Tests for *m*-dependence based on sample splitting methods

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Inference in the presence of m-dependence

- In many situations it is assumed that time series display m-dependence:
- Specification of MA(q) models.
- Differencing methods applied in the presence of white noise errors.
- Testing **predictability** of excess returns in the expectations hypothesis literature involving **maturities** exceeding the sampling interval of the data $(k > 1)$.
- We develop methods to deal with inference in the presence of m-dependence based on **sample splitting**.

m-dependence

- Nonparametric framework:

$$\mathcal{H}_0^{(k)} : \xi_{t+1} \text{ is } (k-1)\text{-dependent}, \quad k \geq 1,$$

allowing for any type of relationship up to $k-1$ lags:

$$\{\ldots, X_{n-1}, X_n\} \text{ is independent of } \{X_{k+n}, X_{k+n+1}, \ldots\} \text{ for any } n.$$

- Parametric linear MA$(k-1)$:

$$H_0^{(k)} : \xi_{t+k} = \alpha_k + \sum_{i=1}^{k} \theta_i e_{t+i},$$

with either,

- $e_t \sim iid\left(0, \sigma^2\right)$.
- $e_t \sim \text{WN}\left(0, \sigma^2\right)$, mixing, and possibly heteroskedastic.

Sample Splitting under m-dependence

- It amounts to **regular sampling** of original data at lower frequencies, so that observations within each subsample are effectively independent, e.g.

$$S_i = \{\xi_i, \xi_{k+i}, \ldots, \xi_{T-k+i}\}.$$

- Standard techniques for checking independence can be applied to individual subsamples.
- But **subsamples** are not independent.
- Information provided by each separate subsample is aggregated, taking into account the correlation between subsamples.
- Our methods do not need to estimate a parametric model, exploiting directly the *m*-dependence, guaranteing exact independence under the null.

Predictability testing

- We recognize that data is $(k-1)$-dependent for some $k \geq 1$ because of data construction, but we want to test for further dependence.
- We can check **linear predictability** of returns at and beyond lag k, through k-step ahead predictive regressions.
- Use dynamic information in excess returns, through higher order autocorrelations:
  - Variance Ratio tests.
  - Portmanteau tests, etc.
- ... or other information on **nonlinear predictability** of levels.
- The properties of these methods are affected by the m-dependence.
- **Independence** testing is not interesting in practice since volatility can be present.

Outline of presentation

1. Variance Ratio tests under independence.
2. Variance Ratio tests under *m*-dependence.
3. Other tests: Portmanteau & Fama-French regressions.
4. Application: expectation hypothesis.
5. Simulation results.
6. Empirical evidence.

Predictive regressions for testing expectations hypothesis

$$\begin{aligned} y_{t+k} &= \alpha_k + \beta_k x_{t|k} + e_{t+k} \\ x_{t|k} &= \varphi x_{t-k|k} + v_{t+k} \end{aligned}$$

- y_{t+k} is k-period ahead spot return and $x_{t|k}$ is the forward premium.
- y_{t+k} is a weighted average of the future changes in the short term interest rate and $x_{t|k}$ is the term spread.
- If $x_{t|k}$ is persistent $(\varphi \approx 1)$, these regressions suffer from finite sample biases and efficiency issues.
- Autocorrelation in e_{t+k} and $x_{t|k}$ should be accounted for.
- **Alternative** to the regression tests: nonparametric tests such as the variance ratio tests which are free of such **biases**.

The Variance Ratio tests

- Have been widely used for testing a **random walk** of asset prices such as stock prices and spot exchange rates.
- But, not been used for testing **predictability** of excess returns in the expectations hypothesis literature.
- VR captures the common restriction of **'uncorrelated'** both in excess returns and in asset returns.
- Testing expectations hypothesis involves **maturities**: when the maturity exceeds the sampling interval of the data ($k > 1$), most distributional results of the variance ratio tests are not directly applicable due to the presence of m-dependence in excess returns.
- Our sampling splitting method allows to deal with this finite dependence, suitable for many test statistics: VR, BP and FF regressions.
- VR's provide further information on the **dependence pattern** useful to identify economic alternatives.

IID returns I

- Under the null hypothesis of $\beta_1 = 1$ in regression (1), one-period excess returns are defined by

$$\mathcal{H}_0^{(1)} : \xi_{t+1} = y_{t+1} - x_{t|1} = \alpha_1 + e_{t+1}, \qquad e_{t+1} \sim iid\left(0, \sigma^2\right).$$

- Define the sample variance ratio as

$$\widehat{VR}_1(q) = \frac{\widehat{\sigma}_b^2(q)}{\widehat{\sigma}_b^2(1)} = 1 + 2\sum_{i=1}^{q-1}\left(1 - \frac{i}{q}\right)\hat{\gamma}_{\xi}(i),$$

where $\widehat{\sigma}_b^2(q) = (qm_1(q))^{-1}\sum_{t=q}^{T}(\xi_t + \cdots + \xi_{t-q+1} - q\widehat{\alpha_1})^2$.

  - $VR_1(q) = 1$ if ξ_t is not serially correlated;
  - $VR_1(q) > 1$ if ξ_t is positively correlated;
  - $VR_1(q) < 1$ if ξ_t is negatively correlated;

IID returns II

- Under $\mathcal{H}_0^{(1)}$,

$$z_q = \sqrt{T}\left(\widehat{VR}(q) - 1\right)\left(\frac{2(2q-1)(q-1)}{3q}\right)^{-\frac{1}{2}} \sim_a N(0,1),$$

  [Lo and MacKinlay (1988)].
- The assumption of independence can be relaxed to martingale difference or uncorrelation+mixing-type conditions, allowing for conditional heteroskedasticity [Lo and MacKinlay (1988)].
- See also Romano and Thombs (1996), Lobato, Nakervis and Savin (2002).

m-dependence

$$\mathcal{H}_0^{(k)} : \xi_{t+1} \text{ is } (k-1)\text{-dependent.}$$

- The VR's have an unknown value for all q (the same for $q \geq k$), and $\hat{\gamma}_{\xi}(i)$ are no longer asymptotically independent under $\mathcal{H}_0^{(k)}$.
- The dependence between ξ_t and $\xi_{t+1}, \ldots, \xi_{t+k-1}$ is unspecified.
- But $\mathcal{H}_0^{(k)}$ rules out any, possibly non-linear, dependence beyond lag k.
- Stationarity is not necessary.
- A direct application of **predictability tests** requires the use of a **single reduced sample** of length T/k to guarantee uncorrelated returns under $\mathcal{H}_0^{(k)}$.
  However, this approach leads to **inefficiencies** due to an effective reduced sample size.

Sample Splitting (k>1)

- Let $S = \{\xi_1, \xi_2, \ldots, \xi_T\}$ be the whole sample and define each of k subsamples by
$$S_i = \{\xi_i, \xi_{k+i}, \ldots, \xi_{T-k+i}\}.$$
- Several approaches to summarize information from **subsamples VR's**:
  - Pooled variance ratios
  - Pooled rank-based variance ratios
  - Bonferroni tests based on maximum and minimum VR
  - Wald variance ratio test
  - Bootstrap-based VR tests
  - Bound methods: Dufour and Torres (1998)

Pooled variance ratios I

- Define the estimates of $Var(\xi_t)$

$$\widehat{\sigma}^2_{a|k} = \frac{1}{k}\sum_{j=1}^{k}\widehat{V}_j(1) \quad \text{and} \quad \widehat{\sigma}^2_{b|k}(q) = \frac{1}{k}\sum_{j=1}^{k}\widehat{V}_j(q),$$

where

$$\widehat{V}_j(q) = \frac{1}{qm_k(q)}\sum_{t=q}^{T/k}\left(\xi_{k(t-1)+j} + \cdots + \xi_{k(t-q)+j} - q\widehat{\alpha_{kj}}\right)^2.$$

- **Lemma**:

Both $\widehat{\sigma}^2_{a|k}$ and $\widehat{\sigma}^2_{b|k}(q)$ are **consistent** and **unbiased** under $\mathcal{H}_0^{(k)}$.

Pooled variance ratios II

- **Pooled VR:**

$$\widehat{VR}_k(q) = \frac{\widehat{\sigma}^2_{b|k}(q)}{\widehat{\sigma}^2_{a|k}}.$$

- **Lemma**:
Under $\mathcal{H}_0^{(k)}$, the asymptotic distributions of the variance ratio of k-period excess returns are given by

$$\frac{\sqrt{T}(\widehat{VR}_k(q) - 1)}{\sqrt{2(q-1)(2q-1)/3q}} \sim_a N\left(0, \Lambda_k(q)\right)$$

where $\Lambda_k(q)$ is derived from covariances between subsamples:

Pooled variance ratios III

- Under $\mathcal{H}_0^{(k)}$,

$$\Lambda_k(q) = \frac{6q}{\sigma_k^4} \sum_{a=1}^{k-1} \sum_{b=1}^{k-1} \sum_{i=1}^{q} \sum_{j=1 \vee i-1}^{q \wedge i+1} \left(1 - \frac{i}{q}\right) \left(1 - \frac{j}{q}\right) \delta^{(a,b)}(i,j),$$

where $\delta^{(a,b)}(i,j) = Cov\left\{\sqrt{T/k}\tilde{\gamma}_a(i), \sqrt{T/k}\tilde{\gamma}_b(j)\right\}$, e.g.

$$\delta^{(a,b)}(i,i) = E\left[\bar{\xi}_0 \bar{\xi}_{a-b} \bar{\xi}_{ik} \bar{\xi}_{ik+a-b}\right] + E\left[\bar{\xi}_0 \bar{\xi}_{a-b-k} \bar{\xi}_{ik} \bar{\xi}_{ik+a-b-k}\right], \; i > 0.$$

- Under $H_0^{(k)}$, MA$(k-1)$ + iid innovations, $\Lambda_k(q) = 1 + \Omega_k(q)$, with

$$\Omega_k(q) = \frac{2}{\sigma_k^4} \sum_{i=1}^{k-1} \frac{k-i}{k} \left\{ \begin{array}{c} E[\bar{\xi}_0 \bar{\xi}_i]^2 + E[\bar{\xi}_0 \bar{\xi}_{k-i}]^2 \\ + \frac{4(q-2)}{(2q-1)} E[\bar{\xi}_0 \bar{\xi}_i] E[\bar{\xi}_0 \bar{\xi}_{k-i}] \end{array} \right\}.$$

Wald VR tests I

- A **Wald** type test, also proposed by Richardson and Smith (1991) for long run regressions, using the joint distribution of individual VR's.
- For individual variance ratio,
$\sqrt{T/k}(\widehat{VR}_k^{(j)}(q) - 1) \sim_a N\left(0, \frac{2(q-1)(2q-1)}{3q}\right), \quad j = 1, \ldots, k.$
- In general,

$$U_k(q) = \frac{\sqrt{T/k}}{\sqrt{\frac{2(q-1)(2q-1)}{3q}}} \left(\widehat{VR}_k^{(1)}(q)\text{-}1, \ldots, \widehat{VR}_k^{(k)}(q)\text{-}1\right)' \sim_a N(0, \Sigma_k(q))$$

- Under $\mathcal{H}_0^{(k)}$,

$$\Sigma_k^{(a,b)}(q) = \frac{1}{\sigma_k^4} \left\{ \begin{array}{c} \sum_{i=1}^{q-1} \left(1 - \frac{i}{q}\right)^2 \delta^{(a,b)}(i,i) + \\ \sum_{i=2}^{q-1} \left(\frac{q-i}{q}\right)\left(\frac{q-i+1}{q}\right)\left[\delta^{(a,b)}(i, i\text{-}1) + \delta^{(a,b)}(i\text{-}1, i)\right] \end{array} \right\}$$

Wald VR tests II

- Under $H_0^{(k)}$, iid innovations, $\delta^{(a,b)}(i, i-1)$ simplify and e.g. $\Sigma_k^{(a,a)}(q) = 1$.
- Then the test statistic is

$$(RU_k(q))' \left(R\hat{\Sigma}_k(q)R'\right)^{-1} (RU_k(q)) \sim_a \chi_r^2$$

  where R depends on the particular null hypothesis we want to test on the vector $V_k(q)$:
  - $R = I_k$: all individual VR's equal to one.
  - $R = (1/k \ \cdots \ 1/k)$: average VR across subsamples equal to one (equivalent to pooled statistics).
  - $R = (1 \ -1 \ 0 \cdots 0 \mid 1 \ 0 \ -1 \ 0 \cdots 0 \mid \cdots)$: equal VR's accross subsamples.
- Richardson and Smith (1991) used similar idea for $k = 1$ and many q's.

Other methods I

- Any nonlinear function of $\left(\widehat{VR}_k^{(1)}(q), \ldots, \widehat{VR}_k^{(k)}(q)\right)$ could be used, but asymptotics may be difficult to apply: median, maximum and minimum deviations from 1, etc.
- **Bonferroni** tests: to exploit information on the sign of the predictability, we can use both the maximum and minimum of $\sqrt{T/k}(\widehat{VR}_k^{(j)}(q) - 1)$, $j = 1, \ldots, k$, adjusting asymptotic critical values using α/k.

Box-Pierce Tests (k>1)

- **Individual subsample Box-Pierce** statistics,

$$Q_k^{(a)}(q) = \frac{T}{k}\sum_{i=1}^{q}\hat{\gamma}_k^{(a)}(i)^2 \sim_a \chi_q^2, \quad a = 1, \ldots, k,$$

- **Pooled Box-Pierce** statistics based on joint estimation of autocorrelations,

$$\widehat{Q}_k(q) = T\sum_{i=1}^{q}\hat{\gamma}_k(i)^2,$$

where

$$\hat{\gamma}_k(i) = \frac{1}{k}\sum_{a=1}^{k}\hat{\gamma}_k^{(a)}(i).$$

Under $H_0^{(k)}$, for $\hat{\gamma}_k = (\hat{\gamma}_k(1), \ldots, \hat{\gamma}_k(q))'$,

$$\widehat{Q}_k(q) = T\hat{\gamma}_k'\Xi_k(q)^{-1}\hat{\gamma}_k \sim_a \chi_q^2$$

$$\Xi_k^{(i,j)}(q) = \frac{1}{k^2}\sum_{a=1}^{k}\sum_{b=1}^{k}\delta^{(a,b)}(i,j), \quad i,j = 1, \ldots, q.$$

Fama French Regressions I

- Fama and French (1988) regress the n-period future returns on the n-period past returns to capture a slowly mean reverting component in stock prices:

  $$\tilde{\xi}^{n,k}_{t+n,k} = \alpha_{n,k} + \beta_{n,k}\tilde{\xi}^{n,k}_{t} + u^{n,k}_{t+n,k}, \quad \text{for positive integer } n,$$

  where

  $$\tilde{\xi}^{n,k}_{t} = \sum_{i=0}^{n-1} \tilde{\xi}_{t-i,k}$$

  is the n-period stock returns between t and $t-(n-1)k$.
- The implication of the null hypothesis tested here is

  $$\beta_{n,k} = 0 \quad \text{for each } n.$$

- The test was designed for testing the **random walk hypothesis** of stock prices so that there is no need to account for the maturity.

Fama French Regressions II

- This test can be used for testing the expectation hypothesis with $k > 1$, taking into account of the mismatch between the forecasting horizon and sampling frequency with adjusted s.e.'s using Hansen and Hodrick (1980) or Newey and West (1987).
- In fact, these regressions are sort of **generalized variance ratios,**

$$\beta_n = \frac{Cov(\widetilde{\xi}_t^n, \widetilde{\xi}_{t+n}^n)}{Var(\widetilde{\xi}_t^n)} = \frac{Var(\widetilde{\xi}_t^n + \widetilde{\xi}_{t+n}^n)}{2Var(\widetilde{\xi}_t^n)} - 1 = VR_1(q = 2n, q' = n) - 1,$$

where

$$VR_1(q, q') = \frac{Var(\sum_{i=0}^{q-1} \xi_{t+i})/q}{Var(\sum_{i=0}^{q'-1} \xi_{t+i})/q'}.$$

- No clear efficiency improvements from $q' > 1$.

Nonparametric Tests: sign and rank tests

- Campbell and Dufour (1995) proposed conditional independence tests based on signs with exact finite sample distribution under the null.
- Wright (2000) proposed Variance Ratio statistics using ranks.
- These tests of are useful in the presence of data with either outliers or important non-normality features that may affect the precision/validity of asymptotics.
- These nonparametric tests are valid under general forms of nonnormality and conditional heteroscedasticity but in the presence of m-dependence can only be applied directly to subsamples, leading to the usual aggregation problem.
- Let $r_j(\xi_t)$ be the **rank** of ξ_t among $\xi_j, \xi_{k+j}, \ldots, \xi_{T-k+j}$ S_j,

$$r_t = \left(r_j(\xi_t) - \frac{T/k+1}{2} \right) \left(\frac{(T/k-1)(T/k+1)}{12} \right)^{-1/2},$$

are standardized with sample mean 0 and variance 1.

Nonparametric Tests: further comments

- **Efficient regression tests:** Campbell and Yogo (2006), Jansson and Moreira (2006).
- Mean prediction (**martingale difference hyp.**): Escanciano and Velasco (2006).
- **Long Horizon Tests:** Finite-sample distribution of VR's and autocorrelation statistics can be quite different from usual asymptotics due to overlap in the returns data, in particular with a small number of non-overlapping multi-year asset returns.
- Richardson and Stock (1989) show that sample VR's are not consistent if q/T approaches some constant and the asymptotic theory with fixed q performs poorly in finite samples.
- Our methods do not solve this problem and should be limited to cases in which $q/(T/k)$ is reasonably small, but our bootstrap alleviate those size distortions in finite samples.
- **Disadvantages of sample splitting methods:** higher order dependence affecting only some specific lags, e.g., over $\gamma(k+1)$, is not observable.
  Solution: sampling at lower frequencies, $k+1$, $k+2$, etc.

Bootstrap tests

- Under $H_0^{(k)}$, we propose a **parametric bootstrap** method under

$$H_0^{(k)} : \xi_{t+k} = \alpha_k + \sum_{i=1}^{k} \theta_i e_{t+i},$$

with either,

1. $e_t \sim iid\left(0, \sigma^2\right)$.
2. $e_t \sim \text{GARCH}(p, q)$.

fitting and resampling from a model for the whole sample.

- Under $\mathcal{H}_0^{(k)}$, it is also possible to use block-bootstrap methods.

Application: Expectation Hypothesis. Econometric Framework I

- The null hypothesis [uncovered interest parity]:

$$H_0 : E_t[s_{t+k}] - s_t = i_{t|k} - i^*_{t|k} = f_{t|k} - s_t \quad \text{for each } k,$$

- The alternative hypothesis:

$$H_a : E_t[s_{t+k}] - s_t = f_{t|k} - s_t - p_{t|k} \quad \text{for each } k,$$

  where p_t are deviations from uncovered interest parity.

- s_t and $f_{t|k}$ are generated from present value models, with different $p_{t|k}$.

Application: Expectation Hypothesis. Econometric Framework II

1. **Time varying risk premium** alternative (negative serial dependence):

$$p_{t|k} = (1-\varphi_k)E[p] + \varphi_k p_{t-k|k} + \nu_t + \cdots + \nu_{t-k+1}$$

2. **Expectations error alternative** (positive serial dependence)
   - Market expectation:

$$E_t^m[s_{t+k}] = (1-\lambda)E_t[s_{t+k}] + \lambda E_t^n[s_{t+k}].$$

   - $p_t = \lambda(E_t[s_{t+k}] - E_t^n[s_{t+k}])$ if $f_{t|k} = E_t^m[s_{t+k}]$
   - Noise traders' expectations are regressive:

$$E_t^n[s_{t+k}] = (1-g_k)s_t + g_k \bar{s}_t.$$

   - $\varpi_t = s_t - \bar{s}_t$:

$$\varpi_{t|k} = E[\varpi] + \psi_k \varpi_{t-k|k} + \eta_t + \cdots + \eta_{t-k+1}.$$

Monte Carlo Simulations

- Model 1:

$$\xi_{t+k} = \xi_{1,t+k} := \sum_{i=1}^{k} \epsilon_{t+i}$$

- Model 2 (the risk premium alternative):

$$\xi_{t+k} = \xi_{1,t+k} + \frac{b}{1 - b\varphi} \sum_{i=1}^{k} \nu_{t+i} - p_{t|k}$$

- Model 3 (the expectation error alternative):

$$\begin{aligned} \xi_{t+k} &= \xi_{1,t+k} + \frac{1 - b(1 + \lambda g_k)}{(1 - \bar{b}\psi_k)(1 - b\lambda)} \sum_{i=1}^{k} \eta_{t+i} \\ &+ \lambda (1 - \lambda) b \left(\frac{\psi_k - 1}{(1 - b\lambda)(1 - b\psi_k)} + \frac{g_k}{1 - b(1 + \lambda g_k)} \right) p_{t|k} \end{aligned}$$

Monte Carlo Simulations

- The data generating mechanism for s_t is formulated at the weekly frequency and return horizons considered are three-month ($k = 13$) and one-year ($k = 52$).
- Sample size: $T = 33 * 52$. However, this sample size is significantly reduced to $T_k = \frac{33*52}{k}$ when using only non-overlapping observations.
- Set $\sigma_\epsilon = \sigma_\nu = \sigma_\eta$ and values to accommodate data properties.
- $p_{t|k}$ are modeled as stationary but persistent processes.
- Use the Warp method of Giacomini, Politis and White (2007) to increase numerical efficiency.
- Tests reported below can be for right-tail and left-tail one-sided and two-sided alternatives.
- 5% significant level.

Summary of results (size)

- There is a size distortion when the test is based on asymptotic critical values: rejections mainly occur at the right-tail for large q.
- The Bootstrap method overcomes this problem: the size is close to the nominal values for all q aggregation values considered.

Size of tests based on asymptotic critical values

Panel A (*iid*). $k = 13$					
q	Pooled method 5%-L	Pooled method 5%-R	Bonferroni max 5%-R	Bonferroni min 5%-L	Wald method 5%
2.00	0.08	0.04	0.02	0.02	0.10
4.00	0.08	0.05	0.03	0.00	0.07
8.00	0.07	0.06	0.03	0.00	0.04
12.00	0.05	0.07	0.04	0.00	0.03
16.00	0.05	0.07	0.04	0.00	0.02
20.00	0.04	0.07	0.04	0.00	0.02
32.00	0.02	0.08	0.05	0.00	0.02
40.00	0.01	0.09	0.05	0.00	0.02

Size of tests based on asymptotic critical values

Panel B (GARCH$(1,1)$). $k = 13$

q	5%-L	5%-R	5%-R	5%-L	5%
	Pooled method		Bonferroni max	Bonferroni min	Wald method
2.00	0.08	0.04	0.03	0.02	0.14
4.00	0.07	0.05	0.04	0.00	0.10
8.00	0.06	0.06	0.04	0.00	0.07
12.00	0.05	0.07	0.04	0.00	0.06
16.00	0.04	0.07	0.04	0.00	0.05
20.00	0.03	0.08	0.05	0.00	0.05
32.00	0.01	0.08	0.05	0.00	0.04
40.00	0.00	0.09	0.05	0.00	0.03

Size of tests based on bootstrap method

Panel A (idd). $k = 13$					
	Pooled VR		max	min	Wald
q	5% L	5% R	5% R	5% L	method
2.00	0.05	0.05	0.05	0.05	0.05
4.00	0.05	0.05	0.05	0.05	0.05
8.00	0.05	0.05	0.05	0.05	0.05
12.00	0.06	0.05	0.05	0.05	0.05
16.00	0.05	0.05	0.05	0.05	0.04
20.00	0.05	0.05	0.05	0.05	0.04
32.00	0.05	0.05	0.04	0.05	0.04
40.00	0.05	0.05	0.04	0.04	0.04

Size of tests based on bootstrap method

Panel B, GARCH$(1,1)$. $k=13$					
	Pooled VR		max	min	Wald
q	5% L	5% R	5% R	5% L	method
2.00	0.05	0.05	0.04	0.05	0.05
4.00	0.05	0.05	0.04	0.05	0.05
8.00	0.06	0.05	0.04	0.05	0.05
12.00	0.06	0.05	0.04	0.05	0.04
16.00	0.06	0.05	0.04	0.05	0.04
20.00	0.05	0.05	0.04	0.04	0.04
32.00	0.05	0.05	0.04	0.04	0.04
40.00	0.05	0.05	0.04	0.04	0.04

Summary of results (power)

- The following aggregation methods produce similar power: pooled rank, pooled, maximum, and minimum methods.
- The minimum test best performs when excess return exhibits negative serial dependence
- The maximum test best performs when excess return exhibits positive serial dependence
- The patterns of power functions are similar in both alternatives: initially increases and then decreases with q (stationary but persistent deviations from null).

Power of tests based on bootstrap: Model 2 (r.p.)

Panel A, *iid*. $k = 13$					
	Pooled VR		max	min	Wald
q	5% L	5% R	5% R	5% L	method
2.00	0.29	0.00	0.00	0.26	0.03
4.00	0.45	0.00	0.00	0.43	0.02
8.00	0.60	0.00	0.00	0.59	0.01
12.00	0.67	0.00	0.00	0.66	0.01
16.00	0.70	0.00	0.00	0.71	0.00
20.00	0.71	0.00	0.00	0.71	0.00
32.00	0.64	0.00	0.00	0.65	0.00
40.00	0.58	0.00	0.00	0.59	0.00

Power of tests based on bootstrap: Model 2 (r.p.)

Panel B, GARCH$(1,1)$. $k=13$

	Pooled VR		max	min	Wald
q	5% L	5% R	5% R	5% L	method
2.00	0.29	0.00	0.00	0.26	0.03
3.00	0.42	0.00	0.00	0.43	0.02
4.00	0.59	0.00	0.00	0.58	0.01
5.00	0.67	0.00	0.00	0.66	0.01
8.00	0.69	0.00	0.00	0.69	0.00
10.00	0.69	0.00	0.00	0.70	0.00
15.00	0.63	0.00	0.00	0.62	0.00
20.00	0.58	0.00	0.00	0.57	0.00

Power of tests based on bootstrap: Model 3 (e.e.)

Panel A, *iid*. $k = 13$					
	Pooled VR		max	min	Wald
q	5% L	5% R	5% R	5% L	method
2.00	0.01	0.28	0.25	0.01	0.09
4.00	0.01	0.37	0.34	0.01	0.09
8.00	0.01	0.45	0.42	0.01	0.12
12.00	0.01	0.47	0.43	0.01	0.16
16.00	0.01	0.47	0.43	0.01	0.21
20.00	0.01	0.46	0.43	0.01	0.22
32.00	0.01	0.43	0.40	0.01	0.23
40.00	0.01	0.41	0.38	0.01	0.23

Power of tests based on bootstrap: Model 3 (e.e.)

Panel B, GARCH$(1,1)$. $k = 13$					
	Pooled VR		max	min	Wald
q	5% L	5% R	5% R	5% L	method
2.00	0.01	0.26	0.22	0.01	0.11
3.00	0.01	0.34	0.30	0.01	0.10
4.00	0.01	0.41	0.37	0.01	0.11
5.00	0.01	0.44	0.39	0.01	0.12
8.00	0.01	0.44	0.40	0.01	0.14
10.00	0.01	0.44	0.39	0.01	0.13
15.00	0.01	0.41	0.37	0.01	0.15
20.00	0.01	0.40	0.35	0.01	0.16

Data

- German deutschmark (GDM), British pound (BRP), Japanese yen (JPY).
- Sample period 1975:1 to 2007:12.
- Log excess returns are annualized as $(5200/k)(s_{t+k} - f_{t|k})$.
- Weekly series for three-month ($k = 13$) and one-year ($k = 52$) returns are obtained from the daily data.
- Data Source: London close bid and ask prices and obtained from the database of Global Insight.

Summary of empirical results

- Foreign excess returns exhibit positive serial dependence for the three-month horizon:
- For the one-year return horizon, the results are mixed: excess returns exhibit positive serial dependence up to five years and negative serial dependence for $q > 5$ years.
- The minimum test provides most rejections for all currencies and return horizons.
- The other three tests such as the maximum, the pooled rank, and the pooled tests are equally powerful for many cases.
- P-values increase with q.

Tests of unbiased hypothesis of three-month forward exchange rates: JPY

q	Variance Ratio	Pooled method	Rank -VR median	Rank -VR max	Rank -VR min
		p-value*	p-value*	p-value*	p-value*
2.00	1.17	0.01	0.00	0.02	0.00
4.00	1.31	0.04	0.01	0.01	0.00
8.00	1.51	0.06	0.01	0.02	0.01
12.00	1.55	0.10	0.02	0.04	0.01
16.00	1.42	0.18	0.07	0.11	0.04
20.00	1.24	0.28	0.13	0.18	0.08
32.00	1.13	0.34	0.22	0.28	0.14
40.00	1.33	0.26	0.16	0.25	0.10
T	1716	1716	1716	1716	1716

Conclusions

- Simple methods to deal with hypothesis testing in the presence of m-dependent data.
- Asymptotic and finite sample properties of the variance ratio tests based on several aggregation methods for the expectation hypothesis are explored when the return horizon typically mismatches with the sampling frequency.
- Those methods include the pooled rank-based, the pooled, the Bonferroni maximum and minimum, Bootstrap based maximum and minimum, and the Wald variance ratio tests.
- The minimum test with bootstrap cv.'s performs best.
- The pooled rank-based, the pooled, and bootstrap based maximum tests are about equally powerful.