an adjustment of
Table IV: Average holdings, trading volume, and collateral premium under constant regulation

<table>
<thead>
<tr>
<th>$m_1$</th>
<th>$\theta^1_1$</th>
<th>$\theta^1_2$</th>
<th>$\phi^1$</th>
<th>$\Delta \theta^1_1$</th>
<th>$\Delta \theta^1_2$</th>
<th>CP</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.6</td>
<td>0.9462</td>
<td>0.9875</td>
<td>-1.277</td>
<td>0.0265</td>
<td>0.0084</td>
<td>3.20%</td>
</tr>
<tr>
<td>0.7</td>
<td>0.9462</td>
<td>0.9922</td>
<td>-1.236</td>
<td>0.0292</td>
<td>0.0063</td>
<td>10.08%</td>
</tr>
<tr>
<td>0.8</td>
<td>0.9490</td>
<td>0.9955</td>
<td>-1.184</td>
<td>0.0284</td>
<td>0.0044</td>
<td>18.67%</td>
</tr>
<tr>
<td>0.9</td>
<td>0.9466</td>
<td>0.9967</td>
<td>-1.144</td>
<td>0.0291</td>
<td>0.0034</td>
<td>27.40%</td>
</tr>
</tbody>
</table>

Average holdings of agent 1, asset trading volume, and collateral premium. $m_1$ = regulated margin level on asset 1, $\theta^j_1$ = agent 1’s average holding of asset $j$, $\phi^1$ = agent 1’s average holding of the risk-free bond, $\Delta \theta^j_1$ = average absolute per-period trading volume of asset $j$, CP = collateral premium of asset 2 over asset 1.

Margin requirements in the regulated market may have stronger effects on the unregulated asset than on the regulated asset itself.

The framework presented in this paper provides avenues for much additional analysis of margin policies. Next, we explore how countercyclical margin regulation in the stock market affects asset return volatility.

### 5.2 Countercyclical Regulation of the Stock Market

In the current policy discussion on margin regulation for repo and securities lending markets, it is often argued that a countercyclical regulation of margin requirements (or haircuts) could be a powerful policy to prevent the build-up of excessive leverage in good times. For example, CGFS (2010) suggests that “a countercyclical add-on to the supervisory haircuts should be used by macroprudential authorities as a discretionary tool to regulate the supply of secured funding, whenever this is deemed necessary.” However, it is ex-ante unclear whether adding a countercyclical dimension to margin regulation would have a quantitatively significant impact on financial market outcomes. To contribute to this ongoing discussion, we therefore explore in our setup of stock market margin regulation whether such countercyclical policies are indeed capable of reducing volatility.

Hence, in contrast to the previous analysis, we now assume that the minimum margin requirement of stocks is state-dependent. In particular, margins are countercyclical: in shocks 1 through 4 the regulating agency sets the minimum margin requirements to 50%, while it raises these margins in the two states with a positive growth rate. For simplicity, we assume that the margin levels are set to the same level in shocks 5 and 6. The solid line in Figure IV shows the resulting return volatility of stocks (regulated asset 1) as a function of the minimum margin requirement in “good” times.

We observe that adjustments to the margin level (in good times) in the range of 50% to 80% again have a negligible impact on the return volatility of the regulated asset. Only once this margin is set to 90% or higher does the return volatility decrease somewhat. For example, margin levels (in good times) of 90 percent lead to a return volatility of 8.0 percent as compared to 8.4 percent when margins are always equal to 50 percent. More importantly, the return volatility...
Standard deviations of asset returns as a function of the countercyclical margin level in states 5 and 6. Margin requirement is 0.5 in states 1–4.

of 8.0 percent with countercyclical margins is considerably lower than the 8.7 percent shown in Figure III for a constant margin of 90 percent (in all six states).

These observations raise the question why a state-dependent regulation with very tight margins in good times reduces return volatility more than a constant regulation. To answer this question, compare a constant regulation of 90 percent with the respective countercyclical regulation that sets margins at 90 percent in good times and at 50 percent in recession and disaster states. Under both policies, the high margin of 90 percent limits the leverage of agent 1 in good times (states 5 and 6), which leaves her with more financial wealth whenever a negative shock hits. However, after a bad shock the two policies differ. Under the countercyclical policy, collateral constraints are looser and agent 1 can retain a larger portion of the asset, which in turn implies a more modest drop in its price. As a result, the regulated asset’s volatility is smaller with countercyclical margins. The effect on unconditional moments is somewhat weak since the states with negative growth rates have small probabilities.

Table V reports agent 1’s average portfolio positions as well as the asset trading volume and the collateral premium of the unregulated asset along long simulations for four different margin
levels. The qualitative effects emerging from Table V for countercyclical regulation are very similar to those for constant regulation emerging from Table IV. The average asset holdings in the regulated asset barely change in response to an increase in the asset’s regulated margin level. Similarly, the asset’s trading volume does not change much. We observe that agent 1’s average position in asset 1 is a little larger under countercyclical than under constant regulation, because she can hold on to a larger portion of the asset after a negative shock. This observation also explains the smaller average trading volume. Moreover, we again observe strong spillover effects on the unregulated asset; its trading volume decreases substantially and its collateral premium rises sharply. The explanation is the same as before. As the margin requirement on the regulated asset is tightened, de-leveraging episodes become less severe and the unregulated asset becomes relatively more attractive as collateral. Consequently, agent 1 has, on average, a larger holding of the unregulated asset and its trading volume declines heavily. The return volatility of the unregulated asset declines monotonically in the margin level of the regulated asset. Actually, the return volatility of the unregulated asset declines much more than the volatility of the regulated asset, see the dashed line in Figure IV.

In sum, countercyclical margin requirements on a regulated asset market may reduce the asset’s return volatility once the margin level is sufficiently tight during good times. However, the reduction is somewhat modest which, in light of the previous analysis, is due to the presence of a second unregulated asset. In fact, an adjustment of countercyclical margin requirements in the regulated market may have stronger effects on the unregulated asset than on the regulated asset itself.

Our analysis so far has revealed the critical role played by the unregulated asset (or asset class) in general equilibrium. The effects of regulatory margin policies on holdings, trading volume and return volatility of a regulated asset are considerably dampened by the presence of an unregulated asset. Clearly, this observation raises the question whether extending margin regulation to all assets in the economy can lead to stronger effects on asset return volatility. This question motivates the next step of our analysis.

Table V: Average holdings, trading volume, collateral premium under countercyclical regulation

<table>
<thead>
<tr>
<th>( m_1 )</th>
<th>( \theta^1_1 )</th>
<th>( \theta^1_2 )</th>
<th>( \phi^1 )</th>
<th>( \Delta \theta^1_1 )</th>
<th>( \Delta \theta^1_2 )</th>
<th>( \text{CP} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.6</td>
<td>0.9529</td>
<td>0.9849</td>
<td>-1.279</td>
<td>0.0192</td>
<td>0.0132</td>
<td>2.50%</td>
</tr>
<tr>
<td>0.7</td>
<td>0.9548</td>
<td>0.9904</td>
<td>-1.244</td>
<td>0.0198</td>
<td>0.0090</td>
<td>7.98%</td>
</tr>
<tr>
<td>0.8</td>
<td>0.9571</td>
<td>0.9949</td>
<td>-1.194</td>
<td>0.0200</td>
<td>0.0049</td>
<td>16.59%</td>
</tr>
<tr>
<td>0.9</td>
<td>0.9545</td>
<td>0.9965</td>
<td>-1.151</td>
<td>0.0232</td>
<td>0.0035</td>
<td>25.90%</td>
</tr>
</tbody>
</table>

Average holdings of agent 1, asset trading volume, and collateral premium. \( m_1 \) = regulated margin level on asset 1 in growth states 5 and 6, \( \theta^1_j \) = agent 1’s average holding of asset \( j \), \( \phi^1 \) = agent 1’s average holding of the risk-free bond, \( \Delta \theta^1_j \) = average absolute per-period trading volume of asset \( j \), CP = collateral premium of asset 2 over asset 1.
5.3 Regulating Margin Requirements in All Markets

We now consider simultaneous margin regulation for both long-lived assets. For simplicity, we assume that both assets are regulated in the same way. Since the two assets have collinear dividend processes, identical margin levels imply that both assets have the same return volatility. Therefore, it suffices to report the return volatility of the overall \(^8\) asset market, see Figure V. The dashed line in Figure V shows the market return volatility as a function of a regulated margin requirement that is constant across all six states; similarly, the solid line shows the market return volatility as a function of a (countercyclical) margin requirement in the growth states 5 and 6 when the margin requirement is fixed at 0.5 in the negative-growth states 1 to 4.

Figure V: Volatilities as a Function of the Margin Requirement on Both Long-Lived Assets

Standard deviations of asset returns as a function of the margin on both long-lived assets. For the case of countercyclical regulation, the margin requirement is 0.5 in states 1–4 and equal to the value on the horizontal axis for the positive-growth states 5 and 6.

For constant margin requirements in the range of 0.5 to 0.9 the asset return volatility varies little. This result is perfectly in line with our analysis of constant margins on stocks alone in Section 5.1. The absence of an unregulated asset makes a big difference only for margins above 90%. Such high margins on all assets have a strong dampening effect on return volatility. If

\(^8\) Again, we can also aggregate the two risk-free bonds and report agent 1's holdings of a single risk-free bond.
agent 1 needs to leverage then she must hold a regulated asset as collateral. For very high margins, deleveraging episodes are both rare and mild resulting in much decreased return volatility. Table VI reports agent 1’s average portfolio holdings and the asset trading volume under constant regulation of all assets. As expected, and as documented above for regulation of only the first

Table VI: Average holdings and trading volume under constant regulation

<table>
<thead>
<tr>
<th>$m$</th>
<th>$\theta^1$</th>
<th>$\phi^1$</th>
<th>$\Delta\theta^1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.6</td>
<td>0.9617</td>
<td>-1.2031</td>
<td>0.0178</td>
</tr>
<tr>
<td>0.7</td>
<td>0.9765</td>
<td>-0.9342</td>
<td>0.0159</td>
</tr>
<tr>
<td>0.8</td>
<td>0.9915</td>
<td>-0.5697</td>
<td>0.0078</td>
</tr>
<tr>
<td>0.9</td>
<td>0.9979</td>
<td>-0.2490</td>
<td>0.0025</td>
</tr>
</tbody>
</table>

Average holdings of agent 1 and asset trading volume. $m =$ regulated margin level on both assets in growth states 5 and 6, $\theta^1 =$ agent 1’s average holding of the two assets, $\phi^1 =$ agent 1’s average holding of the risk-free bond, $\Delta\theta^1 =$ average absolute per-period trading volume of assets.

asset, an increase in the margin level leads to higher average holdings of the long-lived assets, a smaller short position in the bond, and much reduced trading volume in the long-lived assets.

Contrary to constant margin requirements, countercyclical margin regulation on all assets is already effective for margin levels much below 90%. Perhaps even more importantly, applying countercyclical regulation to all assets reduces return volatility much more than a regulation of the stock market alone. For example, countercyclical margins of 90% on all markets leads to a return volatility below 5.4 percent, which is much lower than the aggregate volatility of above 7.1 percent and a stock market volatility of 8.0 percent when the regulation is applied to the stock market only (see Figure IV). Table VII reports agent 1’s average portfolio holdings and the asset trading volume under countercyclical regulation of all assets. We observe the by now well-known

Table VII: Average holdings and trading volume under countercyclical regulation

<table>
<thead>
<tr>
<th>$m$</th>
<th>$\theta^1$</th>
<th>$\phi^1$</th>
<th>$\Delta\theta^1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.6</td>
<td>0.9650</td>
<td>-1.2188</td>
<td>0.0138</td>
</tr>
<tr>
<td>0.7</td>
<td>0.9827</td>
<td>-0.9685</td>
<td>0.0128</td>
</tr>
<tr>
<td>0.8</td>
<td>0.9899</td>
<td>-0.6051</td>
<td>0.0111</td>
</tr>
<tr>
<td>0.9</td>
<td>0.9900</td>
<td>-0.2593</td>
<td>0.0057</td>
</tr>
</tbody>
</table>

Average holdings of agent 1 and asset trading volume. $m =$ regulated margin level on both assets in growth states 5 and 6, $\theta^1 =$ agent 1’s average holding of the two assets, $\phi^1 =$ agent 1’s average holding of the risk-free bond, $\Delta\theta^1 =$ average absolute per-period trading volume of assets.

qualitative changes in response to an increase in margin requirements.

The changes in average holdings and trading volume in Tables VI and VII for uniform and
countercyclical margin regulation, respectively, are qualitatively very similar. An increase in the margin requirement, \( m \), leads to higher average asset holdings and much reduced average leverage for (the much less risk-averse) agent 1; the average trading volume in the assets decreases significantly. Naturally the question arises, why, nevertheless, the volatility effects of the two types of margin regulation documented in Figure V are so different, particularly for large values of \( m \). To answer this question, we need to examine conditional equilibrium quantities. Tables VIII and IX report the average asset price and agent 1’s portfolio holdings conditional on the exogenous shock \( s \) for uniform and countercyclical margin regulation, respectively. (Recall that the growth rate \( g(s) \) is increasing in the state \( s \).)

Table VIII: Average conditional asset price and portfolio holdings under uniform regulation

<table>
<thead>
<tr>
<th>( m )</th>
<th>( s )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.6</td>
<td>( p</td>
<td>s )</td>
<td>2.1531</td>
<td>2.5214</td>
<td>2.9000</td>
<td>3.1860</td>
<td>3.3610</td>
</tr>
<tr>
<td>( \theta^1</td>
<td>s )</td>
<td>0.0888</td>
<td>0.5876</td>
<td>0.8580</td>
<td>0.9684</td>
<td>0.9708</td>
<td>0.9722</td>
</tr>
<tr>
<td>( \phi^1</td>
<td>s )</td>
<td>-0.0746</td>
<td>-0.5750</td>
<td>-0.9768</td>
<td>-1.2165</td>
<td>-1.2201</td>
<td>-1.1868</td>
</tr>
<tr>
<td>0.7</td>
<td>( p</td>
<td>s )</td>
<td>2.2244</td>
<td>2.4962</td>
<td>2.7772</td>
<td>3.0326</td>
<td>3.2297</td>
</tr>
<tr>
<td>( \theta^1</td>
<td>s )</td>
<td>0.4287</td>
<td>0.7147</td>
<td>0.8780</td>
<td>0.9549</td>
<td>0.9847</td>
<td>0.9863</td>
</tr>
<tr>
<td>( \phi^1</td>
<td>s )</td>
<td>-0.2761</td>
<td>-0.5188</td>
<td>-0.7130</td>
<td>-0.8503</td>
<td>-0.9517</td>
<td>-0.9383</td>
</tr>
<tr>
<td>0.8</td>
<td>( p</td>
<td>s )</td>
<td>2.2691</td>
<td>2.4501</td>
<td>2.6408</td>
<td>2.8065</td>
<td>2.9509</td>
</tr>
<tr>
<td>( \theta^1</td>
<td>s )</td>
<td>0.7413</td>
<td>0.8715</td>
<td>0.9479</td>
<td>0.9826</td>
<td>0.9952</td>
<td>0.9963</td>
</tr>
<tr>
<td>( \phi^1</td>
<td>s )</td>
<td>-0.3248</td>
<td>-0.4134</td>
<td>-0.4860</td>
<td>-0.5365</td>
<td>-0.5758</td>
<td>-0.5802</td>
</tr>
<tr>
<td>0.9</td>
<td>( p</td>
<td>s )</td>
<td>2.2106</td>
<td>2.3061</td>
<td>2.4052</td>
<td>2.4883</td>
<td>2.5720</td>
</tr>
<tr>
<td>( \theta^1</td>
<td>s )</td>
<td>0.9156</td>
<td>0.9597</td>
<td>0.9851</td>
<td>0.9959</td>
<td>0.9990</td>
<td>0.9994</td>
</tr>
<tr>
<td>( \phi^1</td>
<td>s )</td>
<td>-0.1949</td>
<td>-0.2133</td>
<td>-0.2286</td>
<td>-0.2393</td>
<td>-0.2501</td>
<td>-0.2576</td>
</tr>
</tbody>
</table>

Average asset price and average holdings of agent 1 conditional on the state \( s \). \( m \) = regulated margin level on both assets in all six states, \( p|s \) = average price of the aggregated long-lived asset, \( \theta^1|s \) = agent 1’s average holding of the two assets, \( \phi^1|s \) = agent 1’s average holding of the risk-free bond.

The results in Table VIII for uniform regulation reveal several patterns. First, for each margin level \( m \), the average conditional price of the aggregated long-lived asset is increasing in the exogenous shock \( s \). Second, agent 1’s average holding of the long-lived asset is increasing in the shock \( s \) for each \( m \). Third, agent 1’s average short position in the bond is increasing in the shock \( s \) until state 5 (again for each \( m \)). In the highest-growth state, state 6, agent 1 is so rich, that she can afford to reduce leverage for \( m \in \{0.6, 0.7\} \). We also observe three more patterns which are the foundation for the unconditional statistics reported in Table VI. Fourth, for each state \( s \), agent 1’s average holding of the long-lived asset is increasing in the margin level \( m \). Fifth, for the states 2 until 6, the average conditional price of the long-lived asset is decreasing in \( m \). Sixth, for the states 2 until 6, agent 1’s average short position in the bond is decreasing in \( m \).

The results in Table IX for countercyclical regulation reveal somewhat different patterns.
Table IX: Average conditional asset price and portfolio holdings under countercyclical regulation

<table>
<thead>
<tr>
<th>m</th>
<th>s</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.6</td>
<td>p</td>
<td>2.1616</td>
<td>2.5554</td>
<td>2.9403</td>
<td>3.2043</td>
<td>3.3798</td>
<td>3.4584</td>
</tr>
<tr>
<td></td>
<td>θ</td>
<td>0.0970</td>
<td>0.7115</td>
<td>0.9235</td>
<td>0.9757</td>
<td>0.9716</td>
<td>0.9728</td>
</tr>
<tr>
<td></td>
<td>φ</td>
<td>-0.1019</td>
<td>-0.8877</td>
<td>-1.1765</td>
<td>-1.2434</td>
<td>-1.2287</td>
<td>-1.1941</td>
</tr>
<tr>
<td>0.7</td>
<td>p</td>
<td>2.2876</td>
<td>2.6280</td>
<td>3.0601</td>
<td>3.2283</td>
<td>3.2810</td>
<td>3.4068</td>
</tr>
<tr>
<td></td>
<td>θ</td>
<td>0.5972</td>
<td>0.9604</td>
<td>0.9944</td>
<td>0.9962</td>
<td>0.9836</td>
<td>0.9873</td>
</tr>
<tr>
<td></td>
<td>φ</td>
<td>-0.6662</td>
<td>-1.1664</td>
<td>-1.1241</td>
<td>-1.0494</td>
<td>-0.9605</td>
<td>-0.9582</td>
</tr>
<tr>
<td>0.8</td>
<td>p</td>
<td>2.7285</td>
<td>2.8737</td>
<td>2.9971</td>
<td>3.0842</td>
<td>3.0563</td>
<td>3.1735</td>
</tr>
<tr>
<td></td>
<td>θ</td>
<td>0.9943</td>
<td>0.9990</td>
<td>0.9999</td>
<td>0.9999</td>
<td>0.9886</td>
<td>0.9935</td>
</tr>
<tr>
<td></td>
<td>φ</td>
<td>-1.0325</td>
<td>-0.8473</td>
<td>-0.7298</td>
<td>-0.6755</td>
<td>-0.5931</td>
<td>-0.6076</td>
</tr>
<tr>
<td>0.9</td>
<td>p</td>
<td>2.7045</td>
<td>2.7338</td>
<td>2.7569</td>
<td>2.7716</td>
<td>2.6208</td>
<td>2.6935</td>
</tr>
<tr>
<td></td>
<td>θ</td>
<td>1.0000</td>
<td>1.0000</td>
<td>1.0000</td>
<td>1.0000</td>
<td>0.9888</td>
<td>0.9919</td>
</tr>
<tr>
<td></td>
<td>φ</td>
<td>-0.4965</td>
<td>-0.4113</td>
<td>-0.3546</td>
<td>-0.3275</td>
<td>-0.2501</td>
<td>-0.2607</td>
</tr>
</tbody>
</table>

Average asset price and average holdings of agent 1 conditional on the state s. m = regulated margin level on both assets in growth states 5 and 6, p|s = average conditional price of the aggregated long-lived asset, θ|s = agent 1’s average holding of the two assets, φ|s = agent 1’s average holding of the risk-free bond.

First, for margin levels m ∈ {0.8, 0.9} the average conditional price of the aggregated long-lived asset does not increase from (the recession) state 4 to (the normal-growth) state 5 but instead decreases. Second, for each margin level m, agent 1’s average holding of the long-lived asset in state 4 is larger than her average holding in the growth states 5 and 6. Moreover, for m ∈ {0.8, 0.9} her average holding of the long-lived asset in all four negative-growth states exceeds her average holding in the normal-growth state 5. Third, for each margin level m, agent 1’s average short position in the bond in state 4 is larger than in states 5 and 6. In fact, for m ∈ {0.7, 0.8, 0.9}, her average short position in states 2, 3, and 4 exceeds those in states 5 and 6. These three patterns, which are not at all present for uniform regulation, reveal the critical impact of countercyclical margins on the economy.

In response to larger margin requirements in the good states, agent 1 must reduce her leverage. For this purpose, she must even sell a small portion of the long-lived asset; selling the risky asset to the risk-averse agent 2 dampens the increase in the conditional price that naturally occurs when agent 1’s relative wealth increases in response to a good shock 5 or 6. In fact, for margin levels m ∈ {0.8, 0.9} the conditional normalized price in state 5 is even smaller than in state 4. This dampening effect on the asset price in the positive-growth states reduces the asset return volatility. Conversely, in response to smaller margin requirements in negative-growth states, agent 1 can actually increase her leverage compared to good states. In particular for m ∈ {0.8, 0.9}, both her average holding of the aggregated long-lived asset and her short position in the bond are larger in the four negative-growth states than in the two positive-growth states. So, on average,
agent 1 buys the long-lived asset in response to a bad shock. As a result, the relative asset price does not decrease as much as it would have otherwise because agent 1's relative wealth decreases in response to a bad shock $s = 1, 2, 3, 4$. This buffer effect on the asset price in the negative-growth states also reduces the asset return volatility. And so, the dampening effect in the good states and the buffer effect in bad states together lead to the drastic decrease in the asset return volatility apparent in the solid line in Figure V.

In sum, equilibrium portfolios and prices exhibit qualitatively different features in an economy with countercyclical margins on all assets than in an economy with uniform margins on all assets or in an economy with an unregulated asset. For sufficiently large margin requirements in the positive-growth states, the much less risk-averse agent 1 reduces leverage in positive-growth states and increases leverage in negative-growth states. A reduction of leverage in good times and increase of leverage in bad times greatly reduces asset return volatility compared to uniform regulation. The countercyclicality of leverage dampens or even reverses those movements in the conditional price that lead to large excess volatility under constant regulation.

6 Discussion

The numerical analysis (in the two previous sections) of margin regulation in the framework of a general equilibrium model has delivered numerous insights. We now want to argue that these insights bear fundamental significance for margin policy. For this purpose, we first explain that the findings on the effects of U.S. Regulation T in the empirical literature are in line with the predictions of the general equilibrium model. Second, we relate the current policy discussion on margin regulation in securities lending and repo markets to the model predictions. Finally, we discuss the assumptions and limitations of our general equilibrium analysis.

6.1 Regulation T

In response to the speculative stock market bubble of 1927–1929 and the subsequent “great crash” of 1929, the United States Congress passed the Securities Exchange Act of 1934 which granted the Federal Reserve Board (FRB) the power to set initial margin requirements on national exchanges. The introduction of this law had three major purposes: the reduction of “excessive” credit in securities transactions, the protection of buyers from too much leverage, and the reduction of stock market volatility, see, for example, Kupiec (1998). Under the mandate of this law, the FRB established Regulation T to set minimum equity positions on partially loan-financed transactions of exchange-traded securities. Figure VI from Fortune (2000) shows the Regulation T margin requirements between 1940 and 2000. While the initial margin ratio has been held constant at 50 percent since 1974 (until today), the FRB frequently changed initial margin requirements in the range of 50 to 100 percent from 1947 until 1974.\footnote{While the Securities Exchange Act of 1934 also granted the Federal Reserve Board to set maintenance margins (see Kupiec (1998)), Regulation T governs initial margin requirements only. Maintenance margins are generally set by security exchanges and broker-dealers.}
The introduction and frequent adjustments of the initial margin ratio prompted the development of a sizable empirical literature on the effects of Regulation T. Already Moore (1966) claimed that the establishment of margin requirements had failed to satisfy any of its objectives. He argued that a major reason for the regulation’s failure was that investors could avoid the regulation by substituting margin loans through other forms of borrowing. Kupiec (1998) provides a comprehensive review of the empirical literature; in particular, he extends the scope of his analysis to account for margin constraints on equity derivative markets. He finds that “there is no substantial body of scientific evidence that supports the hypothesis that margin requirements can be systematically altered to manage the volatility in stock markets. The empirical evidence shows that, while high Reg T margin requirements may reduce the volume of securities credit lending and high futures margins do appear to reduce the open interest in futures markets, neither of these measurable effects appears to be systematically associated with lower stock return volatility.” Furthermore, Kupiec (1998) quotes from an internal 1984 FRB study that “margin requirements were ineffective as selective credit controls, inappropriate as rules for investor protection, and were unlikely to be useful in controlling stock price volatility.” Similarly, Fortune (2001) concludes after a review of 18 papers in the literature as well as some additional analysis that “the literature does provide some evidence that margin requirements affect stock price performance, but the evidence is mixed and it is not clear that the statistical significance found translates to an economically significant case for an active margin policy.” In particular, Fortune (2001) argues that even though some studies suggest that the effect of margin loans on stock return volatility is statistically significant, such effects are much too small to be of economic significance.
recalls the popular sentiment, see Moore (1966), that investors may substitute between margin loans and other debt: “If an investor views margin debt as a close substitute for other forms of debt, changes in margin requirements will shift the type of debt used to finance stock purchases without changing the investors total debt. The investors leverage will be unchanged but altered in form. The risks faced, and the risk exposure of creditors, will be unchanged. Little will be changed but the name of the paper.”

The empirical analysis of Regulation T in Hardouvelis and Theodossiou (2002) provides a notable exception from the mainstream opinion and finds that increasing margin requirements in normal and bull periods significantly lowers stock market volatility and that no relationship can be established during bear periods. The authors’ policy conclusion is to set margin requirements in a countercyclical fashion as to stabilize stock markets.

The main predictions of our general equilibrium analysis of margin requirements are in consonance with the empirical evidence on Regulation T. The model predictions in Sections 5.1 and 5.2 that, in the presence of unregulated asset classes, changes in the margin requirement of a regulated asset (class) have a non-monotone and only weak impact on that asset’s return volatility coincides with the results reported in Kupiec (1998) and Fortune (2001). Also, in consonance with the findings of Kupiec (1998), we do find that the average amount of lending decreases as regulated margin levels increase. Moreover, as the margin requirement on the regulated asset increases, the leveraged agent holds a larger fraction of her wealth in the unregulated asset. That is, she borrows relatively more against the unregulated asset than against the regulated asset, just as argued by Moore (1966) and Fortune (2001). The general equilibrium analysis in Section 5.2 also suggests that countercyclical regulation may have a somewhat larger albeit still small effect on return volatility and thus may provide some support for the suggestion of Hardouvelis and Theodossiou (2002).

6.2 Current Policy Discussion

This section relates our theoretical findings to the current debate on the regulation of margin requirements in repo and securities lending markets. In the aftermath of the financial crisis of 2007–2009, it has been argued that excessively low margin requirements led to a build-up of collateralized borrowing thus exacerbating the subsequent downturn (see, for example, CGFS (2010)). As a consequence, the Financial Stability Board launched a public consultation on a policy framework for addressing risks in securities lending and repo markets (see FSB (2012) or FSB (2013)). Among other things, it explicitly includes a policy proposal to introduce minimum haircuts on collateral for securities financing transactions. On page 12 of FSB (2012), the motivation for regulating margins or haircuts is summarized as follows: “Such a framework would be intended to set a floor on the cost of secured borrowing against risky asset in order to limit the build-up of excessive leverage.” Though our model is foremost suited for the analysis of stock market margin regulation (like Regulation T), we believe that the underlying economic mechanisms yielding our key results also play a major role for margin regulation in other markets featuring collateralized borrowing. In particular, our analysis suggests the following three implications for the ongoing policy debate in repo and securities lending markets:
First, a countercyclical regulation of margin requirements is more effective than a simple constant margin regulation. Our analysis of margin regulation for all asset classes has shown that countercyclical regulation decreases stock market volatility substantially, in strong contrast to uniform regulation. Hence, competent authorities should be given the power to require higher margin requirements in good times.

Second, our analysis shows that only a regulation of all asset classes leads to a quantitatively significant decrease in volatility: if not all collateralizable assets are subject to regulation, our model predicts that the ability to take up leverage with other, unregulated assets would strongly limit the impact of regulation. This suggests that margin regulation should be applied with a very broad scope as to ensure maximal impact on volatility. In other words, “carve-outs” for specific asset classes would be counterproductive and should be largely avoided.

Finally, our general equilibrium analysis also reveals that there might be unintended consequences of margin regulation on other, unregulated asset classes. As the analysis in Section 5 has shown, the volatility of the unregulated asset decreases monotonically as the margin requirement on the other asset is increased. Thus, the effect on the volatility of other assets turns out to be stronger and monotone.\(^{10}\) So, there are strong spillover effects from the margin regulation of the regulated asset on the return volatility of the unregulated asset. Hence, the lesson is that for any quantitative impact study done for margin regulation the effects on similar asset markets have to be considered.

While the regulation of asset markets that we consider is very different from the regulation of capital requirements for banks, these two regulatory tools share a common economic motivation. Therefore, we briefly want to discuss the countercyclical nature of margin regulation in light of the Countercyclical Capital Buffer (CCB) as introduced in the Basel III accord. Remember that the primary goal of setting higher margins in good times is to lean against the build-up of leverage via borrowing against collateral. To have a significant effect on volatility, our analysis has shown that high margins in good times need to be complemented with low regulated margins in bad times. The goals of the CCB are similar, but the ordering is reversed: the main objective of the CCB is to ensure that in bad times “the banking sector in aggregate has the capital on hand to help maintain the flow of credit in the economy without its solvency being questioned” (BIS (2010), p.8). Only as a secondary goal, the CCB is supposed to have a moderating effect on the build-up of excessive credit in good times.

6.3 Discussion of Assumptions and Limitations

The general equilibrium analysis of regulated and endogenous margin requirements in this paper rests on a number of strong assumptions. We now critically review the most important assumptions and the resulting limitations of our analysis.

\(^{10}\)To understand this result we have to recall the mechanisms that are at work when margins on stocks are changed. First, as the margin for the regulated asset increases, the unregulated asset becomes, in relative terms, even better collateral for agent 1. Thus, she now has an even stronger motive to hold on to the unregulated asset after a bad shock. This stabilizes the price of this asset. Second, as agent 1’s ability to leverage decreases deleveraging episodes become less severe which further reduces the volatility of the unregulated asset. Thus, both effects work in the same direction for the unregulated asset.
Our general equilibrium economy is a grossly simplified model of modern markets. The focus on only two asset classes completely ignores the presence of a huge number of security markets with very different features. In particular, contrary to some microeconomic studies of specific markets or sectors such as, for example, repo markets or the banking industry, the GE model ignores institutional details. The model takes a very high-level view and considers financial markets only in an aggregated fashion. Clearly, specific market structure can have a strong impact on market outcomes. Another oversimplification is the restriction to two agents; we consider neither implications of non-participation nor the fact that only agents with margin accounts can buy stocks on margin. Therefore, our model does not consider the many different types of trading restrictions that perhaps could be modeled via many agents with different budget restrictions. Instead, our general equilibrium analysis is meant as a transparent macro-finance study of margin regulation in an economy. In particular, we want to analyze general equilibrium effects of margin regulation. For that reason, we believe that our abstraction from many specific market features and the restriction to two agents facing collateral constraints is justified by the transparent insights into the economic mechanism that we obtain from a general equilibrium analysis.

For the general equilibrium model to be numerically tractable, we have to limit the possible trades that agents can enter into. In the economy, short-sales of the long-lived assets are not permitted. Clearly, this assumption is not satisfied in practice. Investors can enter short positions in the stock market and secure such short sales by holding bonds as collateral. Allowing for such ‘reversed’ portfolios of long bond and short stock positions is certainly important but would render the model (at least currently) intractable. We believe that an important area of future research in financial economics will be the examination of bond-secured short positions in long-lived financial assets or other financial securities. At the same time, we believe that, despite this limitation, our analysis in this paper contributes to the understanding of margin regulation.

As in any quantitative study, our numerical results hinge on the parametrization of the economy. Our parametrization exhibits several special features that are somewhat unrealistic. In the analysis of countercyclical margins, we assume that the regulator chooses the margin level depending on the present growth rate. That appears likely to be unrealistic because the regulating agency may not know the current growth rate; instead it appears to be more likely that regulators would make margins depend on price levels, particularly on the price-dividend ratio, or on total leverage in the economy. With such a regulator in the general equilibrium economy, the margin levels would become dependent on the endogenous state variable. Such a model is much more difficult if not impossible to solve. Another unattractive feature of our model parametrization is that the growth shocks are i.i.d. Certainly it may be more realistic to consider Markovian shocks; particularly after a sharp economic downturn we may expect a higher probability of a strong “recovery”, that is, a higher probability for the good states after a very bad state. And we could certainly include Markovian shocks in our analysis. However, we deliberately chose i.i.d. shocks so that the transition probabilities would not impact the equilibrium and thereby obscure our analysis of the features of margin regulation.

We also assume that labor endowments of the two agents and the dividends paid by the two assets are all collinear with aggregate endowments, and consequently with each other. This
assumption is, of course, also unrealistic. In particular, dividends in our model are less volatile than in the data. This is one reason why return volatility in our simulations is substantially lower than in the data, even though collateralized borrowing causes substantial excess volatility in our model. Despite this drawback, we assume collinearity because it ensures that the differing statistics for the two assets are not driven by their dividend dynamics, but only by their different margins (and to a much smaller extent their different size). It also ensures that the behavior of the two agents is only driven by their different risk aversion and income share, not the hedging demand resulting from a non-trivial correlation structure between labor endowments and dividends. Thus, the collinearity assumption allows for a more transparent analysis of the dynamics within the model and in particular of the effect of margin regulation.

In sum, while our general equilibrium analysis ignores many institutional details, it allows us to examine general equilibrium effects of margin regulation. Moreover, for technical reasons we must impose short-sale constraints on the long-lived assets and let countercyclical margin levels depend on the exogenous shock. Furthermore, for a transparent analysis we choose a rather special model specifications. Undoubtedly, all these assumptions influence the quantitative results. However, the described qualitative effects of margin requirements and regulation are likely to be present in models far beyond the scope of ours.

As a final comment on the limitations of our analysis, we emphasize again that the motivation for this study has been the question whether Regulation T enabled the FRB to reach the aforementioned third goal of the Securities and Exchange Act of 1934, namely the reduction of stock market volatility. Therefore, we focus almost exclusively on asset market volatility in our analysis. In particular, we do not report results on the welfare effects of margin regulation. Such a welfare study would face serious obstacles in our model. First, it is unclear which welfare metric would be most appropriate for an economy with heterogeneous preferences. Second, Epstein et al. (2012) casts serious doubt on the usefulness of Epstein-Zin utility for the study of normative issues.

### 7 Conclusion

In this paper, we have analyzed the quantitative effects of margin regulation on asset return volatility in the framework of a general equilibrium infinite-horizon economy with heterogeneous agents and collateral constraints. There are two assets in the economy which can be used as collateral for short-term loans. We have first analyzed an economy in which a regulating agency imposes a margin requirement on the first asset while the margin requirement for the second asset is determined endogenously in equilibrium. We have shown that the presence of collateral constraints leads to strong excess volatility and a regulation of margin requirements potentially has stabilizing effects. However, we have seen that changes in the regulation of a class of assets may have only small effects on the assets’ return volatility if investors have access to another (unregulated) class of collateralizable assets to take up leverage. Therefore, the predictions of the general equilibrium model are in consonance with the findings of the empirical literature on U.S. Regulation T. In fact, the regulatory changes in the regulated market have much stronger effects on the return volatility of the unregulated asset. We have also shown that margin regulation has a much stronger impact on asset return volatility if all long-lived assets in the economy
are regulated. In such an economy, countercyclical regulation that imposes sufficiently large macroprudential add-ons on margin levels in high-growth states can lead to drastic reductions in asset return volatility.

Appendix

A Sensitivity Analysis

As in any quantitative study, our results above hinge on the parametrization of the economy. In this appendix, we discuss how our results depend on the preferences of the two types of agents. We also check how the presence of disaster risk influences our results. These robustness checks further deepen our understanding of the mechanisms of the model.

A.1 Preferences

As a robustness check for the results on the effectiveness of the various forms of regulations presented in Sections 5, we consider different specifications for the IES, the coefficients of risk aversion, and the discount factor, $\beta$. Obviously, changes in the IES and the risk aversion coefficients affect the risk-free rate. For each specification, we recalibrate $\beta$ to get a risk-free rate of 1.0% for the case of a constant margin of 60 percent. Table X reports changes in stock market volatility for several different combinations of these parameters. For each combination of parameters, we report three numbers: First, the change in stock market volatility if the constant regulation of stock margins is changed from 60 percent to 90 percent. Second, the change in stock market volatility if the countercyclical regulation of stock margins is changed from 60 percent to 90 percent in boom periods. As in Section 5, the margin in shocks 1-4 is set to 50 percent. Finally, we report changes in volatility for countercyclical margin regulation of all assets. We consider a change of boom margins from 60 percent to 90 percent. Note that for this case the reported change in volatility corresponds both to the stock market volatility and the aggregate volatility, as the two assets are now identical. For convenience, we repeat the results for our baseline model, $(IES, RA, \beta) = ((2, 2), (0.5, 7), (0.942, 0.942))$, and report them as the case (P1).

<table>
<thead>
<tr>
<th>$(IES^1, IES^2), (RA^1, RA^2), (\beta^1, \beta^2)$</th>
<th>Constant</th>
<th>Countercyclical</th>
<th>Full market (countercyclical)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(P1): (2,2),(0,5,7),(0,942,0,942)</td>
<td>2.5</td>
<td>-5.3</td>
<td>-35.6</td>
</tr>
<tr>
<td>(P2): (2,2),(0,5,9),(0,880,0,880)</td>
<td>-3.2</td>
<td>-5.9</td>
<td>-35.5</td>
</tr>
<tr>
<td>(P3): (2,2),(0,5,5),(0,975,0,975)</td>
<td>7.7</td>
<td>5.7</td>
<td>-23.5</td>
</tr>
<tr>
<td>(P4): (2,2),(0,1,7),(0,942,0,942)</td>
<td>1.6</td>
<td>-6.5</td>
<td>-36.1</td>
</tr>
<tr>
<td>(P5): (2,2),(1,7),(0,941,0,941)</td>
<td>6.9</td>
<td>-1.4</td>
<td>-33.8</td>
</tr>
<tr>
<td>(P6): (0,75,0,75),(0,5,7),(0,945,0,945)</td>
<td>-11.2</td>
<td>-10.3</td>
<td>-38.4</td>
</tr>
<tr>
<td>(P7): (1,5,1,5),(0,5,7),(0,943,0,943)</td>
<td>1.6</td>
<td>-5.4</td>
<td>-42.4</td>
</tr>
</tbody>
</table>

Table X: Sensitivity analysis for preferences (percentage change in volatility)
We find that the effect of a change in the constant regulation of stock margins is relatively small for all different preference specifications. The same is true for countercyclical regulation. However, as in the baseline, countercyclical regulation tends to be more effective in reducing stock market volatility. Finally, a countercyclical regulation of all assets substantially reduces volatility across all specifications considered.

### A.2 Disaster Shocks

Disaster shocks are a central feature of our calibration. Naturally the question arises how much the reported qualitative and quantitative economic consequences of margin regulation depend on these extreme shocks. To answer this question, we conduct two analyses. We first report results for a model with disaster shocks that are only half as severe (D2) and demonstrate that the results remain qualitatively the same. Second, we scale down the probability of disasters (D3) and find the effect of margin regulation in the case of constant or countercyclical regulation is even quantitatively almost the same as in the baseline model. However, the effect of regulating the full market turns out to be only half as strong.

<table>
<thead>
<tr>
<th></th>
<th>Constant</th>
<th>Countercyclical</th>
<th>Full market (countercyclical)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(D1): Baseline</td>
<td>2.1</td>
<td>-5.3</td>
<td>-35.6</td>
</tr>
<tr>
<td>(D2): Half-Sized Disaster</td>
<td>2.1</td>
<td>-1.4</td>
<td>-11.5</td>
</tr>
<tr>
<td>(D3): Half-Probability of Disaster</td>
<td>1.0</td>
<td>-6.1</td>
<td>-17.6</td>
</tr>
</tbody>
</table>

Table XI: Sensitivity analysis for preferences (percentage change in volatility)

### B Details on Computations

The algorithm used to solve all versions of the model is based on Brumm and Grill (2013). Equilibrium policy functions are computed by iterating on the per-period equilibrium conditions, which are transformed into a system of equations which we solve at each grid point. Policy functions are approximated by piecewise linear functions. By using fractions of financial wealth as the endogenous state variables, the dimension of the state space is equal to the number of agents minus one. Hence with two agents, the model has an endogenous state space of one dimension only. This makes computations much easier than in Brumm and Grill (2013), where two- and three-dimensional problems are solved. In particular, in one dimension reasonable accuracy may be achieved without adapting the grid to the kinks. For the reported results we used 160 grid points. If the number of grid points is increased to a few thousands, then the moments under consideration only change by about 0.1 percent. Hence, using 160 points provides a solution which is precise enough for our purposes. The moments reported in the paper are averages of 50 different simulations with a length of 10,000 periods each (of which the first 100 are dropped). This is enough to let the law of large numbers do its job, even for the rare disasters.
References


CGFS (2010). The role of margin requirements and haircuts in procyclicality. *CGFS Papers 36*.


