

Fight Cartels or Control Mergers?

On the Optimal Allocation of Enforcement Efforts within Competition Policy

Andreea Cosnita-Langlais^{*†} and Jean-Philippe Tropeano[‡]

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Abstract

This paper deals with the optimal enforcement of the competition law in terms of merger and anti-cartel policies. We examine the interaction between these two branches of the competition policy given the cost of resources available to the competition agency and taking into account the ensuing incentives for firms' behavior in terms of choice between cartels and mergers. We are thus able to infer the optimal allocation of enforcement efforts between controlling mergers and fighting cartels, and thereby conclude on their optimal competition policy mix. We show for instance that to the extent that firms may switch from cartel to merger depending on the current focus of the competition law enforcement, applying a stricter merger control only pays if the cartel fighting policy is not too expensive.

Keywords: competition law enforcement, antitrust, merger control, anti-cartel policy

JEL classification: L41, K21, D82

^{*}Université Paris Ouest Nanterre La Défense and EconomiX - FRE CNRS 3257

[†]Corresponding author: acosnita@u-paris10.fr, +33(0)140977798, 200 Av.de la République, 92001 Nanterre, France.

[‡]Université de Paris 1-Paris School of Economics; tropeano@univ-paris1.fr

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1. Introduction

Horizontal Agreements scrutiny by Competition Authorities experienced recently increasing severity. The new European Commission Merger Regulation that tightens the clearing criteria as well as the new European leniency program that undermines cartel stability are clear examples. The effectiveness of a stricter merger control crucially depends on the type of cartel-fighting that can be afforded by the competition agency (CA henceforth). The main argument behind this intuition is the increasing evidence that mergers and cartels are substitutable strategies for industry firms contemplating market coordination, which swap one for another depending on the relative focus of CA on either merger control or cartel fighting. On the one hand, when mergers become harder to accomplish, cartel activities are adopted instead, as Neumann (2001) argues for German industries such as cement, food processing, and machine building. On the other, whenever cartel formation is restricted, firms tend to turn to mergers instead. In the words of Mueller (1996), "The Sherman Act was passed in 1890 as a reaction to the expansion of size and power of large enterprises in the United States, i.e., the rise of the 'trusts'. Ironically, by prohibiting cartels it encouraged firms to combine to avoid the costs of unbridled competition that were expected to follow the dissolution of the cartels, and thus helped precipitate the first great merger wave at the turn of the century." The American example of the Sherman and Clayton Acts is later confirmed in the UK by the outcome of the Restrictive Trade Practices Act (1956), which triggered a merger wave by outlawing cartels (Bittlingmayer (1985), Symeonidis (2002)). Similarly, based on the analysis of duration for a sample of international cartels prosecuted in the 1990s, Evenett et al (2001) argue that joint ventures and mergers are among the different measures adopted by firms for survival, in cartel-prone industries where cartel formation is restricted¹. The choice to resort to merger instead of cartel, or the reverse, depends on the current enforcement of the competition policy, to the extent that the latter modifies the relative profitability of these two options.

In this paper we develop a very simple framework to determine the optimum enforcement effort of both merger control and cartel fighting, depending on the cost of cartel detection, the social loss from market coordination between poorly efficient firms and the social gain from mergers leading to cost savings. For this purpose, we also model the firms' decision to either

¹According to Symeonidis (2002), the 1956 Restrictive Practices Act in the UK, by banning cartels, led to an increase in competition which resulted in lower profits, which in its turn prompted a profit-restoring increase in market concentration through mergers.

join a price-fixing cartel or notify and undertake a horizontal merger. The relative profitability of the two options will depend on the probability for a cartel to be convicted, as well as on a merger's associated cost and its capacity to achieve cost savings.

Cartel fighting is imperfect in our model, not all cartels get punished, and the probability to convict a cartel will depend on the amount of resources allocated to this branch of the competition policy, knowing that resources are costly. The amount of resources used for fighting cartel will be used to model the severity of the fight against price agreements. The enforcement of merger control is also imperfect, at least to the extent that one considers the current policy setting, as we do. Explicitly, the *ex ante* assessment of horizontal mergers (without any *ex post* review) inevitably gives rise to both type of errors, i.e. clearing anti-competitive mergers and banning cost-efficient pro-competitive ones. This occurs because of the asymmetric information between the competition agency and the merging partners on the actual probability for a given notified merger to be a cost-efficient one. To account for this, we model a competition agency dealing with a population of merger projects that differ along two dimensions: the degree of cost savings that can be obtained through the merger, and the riskiness of the project itself (i.e. the probability to actually achieve the cost savings).

The main trade-off that we put forward in this framework is the following. Whenever more resources are invested into fighting cartels, not only will more cartels be detected and punished and thus welfare losses avoided (basically, a detection effect), but also firms will be prompted to abandon cartel formation and undertake horizontal mergers instead (a so-called selection effect). This may improve welfare as long as the newly-triggered mergers are efficient. However, given the imperfect merger control, it may also turn out to be a welfare-decreasing strategy. On the other hand, increasing the severity of merger control - which is not the same as its accuracy - means that the least socially-profitable merger projects will turn into price-fixing cartels, which will lower expected welfare. The opportunity to resort to a more or less lenient cartel-fighting policy and merger control will depend on the cost of available resources and on the level of welfare gains and losses that are likely to obtain from mergers.

Based on this, we are able to provide the following results. First of all, depending on the relative cost of resources allocated to fighting cartels, the public actions towards horizontal mergers and price agreements may be either substitutable or complementary. As long as an intense cartel detection is not affordable, due to a high cost of budget resources, we are able to show that a tougher merger control saves on these costly anti-cartel resources - hence the

substitutability between the two. In turn, when the CA affords to intensify cartel fighting thanks to low-cost resources, we find that the soft merger policy saves on anti-cartel resources, leading to complementarity between merger control and cartel fighting.

In terms of welfare, we are able to identify the optimal competition policy mix between merger control and cartel fighting, according to the different levels of cost of anti-cartel resources. Given the above mentioned detection and selection effects, we obtain that for very expensive anti-cartel action, welfare is maximized by soft merger control coupled with intense cartel fighting. If the latter becomes less expensive, then the soft merger policy is still optimal, but together with lenient anti-cartel action. Finally, for very cheap anti-cartel resources, welfare is maximized by a combination of strict merger policy and very intense cartel fighting.

This is to our knowledge the first research paper to ask the question of the optimal competition law enforcement mix between merger control and cartel fighting. In a related but different context, a similar question was raised by Aubert and Pouyet (2004) concerning the relationship between cartel-fighting and sectorial regulation². As far as antitrust and merger control go, the only contribution, albeit from a positive perspective, is that of Mehra (2008), which formalizes the firms' choice between merger and cartel depending on the private cost incurred on account of the current law enforcement (the fine in case the cartel is detected).

The rest of the paper is structured as follows. We first present the framework of our analysis, then provide the results starting with the private choice by industry firms between cartel and horizontal merger. Next we infer the optimal public choice in terms of cartel deterrence effort, and conclude on the optimal competition policy mix between cartel fighting and merger control. The paper closes with some ideas for enriching the basic framework retained and thus extend our results. All formal proofs are grouped in a final technical appendix.

2. Model

2.1. Framework

The analysis will involve two risk-neutral agents, a group of two firms on the one hand and the competition agency on the other. The former choose between either horizontal merger or

²See also Bensaid et al. (1995), which investigate the optimality of having a unique antitrust authority to deal with both cartel and mergers, or whether it is on the contrary best to separate the two on account of information and incentives issues.

cartel, whereas the latter decides on the amount of resources dedicated to fighting cartels, as well as which mergers are to be cleared or banned.

Consider for this purpose the following reduced-form setting, in which two firms may coordinate in order to improve profitability either by colluding or by engaging in a horizontal merger³. If the cartel is not detected, it provides a joint collusive payoff of π^C . It may however be punished with probability $a(\Delta)$, $a(0) = 0$, where $\Delta \in [0, \bar{\Delta}]$ stands for the amount of resources spent by the CA on fighting cartels. If so, then the ensuing payoff for the firms will be the status-quo competition joint revenue π . We assume that the total cost incurred by the CA to fight against collusive agreements is $k\Delta$, where k is the constant marginal opportunity cost of funds used to fight cartels. Also, the punishment technology is linearly increasing with the amount of available resources: $a'(\Delta) > 0$ and $a''(\Delta) = 0$, with $a(0) = 0$ and $a(\bar{\Delta}) = 1$. Basically, the cost and the effectiveness of the anti-cartel fighting (represented by $a(\Delta)$) exhibit constant returns to scale.

The horizontal merger on the other hand is not only a legal means to achieve coordination, but also a source of cost savings. These efficiency gains, denoted e , will be achieved with probability ρ . In that case, the profit earned is equal to $\pi^M(e)$. In case of no efficiency gains, the profit is $\pi^M \leq \pi^C$. The probability ρ is distributed on $[0, 1]$ according to a uniform cumulative distribution function $G(\rho)$ and density $g(\rho)$. For simplicity, we assume that there are only two types of cost savings, either high or low, $\bar{e} > \underline{e}$, both occurring with equal probability. We assume that this efficiency gains parameter e is observable, whereas the probability of achieving the cost savings is not. In other words, only firms know the precise value of ρ , the CA only knows its distribution⁴. Finally, industry firms incur a fixed cost K in order to merge. This assumption summarizes the fact that coordination through merger is likely to be costlier than through collusion, to the extent that the price-fixing does not require a structural change in the organization of the partners, but also the fact that there are legal constraints (notification and lawyers' fees) to be obeyed and legal costs to be incurred for a merger to be started, whatever its outcome⁵.

³Our results do not depend on the type of competition (price or quantities) prevailing on the market.

⁴In practice, merging parties typically provide to the CAs estimations of the cost savings generated by their merger. However, proving convincingly and beforehand to the CA that the cost savings will indeed materialize afterwards is altogether a different matter. Our framework basically focuses on this latter uncertainty.

⁵Basically, we normalize to zero the private costs of undertaking cartels (establishing productions quotas, monitoring members, punishing deviations and so on). Thus K represents the relative cost of mergers w.r.t. cartels. We wish to stress that mergers require fixed and sunk costs rather than reoverable or variable ones.

In terms of competition policy, the CA maximizes the expected Consumers' Surplus from both fighting cartels and controlling mergers, net of the cost of resources. Let W^C denote consumers' welfare following a successful cartel without merger, and W the status-quo competition welfare without any coordinated behavior and without merger with $W^C < W$. Concerning the merger policy, let the post-merger consumers' welfare be either $W^M(e)$ or W^M , depending on whether the cost savings materialize or not with $W^M(e) > W^M$. Furthermore we assume that $W^M(\bar{e}) > W^M(\underline{e})$, meaning that the more efficiency gains the higher the consumers' surplus. In practice, mergers get cleared or banned depending on the expected competitive impact, which is basically assessed in terms of expected post-merger price variation. Note however that due to the imperfect information on the probability for the cost savings to actually occur, and the firms' possible choice of cartel instead of merger, merger control may end up clearing welfare-lowering mergers.

The timing of the game will be the following:

At the first stage the CA chooses the amount of resources Δ available for fighting against cartels, and also the severity of the merger control policy it will apply. Explicitly, it sets the threshold $\hat{e}, \hat{e} \in \{\bar{e}, \underline{e}\}$ such that it will only clear mergers with $e \geq \hat{e}$. Two cases may occur. According to the "soft" merger policy all mergers will actually be cleared, because $e \geq \hat{e} = \underline{e}$, for any e . Under the "strict" merger policy, only the high-efficiency mergers get cleared, as implied by $\hat{e} = \bar{e}$.

At the second stage, industry firms make their coordination choice between horizontal merger and collusive behavior. If merger is chosen, they notify it to the CA.

At the final stage, notified mergers are cleared or banned according to the threshold set at the first stage. If there was no notification, then the cartel is convicted with probability $a(\Delta)$ and the market is forced back to its status-quo competition situation. Otherwise, the industry ends up with the collusive market outcome.

The relevant equilibrium concept is the subgame perfect Nash equilibrium, and in what follows we solve the game backwards.

2.2. The private choice between merger and cartel

When deciding how best to achieve profitable coordination, the firms anticipate the outcome of the CA's decision at the final stage of the game. That means that the choice between horizontal merger and cartel is determined by the probability for a cartel to be detected on the one hand, and the type of merger control, more or less severe, on the other. The resulting expected profit will therefore lead to a self-selection effect according to which the merger will be chosen only if the probability to achieve cost savings is sufficiently high. In the Appendix we show that:

Lemma 1. *For any $e \in \{\bar{e}, \underline{e}\}$, there exists a probability threshold $\hat{\rho}_e(\Delta, \hat{e})$ decreasing with Δ , such that the merger is notified iff $\rho \geq \hat{\rho}_e(\Delta, \hat{e})$. We set $\hat{\rho}_e(\Delta, \hat{e}) = 1$ for any $e < \hat{e}$.*

The intuition is straightforward: merger is possibly costlier than the cartel as a means to increase joint profits, so it takes a high enough probability of achieving the cost savings for the industry firms to prefer it to collusion. In addition, the higher the resources allocated to fighting cartels, and thus the higher the detection probability in case of cartel, the higher the incentives for firms to prefer the merger instead. Finally, for a given amount of anti-cartel resources, there will be relatively more highly efficient mergers taking place rather than low efficient ones, because the relevant probability threshold is lower ($\hat{\rho}_{\underline{e}}(\Delta, \hat{e}) > \hat{\rho}_{\bar{e}}(\Delta, \hat{e})$). Basically, the higher the efficiency gains, the likelier the firms are to choose merger instead of cartel.

The level of K the merger fixed cost is naturally crucial for the trade-off to occur or not between merger and cartel from the firms' point of view. Otherwise, either cartel or merger is a dominant strategy for industry firms. For this reason we actually restrict the analysis to the relevant range for K , defined in the Appendix.

3. Optimal competition policy mix

Going back to the previous stage, we discuss in this section the outcome of the CA's simultaneous choice of how many resources to allocate to fighting cartels and which mergers to ban. The CA anticipates the self-selection effect highlighted by Proposition 1, so it knows that its choice of a merger efficiency threshold and amount of anti-cartel resources will impact on the populations of possible cartels and mergers.

Taking into account this double choice, as well as the expected net consumers' welfare maximization objective and the cost of cartel-fighting resources, the general programme for the CA

writes as follows:

$$\begin{aligned} & \max_{\substack{\Delta \in [0, \bar{\Delta}] \\ \hat{e} \in \{\underline{e}, \bar{e}\}}} W(\hat{e}, \Delta) \\ = & \sum_e \frac{1}{2} \left\{ G(\hat{\rho}_e(\Delta, \hat{e})) \cdot [a(\Delta)W + (1 - a(\Delta))W^C] + \int_{\hat{\rho}_e(\Delta, \hat{e})}^1 [\rho W(e) + (1 - \rho)W^M] g(\rho) d\rho \right\} \\ & - k\Delta \end{aligned}$$

This expression summarizes the following trade-off on behalf of the CA.

The CA's strategy to fight cartels will deter part of the population of possible mergers from colluding, and from lemma 1 we know that it is the merger projects most likely to achieve the cost savings which will actually be notified. At the same time, the CA's choice of a merger control threshold implies that some merger projects may never get approved. Explicitly, given the binary setting for the cost savings parameter in our setting, the CA may allow either all mergers or only the highly efficient ones. The latter case corresponds as before mentioned to a stricter merger control which, although improving on average the outcome of merger control, will push the less efficient merger projects to become cartels instead, and hence may lower the final welfare outcome of the competition policy. The CA will solve this trade-off between the severity of merger control and the intensity of effort put into fighting cartels by identifying the optimal combination $(e, \Delta(e))$ in terms of merger threshold and amount of anti-cartel resources.

We start by examining the optimal amount of resources allocated to cartel fighting for a given threshold \hat{e} that governs merger control.

3.1. The optimal choice of anti-cartel resources

We characterize the optimal level of resources allocated to cartel fighting and we prove the following intermediary result:

Lemma 2. *For any type of merger control policy $\hat{e} \in \{\underline{e}, \bar{e}\}$, the optimal amount of resources to be allocated to fighting cartels, denoted by $\Delta^*(\hat{e})$, is such that:*

$$\sum_{e \in \{\underline{e}, \bar{e}\}} \frac{1}{2} \left\{ \underbrace{G(\hat{\rho}_e(\Delta, \hat{e})) \cdot [a'(\Delta)(W - W^C)]}_{\text{detection effect}} - \underbrace{\hat{\rho}'_e(\Delta, \hat{e}) [\hat{\rho}_e(\Delta, \hat{e})W(e) + (1 - \hat{\rho}_e(\Delta, \hat{e}))W^M - a(\Delta)W - (1 - a(\Delta))W^C]}_{\text{Selection effect}} \right\} = k$$

This means that at the optimum, whatever the type of merger control, the marginal welfare gain from increasing the amount of resources for fighting cartels needs to equal the marginal cost of this increase in resources.

The marginal benefit from enhancing cartel fighting can be decomposed into two effects. On the one hand, the more money is spent to convict cartels, more cartels get punished and thus more welfare losses are avoided. This is what we call a detection effect, which trivially improves expected welfare whatever the type of merger policy. On the other hand, there is also a possibly ambiguous selection effect, according to which some cartels being now deterred become mergers. This explains why in general $\Delta^*(\underline{e}) \neq \Delta^*(\bar{e})$. Indeed, the size of this selection effect depends on the type of merger control. With a soft merger policy, it concerns both possible types of merger, \underline{e} and \bar{e} , but with the strict merger control, only the \bar{e} -types will possibly react in this way, because the \underline{e} -ones will not be allowed to merge, however large the incentives provided by the stronger anti-cartel action. Moreover, it can take both signs, to the extent that the deterred cartels are not necessarily pro-competitive mergers, i.e. they do not necessarily achieve the cost savings often enough to actually improve expected welfare on average.

Next we investigate the outcome of a change in merger policy (from soft to strict) in case the amount of resources allotted to cartel-fighting goes up:

Lemma 3. *The marginal welfare gain from enhancing the severity of the merger policy increases with the amount of resources allocated to fighting collusion: $\frac{\partial}{\partial \Delta}(MB(\bar{e}, \Delta) - MB(\underline{e}, \Delta)) > 0$.*

Lemma 3 basically presents a single-crossing property, whose intuition builds on both the selection effect and the detection effect before mentioned.

To start with, consider the selection effect under the soft merger control. As cartel fighting gets tougher and tougher through higher resources spending, the less likely the newly triggered mergers to achieve the cost savings, simply because the opportunity cost to undertake a merger is lower. This follows from Lemma 1. Consequently, the more resources are spent on fighting cartels, the lower the marginal social benefit of investing in cartel fighting. Under tougher merger control, this decreasing selection effect is lower since the less efficient mergers are prohibited. Regarding the detection effect, any increase in cartel fighting has an increasing marginal impact under tough merger control since the probability for the firm to merger is lower. On the whole, the benefit from enhancing the severity of merger control (i.e. giving up on \underline{e} -mergers) increases with the amount of resources used to fight cartels.

Based on these two intermediary results, we have the following:

Proposition 1. *There exists a marginal cost of resources threshold \tilde{k} such that:*

- (i) *for $k \geq \tilde{k}$, harder merger control leads to lenient cartel fighting ($\Delta^*(\underline{e}) > \Delta^*(\bar{e})$)*
- (ii) *for $k < \tilde{k}$, harder merger control leads to intense cartel fighting ($\Delta^*(\underline{e}) < \Delta^*(\bar{e})$).*

Proposition 1 basically concludes on the relationship between the optimal intensity of cartel fighting and the severity of merger control that is applied. It states that for relatively high costs of anti-cartel resources, cartel fighting and merger control are substitutable, whereas for relatively low costs, the two branches of the competition policy are complementary.

Proposition 1 orders actually the optimum levels of anti-cartel resources, one for each type of merger policy, depending on the marginal cost of these resources. It establishes two regimes, each corresponding to the relatively high or low cost of resources. The difference between the two regimes, which explains the change in the ranking of the two Δ^* , follows from Lemma 3, according to which the stricter the merger control, the higher the benefit from intensifying cartel fighting. The latter naturally depends on the cost of resources used.

To start with, for a high marginal cost of anti-cartel fighting, the optimal amount of resources used for this purpose will be low. In this case, the marginal social gain from intensifying anti-cartel action is higher under the soft merger policy, following Lemma 3. Equivalently, for the same marginal cost of anti-cartel resources, it takes a higher amount of resources to optimally fight against cartels under the soft merger policy. In other words, whenever cartel deterrence is quite expensive, the strict merger policy saves on the optimal amount of anti-cartel resources, which means that optimal cartel fighting and merger control enforcement are substitutes. The intuition stems from the above-mentioned detection and selection effects. Any increase in the amount of resources used against cartels always yields a positive detection effect (more cartels are punished, and more welfare losses are avoided). But with a high cost of resources used, the so-called selection effect is also positive. Indeed, the high unit cost of resources affords few such resources, which implies that there will be relatively few mergers notified but with a high probability of cost savings, and relatively many cartels, under both merger policies (Lemma 1). Any supplementary euro invested into anti-cartel fighting will yield a positive selection effect, because it will trigger some more mergers, which are highly likely to achieve the cost savings and are therefore welfare-improving. Finally, this positive selection effect is enhanced under the soft merger policy, because it concerns both types of mergers, whereas the strict merger

control restricts this effect to the only \bar{e} -type. To put it differently, for a high cost of resources used, investing one more euros into fighting cartels improves more welfare under the soft merger policy. Given that the marginal benefit from this under either merger policy is decreasing, it takes more resources for a given unit cost to equal it with the higher marginal social gain under the soft merger policy, or less such resources under the strict one, which in turn yields a lower marginal social gain. Hence the substitutability between the enforcement of the two branches of competition policy.

As the unit cost of resources goes down, the selection effect will also go down, and even becomes negative, unlike the detection effect, which is always positive. Indeed, the lower the unit cost of anti-cartel resources, the more of them are available, so more mergers get notified instead of cartels. But the social marginal benefit from having more mergers submitted under the soft merger policy diminishes and eventually turns negative, because the newly-triggered mergers are less and less likely to achieve the cost savings and thereby to improve welfare. Moreover, the strict merger control prevents this negative selection effect associated with the \underline{e} -mergers because it blocks them altogether. Consequently, for a low unit cost of anti-cartel resources, making available many such resources, the same slight increase in their amount will yield a higher welfare gain under the strict merger policy. This means that for the same marginal cost of anti-cartel resources, it takes more money to optimally fight cartels under the strict merger control, i.e. to equalize the higher marginal gain of these resources with their marginal cost. By the same token, the same marginal welfare increase may be achieved under the soft merger regime with a lower anti-cartel effort. To sum up, whenever fighting cartels is relatively cheap, we find that the enforcement of merger control and the optimal anti-cartel effort are complementary.

3.2. Optimal competition policy mix

At the first stage of the game the CA makes the choice of the optimal competition policy mix in terms of merger control and cartel fighting. Following Proposition 1, if the cost of anti-cartel resources is rather high, the CA will choose between either the soft merger control combined with tough cartel deterrence or the strict merger policy combined with a more lenient anti-cartel action. On the contrary, if the cost of anti-cartel resources is rather low, the CA will balance the combination of soft merger control with lenient cartel fighting against that of strict merger control with tough cartel action. Below we provide the outcome of this trade-off:

Proposition 2. *There exists $k^* \in (0, \tilde{k})$ such that the optimum competition policy is the following:*

- (i) *strict merger control and intense cartel fighting for $k < k^*$*
- (ii) *soft merger control and lenient cartel fighting for $k^* \leq k < \tilde{k}$*
- (iii) *soft merger control and intense cartel fighting for $\tilde{k} \leq k$.*

The merger control should become tough for a cost of resources low enough. Moreover it worth stressing that the switch from soft merger control to tough merger control does not occur when the marginal benefit of cartel fighting becomes lower under the soft merger control than under tough merger control.

Let us give the intuition for this policy-making result.

When cartel fighting is very costly, there is only a limited amount of resources to be spent for it. This implies in return that the relevant probability thresholds for mergers to be preferred are quite high, so basically very few mergers, of either \underline{e} or \bar{e} type, will actually be submitted so that they will all be very likely to achieve the cost savings. More of them are submitted under the soft merger policy. In short, with a high marginal cost of anti-cartel resources, the before mentioned selection effect is positive under both merger policies, and enhanced under the soft one. In addition, following Proposition 2, merger control and optimal cartel fighting are substitutable over the range $k \geq \tilde{k}$ so that the CA will always apply the toughest possible anti-cartel action under the soft merger control.

In turn, for a low cost of anti-cartel resources ($k \leq \tilde{k}$), the two branches of the competition policy are complementary, as indicates Proposition 1.

Given the large amount of anti-cartel resources available thanks to their low marginal cost, the choice of firms will be biased towards mergers under both merger policies (the relevant efficiency probability thresholds are very low, so a lot of either type of mergers tend to be submitted). This means that many of them will likely reduce welfare, because the latest among them are unlikely to achieve the cost savings. This is particularly true under the soft merger policy, which allows the \underline{e} -mergers to take place. Basically, for $k < k^*$ the selection effect on \underline{e} -mergers is negative and large. This welfare loss may be prevented however if the strict merger control were applied. The trade-off over the severity of merger control is unambiguously solved in favour of the strict merger policy when the opportunity cost of giving up on \underline{e} -mergers is low, i.e. the social benefit of such merger is low. Thus, for $k < k^*$, the complementarity between merger control and optimal cartel fighting comes down to a self-enforcing relationship: the strict

merger control is better because it prevents the negative selection effect on \underline{e} -mergers. However, it also leaves no choice to these \underline{e} -mergers but to collude, so it takes an intense cartel fighting to optimally cope with them.

Finally, Proposition 2 identifies a medium range $\tilde{k} > k \geq k^*$ for the unit cost of anti-cartel resources. In this case, merger control and cartel are still complementary, as stated by Proposition 1. So the CA has basically the choice between the soft merger control with lenient cartel fighting and the strict merger policy with intense cartel fighting. But should the strict merger control be applied, it would forbid all \underline{e} -mergers. And for this to be better in terms of welfare, an intense cartel fighting would be required, so as to deal with all the \underline{e} -types which would consequently undertake cartel. Yet, the unit cost of resources within this medium range is still too high to afford enough money for a sufficiently tough anti-cartel action. Equivalently, the welfare loss from giving up on \underline{e} -mergers would not be covered by the welfare gain from the stronger cartel fighting, given the cost of the latter. In other words, the opportunity cost of giving up on \underline{e} -mergers through the strict merger policy is still too high, which means that the so-called selection effect of \underline{e} -mergers is still positive, and thus playing against the strict merger control, even though this soft merger policy leads to clear anticompetitive mergers. This explains why the combination of soft merger policy and lenient cartel fighting is optimal over this medium range of cost of resources.

4. Post merger cartel

We show in that section that our results remain even though the merging firms can form a cartel after the merger. We consider the following extension of our previous model. We assume that after the merger a cartel is detected with the same probability $a(\Delta)$. If the cartel is not detected the profit earned by the merging firms is denoted by π^{MC} without efficiency gains and $\pi^{MC}(e)$ in case of efficiency gains. We make the two following assumptions on profits:

$$\text{A1 } \pi^{MC} - \pi^M < \pi^C - \pi$$

This assumption means that the cartel leads to a higher profit increase if the firms do not merge. This is true as long as the cartel profit if the firms merge (π^{MC}) is not too high with respect to the cartel profit without merger (π^C). For instance, as long as the same cartel could be sustained after and before the merger the result is true.

$$\text{A2 } \pi^M(e) - \pi^M > \pi^{MC}(e) - \pi^{MC}$$

In other words, we assume here that higher efficiency gains lead to a higher profit increase if

the merging firms do not form a cartel. Indeed, the output is higher without cartel so that the positive impact of lower variable cost is higher without cartel.

The consumer surplus is denoted by W^{MC} if the merging firms without efficiency gains form a cartel and is denoted by $W^{MC}(e)$ in case of efficiency gains.

Basically our main results summarized in proposition 2 are based on the negative impact of the resources Δ invested in cartel fighting on the threshold $\hat{\rho}$ (lemma 1) as well as on the single crossing property (lemma 2). We show that both results remain valid under assumptions A1 and A2 and we derive the following proposition.

Proposition 3. *Under assumptions A1 and A2, lemma 1 and 2 hold so that the optimal policy mix is given by the following rule: there exists k^{**} such that the optimum competition policy is the following:*

- (i) *strict merger control and intense cartel fighting for $k < k^*$*
- (ii) *soft merger control and lenient cartel fighting for $k^* \leq k < \tilde{k}$*
- (iii) *soft merger control and intense cartel fighting for $\tilde{k} \leq k$.*

The result could be explained in two steps. First, under assumptions A1 and A2, the cartel fighting is more detrimental to profits when the firms do not merge. Indeed, if the firms merge, in case of efficiency gains, the cartel is relatively less profitable and without efficiency gains, higher market power is ensured by the merger even though the cartel is detected. As a result, an increase in the cartel fighting resources rises the incentive to merge. The threshold $\hat{\rho}$ decreases with Δ as a result. Second, regarding the optimal policy, the introduction of post merger cartel does not affect the basic trade-off between the hard merger control and the soft merger control. As before, to be socially beneficial a tougher merger control requires a high investment in cartel fighting in order control efficiently the prohibited mergers. The CA accepts to incur that high investment for a low cost of resources only.

5. Concluding remarks

This paper focuses on the optimal enforcement of the competition law in terms of merger control and anti-cartel policies. The observation of real-life market behavior indicates that often firms switch from cartels to mergers, or the reverse, depending on the current focus of competition agencies. To put it short, when fighting cartels became the top-ranking objective,

markets have been known experience a surge in merger activity. Starting from this, we examined the interaction between the enforcement of the two branches of the competition policy, the merger control and the anti-cartel fighting. In so doing, we accounted for the resulting incentives for firms, which play an essential part in the final welfare outcome of the competition law enforcement. Given the cost of resources available for fighting cartels, we infer the optimal competition policy mix between controlling mergers and fighting cartels. We show for instance that applying a stricter merger control only pays in terms of welfare if the anti-cartel effort is not too expensive. The results would nevertheless benefit from enriching the current simple framework. Several alternative assumptions could be explored. Introducing a public cost for controlling mergers, or equivalently a total amount of resources available for both fighting cartel and controlling mergers, would enable a true resource-allocation analysis for these branches of the competition policy. On a more general level, the case of complementarity between the strategies of horizontal merger and cartel on behalf of industry firms is equally worth integrating into the framework. Such extension as well as several other alternative technical assumptions on the shape of cost functions and probability distributions are left for future research.

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6. Appendix

Proof of Lemma 1.

Horizontal merger is preferred to cartel as long as $\rho\pi^M(e) + (1-\rho)\pi^M - K \geq a(\Delta)\pi + (1-a(\Delta))\pi^C$
 $\Leftrightarrow \rho \geq \frac{a(\Delta)\pi + (1-a(\Delta))\pi^C - \pi^M + K}{\pi^M(e) - \pi^M} = \hat{\rho}_e(\Delta, \hat{e})$. In addition, $\frac{\partial}{\partial \Delta} \hat{\rho}_e(\Delta, \hat{e}) = a'(\Delta) \frac{\pi - \pi^C}{\pi^M(e) - \pi^M} < 0$,
and also $\hat{\rho}_{\bar{e}}(\Delta) < \hat{\rho}_{\underline{e}}(\Delta)$ due to $\pi^M(\bar{e}) > \pi^M(\underline{e})$. ■

Proof - Define the relevant range for K .

To avoid trivial cases, we assume $\hat{\rho}_e(\bar{\Delta}, \underline{e}) < 1$. This is equivalent to $K \leq (\pi^M(e) - \pi)$. ■

Proof of Lemma 2.

To prove this result we need first explicit the expected welfare depending on the possible values of $\hat{e} \in \{\underline{e}, \bar{e}\}$:

The expected welfare for the soft merger policy writes

$$W(\underline{e}) = \frac{1}{2} \left\{ G(\hat{\rho}_{\underline{e}}(\Delta)) \cdot [a(\Delta)W + (1-a(\Delta))W^C] + \int_{\hat{\rho}_{\underline{e}}(\Delta)}^1 [\rho W^M(\underline{e}) + (1-\rho)W^M] g(\rho) d\rho \right\} \\ + \frac{1}{2} \left\{ G(\hat{\rho}_{\bar{e}}(\Delta)) \cdot [a(\Delta)W + (1-a(\Delta))W^C] + \int_{\hat{\rho}_{\bar{e}}(\Delta)}^1 [\rho W(\bar{e}) + (1-\rho)W^M] g(\rho) d\rho \right\} - k\Delta$$

whereas that for the strict merger policy writes

$$W(\bar{e}) = \frac{1}{2} [a(\Delta)W + (1-a(\Delta))W^C] \\ + \frac{1}{2} \left\{ G(\hat{\rho}_{\bar{e}}(\Delta)) \cdot [a(\Delta)W + (1-a(\Delta))W^C] + \int_{\hat{\rho}_{\bar{e}}(\Delta)}^1 [\rho W^M(\bar{e}) + (1-\rho)W^M] g(\rho) d\rho \right\} - k\Delta$$

Denote $MB(e)$ the first derivative for the expected welfare. One has:

$$MB(\underline{e}) \\ = \frac{1}{2} \left\{ \begin{aligned} & [G(\hat{\rho}_{\underline{e}}(\Delta)) + G(\hat{\rho}_{\bar{e}}(\Delta))] \cdot a'(\Delta) \cdot [W - W^C] + \\ & g(\hat{\rho}_{\underline{e}}(\Delta)) \cdot \hat{\rho}'_{\underline{e}}(\Delta) \cdot [a(\Delta)W + (1-a(\Delta))W^C - \hat{\rho}_{\underline{e}}(\Delta)W^M(\underline{e}) - (1-\hat{\rho}_{\underline{e}}(\Delta))W^M] + \\ & g(\hat{\rho}_{\bar{e}}(\Delta)) \cdot \hat{\rho}'_{\bar{e}}(\Delta) \cdot [a(\Delta)W + (1-a(\Delta))W^M - \hat{\rho}_{\bar{e}}(\Delta)W^M(\bar{e}) - (1-\hat{\rho}_{\bar{e}}(\Delta))W^M] \end{aligned} \right\} \\ -k$$

and

$$MB(\bar{e}) \\ = \frac{1}{2} \left\{ \begin{aligned} & [1 + G(\hat{\rho}_{\bar{e}}(\Delta))] \cdot a'(\Delta) \cdot [W^C - W] + \\ & g(\hat{\rho}_{\bar{e}}(\Delta)) \cdot \hat{\rho}'_{\bar{e}}(\Delta) \cdot [a(\Delta)W + (1-a(\Delta))W^M - \hat{\rho}_{\bar{e}}(\Delta)W^M(\bar{e}) - (1-\hat{\rho}_{\bar{e}}(\Delta))W^M] \end{aligned} \right\} \\ -k$$

For each type of merger policy and for a given k a solution $\Delta^*(e)$ obtains from $MB(e) = 0$.

The solution $\Delta^*(e)$ is unique in each case, thanks to the following (the strict monotonicity of the MB functions):

$$\begin{aligned} & \frac{\partial}{\partial \Delta} MB(\underline{e}) \\ = & \frac{1}{2} \left\{ \begin{array}{l} \left[g(\hat{\rho}_{\underline{e}}(\Delta)) \cdot \underbrace{\tilde{\rho}'_{\underline{e}}(\Delta)}_{<0} + g(\hat{\rho}_{\bar{e}}(\Delta)) \cdot \underbrace{\tilde{\rho}'_{\bar{e}}(\Delta)}_{<0} \right] \cdot \underbrace{a'(\Delta)}_{>0} \cdot \underbrace{[W - W^C]}_{>0} + \\ g(\hat{\rho}_{\underline{e}}(\Delta)) \cdot \underbrace{\tilde{\rho}'_{\underline{e}}(\Delta)}_{<0} \cdot \underbrace{[a'(\Delta)(W - W^C) - \tilde{\rho}'_{\underline{e}}(\Delta)(W^M(\underline{e}) - W^M)]}_{>0} + \\ g(\hat{\rho}_{\bar{e}}(\Delta)) \cdot \underbrace{\tilde{\rho}'_{\bar{e}}(\Delta)}_{<0} \cdot \underbrace{[a'(\Delta)(W - W^C) - \tilde{\rho}'_{\bar{e}}(\Delta)(W^M(\bar{e}) - W^M)]}_{>0} \end{array} \right\} < 0 \\ \text{and } & \frac{\partial}{\partial \Delta} MB(\bar{e}) \\ = & \frac{1}{2} \left\{ \begin{array}{l} \left[g(\hat{\rho}_{\bar{e}}(\Delta)) \cdot \underbrace{\tilde{\rho}'_{\bar{e}}(\Delta)}_{<0} \right] \cdot \underbrace{a'(\Delta)}_{>0} \cdot \underbrace{[W - W^C]}_{>0} + \\ g(\hat{\rho}_{\bar{e}}(\Delta)) \cdot \underbrace{\tilde{\rho}'_{\bar{e}}(\Delta)}_{<0} \cdot \underbrace{[a'(\Delta)(W - W^C) - \tilde{\rho}'_{\bar{e}}(\Delta)(W^M(\bar{e}) - W^M)]}_{>0} \end{array} \right\} < 0. \blacksquare \end{aligned}$$

Proof of Lemma 3.

The marginal welfare gain from increasing the severity of the merger policy from \underline{e} to \bar{e} writes:

$$MB(\bar{e}) - MB(\underline{e}) = \frac{1}{2} \left\{ \begin{array}{l} a'(\Delta) \cdot [W - W^C] \cdot [1 - G(\hat{\rho}_{\underline{e}}(\Delta))] \\ -g(\hat{\rho}_{\underline{e}}(\Delta)) \cdot \tilde{\rho}'_{\underline{e}}(\Delta) \cdot [a(\Delta)W + (1 - a(\Delta))W^C - \hat{\rho}_{\underline{e}}(\Delta)W^M(\underline{e}) - (1 - \hat{\rho}_{\underline{e}}(\Delta))W^M] \end{array} \right\}$$

Taking the first derivative on the marginal benefit differential yields:

$$\begin{aligned} & \frac{\partial}{\partial \Delta} (MB(\bar{e}) - MB(\underline{e})) \\ = & \frac{1}{2} \left\{ \begin{array}{l} \underbrace{a'(\Delta)}_{>0} \cdot [W - W^C] \cdot \underbrace{(-g(\hat{\rho}_{\underline{e}}(\Delta)) \cdot \tilde{\rho}'_{\underline{e}}(\Delta))}_{>0} \\ -g(\hat{\rho}_{\underline{e}}(\Delta)) \cdot \underbrace{\tilde{\rho}'_{\underline{e}}(\Delta)}_{<0} \cdot \underbrace{[a'(\Delta)(W - W^C) - \tilde{\rho}'_{\underline{e}}(\Delta)(W^M(\underline{e}) - W^M)]}_{>0} \end{array} \right\} > 0 \end{aligned}$$

q.e.d. \blacksquare

Proof of Proposition 1.

Define $\tilde{\Delta}$ such that $MB(\bar{e}) - MB(\underline{e}) = 0$. Let then $\tilde{k} = k(\tilde{\Delta})$.

In order to show that $\tilde{\Delta}$ is an interior solution, we evaluate $[MB(\bar{e}) - MB(\underline{e})]$ for $\Delta \rightarrow \bar{\Delta}$ and for $\Delta \rightarrow 0$:

- if $\Delta \rightarrow \bar{\Delta}$ then $a(\Delta) \rightarrow 1$. If $\hat{\rho}_{\underline{e}}(\Delta)$ is close to 0, we have $[MB(\bar{e}) - MB(\underline{e})] > 0$
- if $\Delta \rightarrow 0$ then $a(\Delta) \rightarrow 0$. If $\hat{\rho}_{\underline{e}}(\Delta)$ is close to 1, therefore we have $[MB(\bar{e}) - MB(\underline{e})] < 0$
Given $\frac{\partial}{\partial \Delta} (MB(\bar{e}) - MB(\underline{e})) > 0$ from Lemma 3, one has that $\tilde{\Delta}$ such that $MB(\bar{e}) - MB(\underline{e}) = 0$ belongs to $(0, \bar{\Delta})$. ■

Proof of Proposition 2.

Let $W(e, \Delta^*(e))$ denote the maximum net consumers' welfare function for any merger policy type, $e \in \{\underline{e}, \bar{e}\}$. Explicitly, this yields:

$$\begin{aligned} & W(\underline{e}, \underline{\Delta}^*(\underline{e})) \\ &= \frac{1}{2} \left\{ G(\hat{\rho}_{\underline{e}}(\underline{\Delta}^*)) \cdot [a(\underline{\Delta}^*)W + (1 - a(\underline{\Delta}^*))W^C] + \int_{\hat{\rho}_{\underline{e}}(\underline{\Delta}^*)}^1 [\rho W^M(\underline{e}) + (1 - \rho)W^M] g(\rho) d\rho \right\} \\ &+ \frac{1}{2} \left\{ G(\hat{\rho}_{\bar{e}}(\underline{\Delta}^*)) \cdot [a(\underline{\Delta}^*)W + (1 - a(\underline{\Delta}^*))W^C] + \int_{\hat{\rho}_{\bar{e}}(\underline{\Delta}^*)}^1 [\rho W^M(\bar{e}) + (1 - \rho)W^M] g(\rho) d\rho \right\} - \\ & k \underline{\Delta}^* \\ & \text{and } W(\bar{e}, \bar{\Delta}^*(\bar{e})) = \frac{1}{2} [a(\bar{\Delta}^*)W + (1 - a(\bar{\Delta}^*))W^C] \\ &+ \frac{1}{2} \left\{ G(\hat{\rho}_{\bar{e}}(\bar{\Delta}^*)) \cdot [a(\bar{\Delta}^*)W + (1 - a(\bar{\Delta}^*))W^C] + \int_{\hat{\rho}_{\bar{e}}(\bar{\Delta}^*)}^1 [\rho W^M(\bar{e}) + (1 - \rho)W^M] g(\rho) d\rho \right\} - \\ & k \bar{\Delta}^* \end{aligned}$$

First of all, note that the maximum net consumers' welfare functions are both decreasing with the cost of anti-cartel resources: $\frac{\partial}{\partial k} W(\underline{e}, \underline{\Delta}^*) = -\underline{\Delta}^*$ and $\frac{\partial}{\partial k} W(\bar{e}, \bar{\Delta}^*) = -\bar{\Delta}^*$ thanks to the envelope theorem.

The optimal choice in terms of competition policy is based on the following differential $W(\underline{e}, \underline{\Delta}^*(\underline{e})) - W(\bar{e}, \bar{\Delta}^*(\bar{e}))$.

Next we establish the variation of this maximum welfare differential with the cost of anti-cartel resources: $\frac{\partial}{\partial k} [W(\underline{e}, \underline{\Delta}^*(\underline{e})) - W(\bar{e}, \bar{\Delta}^*(\bar{e}))] = \bar{\Delta}^* - \underline{\Delta}^*$

According to the two regimes defined in Proposition 1, one has that:

- for $k < \tilde{k}$, $\Delta^*(\underline{e}) < \Delta^*(\bar{e})$, thus $\frac{\partial}{\partial k} [W(\underline{e}, \underline{\Delta}^*(\underline{e})) - W(\bar{e}, \bar{\Delta}^*(\bar{e}))] > 0$
- for $k \geq \tilde{k}$, $\Delta^*(\underline{e}) > \Delta^*(\bar{e})$, therefore $\frac{\partial}{\partial k} [W(\underline{e}, \underline{\Delta}^*(\underline{e})) - W(\bar{e}, \bar{\Delta}^*(\bar{e}))] < 0$.

Moreover, we can evaluate the maximum welfare differential in $k = \tilde{k}$, and for $k \rightarrow 0$ and also for k so large that $\Delta^* \rightarrow 0$:

(i) for $k = \tilde{k}$, by definition $MB(\bar{e}) = MB(\underline{e})$, therefore $\Delta^*(\underline{e}) = \Delta^*(\bar{e})$. Consequently,

$$\begin{aligned} & [W(\underline{e}, \underline{\Delta}^*(\underline{e})) - W(\bar{e}, \bar{\Delta}^*(\bar{e}))]_{\Delta^*(\underline{e}) = \Delta^*(\bar{e})} \\ &= \int_{\hat{\rho}_{\underline{e}}(\underline{\Delta}^*)}^1 [\rho W^M(\underline{e}) + (1 - \rho)W^M - a(\bar{\Delta}^*)W - (1 - a(\bar{\Delta}^*))W^C] g(\rho) d\rho > 0 \text{ because for } \tilde{k}, \\ & MB(\bar{e}) - MB(\underline{e}) = 0 \text{ which implies } [\rho W^M(\underline{e}) + (1 - \rho)W^M - a(\bar{\Delta}^*)W - (1 - a(\bar{\Delta}^*))W^C] > 0. \end{aligned}$$

(ii) for k so large that $\Delta^* \rightarrow 0, \forall e$, then $a(\Delta^*) \rightarrow 0$, meaning that under both merger policies, there is actually no fighting against cartels whatsoever. As a result, $W(\underline{e}, \underline{\Delta}^*(\underline{e})) - W(\bar{e}, \bar{\Delta}^*(\bar{e})) > 0$.

(iii) for $k \rightarrow 0$, both $\underline{\Delta}^*(\underline{e})$ and $\bar{\Delta}^*(\bar{e})$ tend to their maximum values, but still $\underline{\Delta}^*(\underline{e}) \leq \bar{\Delta}^*(\bar{e})$ thanks to Proposition 1. Given the monotonicity of $a(\Delta)$ and $\hat{\rho}_e(\Delta)$ with Δ , one has $a(\underline{\Delta}^*) \leq a(\bar{\Delta}^*)$ and $\hat{\rho}_{\bar{e}}(\bar{\Delta}^*) < \hat{\rho}_{\underline{e}}(\underline{\Delta}^*)$. This yields the following when evaluating the maximum welfare differential for or $k \rightarrow 0$:

$$\begin{aligned} & \left[W(\underline{e}, \underline{\Delta}^*(\underline{e})) - W(\bar{e}, \bar{\Delta}^*(\bar{e})) \right]_{k \rightarrow 0} = \\ & \frac{1}{2} \left(\begin{aligned} & G(\hat{\rho}_{\underline{e}}(\underline{\Delta}^*)) \cdot \underbrace{(a(\underline{\Delta}^*) - a(\bar{\Delta}^*))}_{<0} \cdot (W - W^C) \\ & + \int_{\hat{\rho}_{\underline{e}}(\underline{\Delta}^*)}^1 \left[\rho W^M(\underline{e}) + (1 - \rho)W^M - a(\bar{\Delta}^*)W - (1 - a(\bar{\Delta}^*))W^C \right] g(\rho) d\rho \end{aligned} \right) \\ & + \frac{1}{2} \left(\begin{aligned} & (W - W^C) \cdot \underbrace{(a(\underline{\Delta}^*) - a(\bar{\Delta}^*))}_{<0} \cdot \underbrace{(G(\hat{\rho}_{\bar{e}}(\underline{\Delta}^*)) - G(\hat{\rho}_{\bar{e}}(\bar{\Delta}^*)))}_{>0} \\ & + \underbrace{\int_{\hat{\rho}_{\bar{e}}(\underline{\Delta}^*)}^{\hat{\rho}_{\bar{e}}(\bar{\Delta}^*)} \left[\rho W^M(\bar{e}) + (1 - \rho)W^M \right] g(\rho) d\rho}_{<0} \end{aligned} \right) \end{aligned}$$

The only term that needs signing is $\int_{\hat{\rho}_{\underline{e}}(\underline{\Delta}^*)}^1 \left[\rho W^M(\underline{e}) + (1 - \rho)W^M - a(\bar{\Delta}^*)W - (1 - a(\bar{\Delta}^*))W^C \right] g(\rho) d\rho$.

Since $\frac{\partial}{\partial W^M(\underline{e})} \left[\rho W^M(\underline{e}) + (1 - \rho)W^M - a(\bar{\Delta}^*)W - (1 - a(\bar{\Delta}^*))W^C \right] > 0$, for low enough $W^M(\underline{e})$ one has that $\int_{\hat{\rho}_{\underline{e}}(\underline{\Delta}^*)}^1 \left[\rho W^M(\underline{e}) + (1 - \rho)W^M - a(\bar{\Delta}^*)W - (1 - a(\bar{\Delta}^*))W^C \right] g(\rho) d\rho < 0$.

Thus, a low enough $W^M(\underline{e})$ is a sufficient conditions for the existence of $k^* \in (0, \tilde{k})$ such that $W(\underline{e}, \underline{\Delta}^*(\underline{e})) - W(\bar{e}, \bar{\Delta}^*(\bar{e})) = 0$.

The figure below gives the profile of the maximum welfare functions according to the above results. Proposition 2 merely states the obvious result of the comparison between $W(\underline{e}, \underline{\Delta}^*(\underline{e}))$ and $W(\bar{e}, \bar{\Delta}^*(\bar{e}))$ for the different levels of k . ■

Proof of proposition 3. The proof of proposition 3 is similar to the proof of proposition 2 and proceeds in three steps.

First, we show a result similar to lemma 2.

The marginal welfare gain from increasing the severity of the merger policy from \underline{e} to \bar{e} writes:

$$MB(\bar{e}) - MB(\underline{e})$$

$$= \frac{1}{2} \left\{ \begin{array}{l} a'(\Delta) \cdot [W - W^C] \cdot [1 - G(\hat{\rho}_{\underline{e}}(\Delta))] \\ -g(\hat{\rho}_{\underline{e}}(\Delta)) \cdot \hat{\rho}'_{\underline{e}}(\Delta) \\ \cdot (a(\Delta)W + (1 - a(\Delta))W^C) \\ - (\hat{\rho}_{\underline{e}}(\Delta) (a(\Delta)W^M(\underline{e}) + (1 - a(\Delta))W^{MC}(\underline{e}) + (1 - \hat{\rho}_{\underline{e}}(\Delta))(a(\Delta)W^M + (1 - a(\Delta))W^{MC})) \end{array} \right\}$$

Taking the first derivative on the marginal benefit differential yields:

$$\begin{aligned} & \frac{\partial}{\partial \Delta} (MB(\bar{e}) - MB(\underline{e})) \\ = & \frac{1}{2} \left\{ \begin{array}{l} \underbrace{a'(\Delta) \cdot [W - W^C]}_{>0} \cdot \underbrace{(-g(\hat{\rho}_{\underline{e}}(\Delta)) \cdot \hat{\rho}'_{\underline{e}}(\Delta))}_{>0} \\ -g(\hat{\rho}_{\underline{e}}(\Delta)) \cdot \underbrace{\hat{\rho}'_{\underline{e}}(\Delta)}_{<0} \cdot \\ \underbrace{[a'(\Delta)(W - W^C) - \hat{\rho}'_{\underline{e}}(\Delta) (a(\Delta)W^M(\underline{e}) + (1 - a(\Delta))W^{MC}(\underline{e}) - (a(\Delta)W^M + (1 - a(\Delta))W^{MC}))]}_{>0} \\ [+a'(\Delta) (\hat{\rho}_{\underline{e}}(\Delta) W^M(\underline{e}) + (1 - \hat{\rho}_{\underline{e}}(\Delta))W^M - \hat{\rho}_{\underline{e}}(\Delta) W^{MC}(\underline{e}) - (1 - \hat{\rho}_{\underline{e}}(\Delta))W^{MC})] \end{array} \right\} \\ > & 0 \end{aligned}$$

Second, there is a threshold of k similar to the one of proposition 1.

Indeed,

- if $\Delta \rightarrow \bar{\Delta}$ then $a(\Delta) \rightarrow 1$ so that there is no cartel at all which leaves as before $[MB(\bar{e}) - MB(\underline{e})]_{\Delta \rightarrow \bar{\Delta}} > 0$

- if $\Delta \rightarrow 0$ then $a(\Delta) \rightarrow 0$, therefore $[MB(\bar{e}) - MB(\underline{e})]_{\Delta \rightarrow 0} =$

$-g(\hat{\rho}_{\underline{e}}(\Delta)) \cdot \hat{\rho}_{\underline{e}}(\Delta) (a(\Delta)W^M + (1 - a(\Delta))W^{MC} - a(\Delta)W^M(\underline{e}) - (1 - a(\Delta))W^{MC}(\underline{e})) < 0$

Third, in order to use the proof of proposition 2, it is sufficient to consider the consumer surplus after merger without efficiency gains as $a(\Delta)W^M + (1 - a(\Delta))W^{MC}$ instead of W^M and in the same way the consumer surplus after merger with efficiency gains as $a(\Delta)W^M(\underline{e}) + (1 - a(\Delta))W^{MC}(\underline{e})$ instead of $W^M(\underline{e})$.

Then all the results are preserved. The final sufficient condition amounts to a level of $a(\underline{\Delta}^*)W^M(\underline{e}) + (1 - a(\underline{\Delta}^*))W^{MC}(\underline{e})$ low enough. ■