

# Currency Misalignments and Optimal Monetary Policy: A Reexamination

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## Abstract

This paper examines optimal monetary policy in an open-economy two-country world with sticky prices under pricing to market. We show that currency misalignments are inefficient and lower world welfare. We find that optimal policy must target consumer price inflation, the output gap, and the currency misalignment. The paper derives the loss function of a cooperative monetary policymaker and the optimal targeting rules. The model is a modified version of Clarida, Gali, and Gertler's (JME, 2002). The key change is that we allow pricing to market or local-currency pricing and consider the policy implications of currency misalignments. Besides highlighting the importance of the currency misalignment, our model also gives a rationale for targeting CPI, rather than PPI, inflation.

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Exchange rates among large economies have fluctuated dramatically over the past 30 years. The dollar/euro exchange rate has experienced swings of greater than 60%, and even the Canadian dollar/U.S. dollar has risen and fallen by more than 35% in the past decade, but inflation rates in these countries have differed by only a percentage point or two per year. Should these exchange rate movements be a concern for policymakers? Or would it not be better for policymakers to focus on output and inflation and let a freely floating exchange rate settle at a market determined level?

Empirical evidence points to the possibility of “local-currency pricing” (LCP) or “pricing to market”.<sup>1</sup> Exporting firms may price discriminate among markets, and/or set prices in the buyers’ currencies. A currency could be overvalued if consumer prices are generally higher at home than abroad when compared in a common currency, or undervalued if these prices are lower at home.<sup>2</sup> Currency misalignments can be very large even in advanced economies.

In a simple, familiar framework, this paper draws out the implications for monetary policy when currency misalignments are possible. Currency misalignments lead to inefficient allocations for reasons that are analogous to the problems with inflation in a world of staggered price setting. When there are currency misalignments, households in the Home and Foreign countries may pay different prices for the identical good. A basic tenet of economics is that violations of the law of one price are inefficient – if the good’s marginal cost is the same irrespective of where the good is sold, it is not efficient for the good to sell at different prices. We find that these violations lead to a reduction in world welfare and that optimal monetary policy trades off targeting these misalignments with inflation and output goals. In our model, because there are no transportation costs or distribution costs, any deviation from the law of one price would be inefficient. More generally, if those costs were to be included, then pricing would be inefficient if consumer prices in different locations do not reflect the underlying resource costs of producing and distributing the goods.

These misalignments arise even when foreign exchange markets are efficient. The currency misalignment distortion that concerns policymakers arises in the goods market – from price setting – and not in the foreign exchange market. The model of this paper determines the foreign exchange rate in an efficient currency market as a function of fundamental economic variables.

Clarida, Gali, and Gertler (2002), Gali and Monacelli (2005), and others have emphasized the important role of exchange rate adjustment in a model that assumes firms set prices in their own currency (PCP, for “producer-currency pricing”). In the PCP framework, a change in the nominal exchange rate automatically translates into a change in the price of imported goods relative to local goods, and so plays

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<sup>1</sup> Many studies have found evidence of violations of the law of one price for consumer prices. Two prominent studies are Engel (1999) and Atkeson and Burstein (2008). The literature is voluminous – these two papers contain many relevant citations.

<sup>2</sup> A precise definition of our use of the term “currency misalignment” appears in section 3 below.

an important role in achieving nearly efficient outcomes. Why, then, is it optimal to target currency misalignments when there is pricing to market? In this setting, the exchange rate does not play the role of automatically adjusting relative prices facing households because imported goods prices are set by the producer and do not respond automatically to exchange rate changes. However, if prices are sticky in the importer's currency, the change in the exchange rate can lead to inefficient movements in the price of the same good sold in different countries. Hence there is a need to target currency misalignments.<sup>3</sup>

When firms price to market, policymakers should target consumer price inflation, rather than producer price inflation in the international setting. Inflation is inefficient under staggered price setting because it leads to distorted relative prices. If consumers face relative prices that do not reflect underlying relative costs, then demand for the products and hence resources used to produce the products will be distorted. In models in which the law of one price holds, the relative prices of goods produced in one country are identical in both countries. The relative price distortions are identical, and eliminating producer price inflation eliminates all relative price distortions. In fact, it may be undesirable in this setting to tame consumer price inflation. The local price inflation of imported goods is influenced by changes in the exchange rate. Those changes can lead to efficient changes in the relative price of imports to locally-produced goods. But when there is pricing to market or LCP, the relative prices of goods produced in one country will not be the same in both countries. Policymakers must target the consumer price inflation of those goods in both countries to eliminate the relative price distortions.

We examine a two-country model, and proceed in two steps. After setting out the objectives of households and firms, the production functions, and the market structure, we derive a global loss function for cooperative monetary policymakers. We can derive the loss function without making any assumptions about how goods prices or wages are set.<sup>4</sup> We find that in addition to Home and Foreign output gaps, and the cross-sectional dispersion of consumer goods prices within each country, the loss also depends on the currency misalignment. This loss function evaluates the welfare costs arising because firms set different prices in the Home and Foreign country (assuming the costs of selling the good in both countries are identical), and does not depend on whether the price differences arise from local-currency price stickiness, from price discrimination, or for some other reason.

We then assume a Calvo mechanism for price setting, but allowing for the possibility of local-currency pricing. We derive optimal policy under commitment and discretion. We consider only optimal cooperative policy. Our goal is to appraise the global loss from currency misalignments, which we can

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<sup>3</sup> This does not, however, imply that fixed exchange rates are optimal.

<sup>4</sup> Except that we do assume that all households (which are identical) set the same wage. As we note later, this rules out a model of staggered wage setting such as in Erceg, Henderson, and Levin (2000).

see by deriving the loss function for a policymaker that aims to maximize the sum of utilities of Home and Foreign households.

## 1. Related Literature

To understand the contribution of this paper, it is helpful to place it relative to two sets of papers. First, Clarida, Gali, and Gertler (2002) develop the canonical model for open-economy monetary policy analysis in the New Keynesian framework. Their two-country model assumes PCP, and that Home and Foreign households have identical preferences. These two assumptions lead to the conclusion that purchasing power parity holds at all times – the consumption real exchange rate is constant.<sup>5</sup>

This paper introduces local-currency pricing into CGG's model. We derive simple rules for monetary policy that are similar to CGG's. This paper also allows Home and Foreign households to have different preferences. They can exhibit a home bias in preferences – a larger weight on goods produced in a household's country of residence.<sup>6</sup> This generalization does not change the optimal target criteria at all in the CGG framework, but as we now explain, is helpful in developing a realistic LCP model.

The second set of papers includes Devereux and Engel (2003), which explicitly examines optimal monetary policy in a two-country framework with LCP. Corsetti and Pesenti (2005) extend the analysis in several directions. However, neither of these studies can answer the question posed above: is currency misalignment a separate concern of monetary policy, or will the optimal exchange-rate behavior be achieved through a policy that considers inflation and the output gap?

These models make several assumptions that render them unsuited to answering this question. First, inflation per se has no welfare cost in these models. Prices are sticky, but all are set one period in advance, so inflation does not lead to any price dispersion within countries, as it does under staggered price setting.

Second, like CGG, they assume identical preferences in both countries. Identical preferences imply that the optimal policy should fix the nominal exchange rate. But when there is home bias in preferences, for example, policymakers need to trade off the costs of currency misalignments with the objectives of stimulating output differentially across countries in response to productivity shocks.<sup>7</sup>

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<sup>5</sup> Benigno and Benigno (2003, 2006) are important contributions that use models similar to Clarida, Gali, and Gertler's but consider optimal policy when the optimal subsidies to deal with monopoly distortions are not present in steady state.

<sup>6</sup> de Paoli (2009) allows for home bias in preferences in a small open economy model. There is home bias in the sense that while the country is small, the limit of the ratio of expenditure share on home goods to population share is not equal to one. Faia and Monacelli (2008) examine optimal monetary policy in a small open economy model with home bias, using a Ramsey style analysis. Pappa (2004) considers a two-country model with home bias. However, the second-order approximation to the welfare function is expressed in terms of deviations of consumption from its efficient level, rather than in terms of the output gap, so the analysis is not strictly comparable to ours.

<sup>7</sup> See Duarte and Obstfeld (2008), who emphasize this point.

Third, price stickiness is the only distortion in the economy in these papers. In Devereux and Engel (2003), the optimal monetary policy under LCP simultaneously eliminates the currency misalignment and sets world aggregate demand at the efficient level. The paper is not clear on whether currency misalignment and the world output gap are separate objectives for policymakers, or whether the loss from currency misalignment occurs purely because it leads to an inefficient level of world output. This paper shows that they are separate concerns – even if output gaps in both countries were somehow eliminated, consumption is misallocated if there are currency misalignments.<sup>8</sup>

This paper derives optimal policy in a framework that is consonant with the bulk of New Keynesian models of monetary policy analysis.<sup>9</sup> Here we adopt the standard Calvo price-setting technology, which allows for asynchronized price setting. Staggering of prices leads to inefficient internal relative prices – an important cost from inflation that is missing from models in which prices are all set simultaneously. Also, the previous papers assumed that the money supply was the instrument of monetary policy. This paper follows CGG and most of the modern literature in assuming that the policymakers directly control the nominal interest rate in each country.<sup>10</sup>

Because of these modifications to the previous literature with LCP we can first explicitly show that optimal policy involves a tradeoff among inflation, output gap and currency misalignment objectives, and we can demonstrate that CPI inflation is the relevant inflation target for policymakers.

There are many papers that numerically solve rich open economy models, and examine optimal policy. Some of these papers allow for local currency pricing, but are in the framework of a small open economy, and so do not specifically account for the global misallocation of resources that occurs with currency misalignments.<sup>11</sup> Moreover, many use ad hoc welfare criteria for the policymaker or approximations that are not strictly derived from household welfare.<sup>12</sup> One of the main contributions of this paper is to derive the role of the currency misalignment in the policymaker's loss function.

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<sup>8</sup> The contribution of Sutherland (2005) merits attention. His two-country model allows for imperfect pass-through, and for differences in Home and Foreign preferences. His model is static, and he derives a welfare function in which the variance of the exchange rate appears. However, the other terms in the welfare function are prices, so it is not clear how this function relates to standard quadratic approximations that involve output gaps and inflation levels. Moreover, Sutherland does not derive optimal monetary policy in his framework.

<sup>9</sup> A sophisticated extension of the earlier work is the recent paper by Corsetti, Dedola, and Leduc (2010). That paper extends earlier work in several dimensions, including staggered price setting. But it does not directly address the issue of whether currency misalignments belong in the targeting rule along with output gaps and inflation.

<sup>10</sup> While the model of this paper adheres strictly to the set-up of CGG, changing only the assumptions of identical preferences and LCP instead of PCP price setting, the model is very similar to that of Benigno's (2004). Woodford (2010) also considers the LCP version of CGG (though not for optimal monetary policy analysis) and makes the connection to Benigno's paper. Monacelli (2005) considers a small-open economy model with local-currency pricing, and examines optimal monetary policy using an ad hoc welfare criterion.

<sup>11</sup> See, for example, Kollmann (2002), Smets and Wouters (2002), Ambler, Dib, and Rebei (2004), and Adolfson, et. al. (2008). See also Leith and Wren-Lewis (2006) who examine a small-open economy model with non-traded goods (but with PCP for export pricing.)

<sup>12</sup> For example, Smets and Wouters (2002), Ambler, Dib, and Rebei (2004), and Adolfson, et. al. (2008).

Some papers have considered whether it is beneficial to augment the interest rate reaction function of central banks with an exchange-rate variable.<sup>13</sup> But our focus here is on the tradeoffs faced by policymakers in setting policy. In the terminology of Svensson (1999, 2002), we place emphasis on the “targeting rule” rather than the “instrument rule”. There are many instrument rules that could potentially support the optimal targeting rule, though the precise form that any instrument rule takes depends on the stochastic process followed by the exogenous shocks (while the form of the targeting rule does not.)<sup>14</sup>

## 2. The Model

The model is nearly identical to CGG’s. There are two countries of equal size, while CGG allow the population of the countries to be different. Since the population size plays no real role in their analysis, we simplify along this dimension. But we make two significant generalizations. First, we allow for different preferences in the two countries. Home agents may put a higher weight in utility on goods produced in the Home country. Home households put a weight of  $\frac{\nu}{2}$  on Home goods and  $1 - \frac{\nu}{2}$  on Foreign goods (and vice-versa for Foreign households.) This is a popular assumption in the open-economy macroeconomics literature, and can be considered as a short-cut way of modeling “openness”. A less open country puts less weight on consumption of imported goods, and in the limit the economy becomes closed if it imports no goods. The second major change is to allow for goods to be sold at different prices in the Home and Foreign countries.<sup>15</sup>

The model assumes two countries, each inhabited with a continuum of households, normalized to a total of one in each country. Households have utility over consumption of goods and disutility from provision of labor services. In each country, there is a continuum of goods produced, each by a monopolist. Households supply labor to firms located within their own country, and get utility from all goods produced in both countries. Each household is a monopolistic supplier of a unique type of labor to firms within its country. We assume that there is trade in a complete set of nominally-denominated contingent claims

Monopolistic firms produce output using only labor, subject to technology shocks.

At this stage, we will not make any assumptions on how monopolistic households set wages or monopolistic firms set prices. In particular, prices and wages may be sticky, and there may be LCP or

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<sup>13</sup> In a small open economy, see Kollmann (2002) and Leitimo and Soderstrom (2005). In a two-country model, see Wang (2009).

<sup>14</sup> Coenen, Lombardo, Smets, and Straub (2010) examine optimal monetary policy in a two-country model that exhibits incomplete pass-through. However, the numerical analysis does not allow the reader to see explicitly the role of currency misalignments.

<sup>15</sup> One other minor change is merely a matter of labeling. In the presentation of the model here, households have utility over a continuum of goods, which can be represented by aggregate consumption. In CGG, households have utility over a single good which is costlessly assembled from a continuum of intermediate goods.

PCP for firms. We derive the loss function for the policymaker, which expresses the loss (relative to the efficient outcome) in terms of within-country and international price misalignments and output gaps. This loss function applies under various assumptions about how prices are actually set, and so is more general than the policy rules we subsequently derive which depend on the specifics of price and wage setting.

All households within a country are identical. In each period, their labor supplies are identical. This assumption rules out staggered wage setting as in Erceg, Henderson, and Levin (2000), because in that model there will be dispersion in labor input across households that arises from the dispersion in wages set. Our set-up is consistent with sticky wages, but not wage dispersion. However, it is straightforward to generalize the loss functions we derive to allow for wage dispersion following the steps in Erceg, Levin, and Henderson. We do not do that because we want our model to be directly comparable to CGG.

Most equations of the model are relegated to an Appendix available on the author's Web page. Because the model is almost identical to CGG's, including the notation, the reader can also refer to that paper for details.

## 2.a. Households

The representative household in the Home country maximizes

$$(1) \quad U_t(h) = E_t \left\{ \sum_{j=0}^{\infty} \beta^j \left[ \frac{1}{1-\sigma} C_{t+j}(h)^{1-\sigma} - \frac{1}{1+\phi} N_{t+j}(h)^{1+\phi} \right] \right\}, \quad \sigma > 0, \phi \geq 0$$

$C_t(h)$  is the consumption aggregate. Household  $h$  has Cobb-Douglas preferences defined over an aggregate of Home-produced and Foreign-produced goods,  $C_{Ht}(h)$  and  $C_{Ft}(h)$ . Home households put a weight of  $\frac{\nu}{2}$  on Home goods and  $\frac{2-\nu}{2}$  on Foreign goods, with  $0 \leq \nu \leq 2$ . When  $\nu > 1$ , Home households exhibit home-bias in their preferences. Foreign households' preferences are symmetric, with weight  $\frac{\nu}{2}$  on Foreign goods.

In turn,  $C_{Ht}(h)$  and  $C_{Ft}(h)$  are CES aggregates over a continuum of goods produced in each country, with elasticity of substitution among Home and Foreign varieties both equal to  $\xi$ . Foreign households have the same elasticity of substitution among these varieties.

$N_t(h)$  is an aggregate of the labor services that the Home household sells to each of a continuum of firms located in the Home country. Households receive wage income,  $W_t(h)N_t(h)$ , and a share of aggregate profits from Home firms. They pay lump-sum taxes each period. Each household can trade in a complete market in contingent claims denominated (arbitrarily) in the home currency.

Foreign households have analogous preferences and face an analogous budget constraint.

## 2.b. Firms

Each Home good,  $Y_t(f)$  is made according to a production function that is linear in the labor input:

$$Y_t(f) = A_t N_t(f).$$

The productivity shock,  $A_t$ , is common to all firms in the Home country.  $N_t(f)$  is a CES composite of individual home-country household labor. The elasticity of substitution among varieties of Home labor,  $\eta_t$ , is stochastic and common to all Home firms.

Profits are given by:

$$\Gamma_t(f) = P_{Ht}(f)C_{Ht}(f) + E_t P_{Ht}^*(f)C_{Ht}^*(f) - (1 - \tau_t)W_t N_t(f).$$

In this equation,  $P_{Ht}(f)$  is the Home-currency price of the good when it is sold in the Home country.  $P_{Ht}^*(f)$  is the Foreign-currency price of the good when it is sold in the Foreign country.  $\tau_t$  is a subsidy to the Home firm from the Home government.  $C_{Ht}(f)$  represents aggregate sales of the Home good in the Home country and  $C_{Ht}^*(f)$  are aggregate sales of the Home good in the Foreign country. It follows that  $Y_t(f) = C_{Ht}(f) + C_{Ht}^*(f)$ .

There are analogous equations for  $Y_t^*(f)$ , with the foreign productivity shock given by  $A_t^*$ , the foreign technology parameter shock given by  $\eta_t^*$ , and foreign subsidy given by  $\tau_t^*$ .

## 2.c Equilibrium

Goods market clearing conditions in the Home and Foreign country are given by:

$$(2) \quad Y_t = C_{Ht} + C_{Ht}^* = \frac{\nu}{2} \frac{P_t C_t}{P_{Ht}} + \left(1 - \frac{\nu}{2}\right) \frac{P_t^* C_t^*}{P_{Ht}^*} = k^{-1} \left( \frac{\nu}{2} S_t^{1-(\nu/2)} C_t + \left(1 - \frac{\nu}{2}\right) (S_t^*)^{-\nu/2} C_t^* \right),$$

$$(3) \quad Y_t^* = C_{Ft} + C_{Ft}^* = \left(1 - \frac{\nu}{2}\right) \frac{P_t C_t}{P_{Ft}} + \frac{\nu}{2} \frac{P_t^* C_t^*}{P_{Ft}^*} = k^{-1} \left( \frac{\nu}{2} (S_t^*)^{1-(\nu/2)} C_t^* + \left(1 - \frac{\nu}{2}\right) S_t^{-\nu/2} C_t \right).$$

In this equation,  $P_t$  is the exact price index for consumption, given by:

$$P_t = k^{-1} P_{Ht}^{\nu/2} P_{Ft}^{1-(\nu/2)}, \quad k = (1 - (\nu/2))^{1-(\nu/2)} (\nu/2)^{\nu/2}.$$

The Foreign consumer price index,  $P_t^*$  is defined analogously.  $P_{Ht}$  is the Home-currency price of the Home aggregate good and  $P_{Ft}$  is the Home currency price of the Foreign aggregate good when purchased in the Home country.  $P_{Ht}$  and  $P_{Ft}$  are the usual CES aggregates over prices of individual varieties.  $P_{Ht}^*$  and  $P_{Ft}^*$  are the Foreign-currency prices of these aggregates in the Foreign country. We have used  $S_t$  and

$S_t^*$  to represent the price of imported to locally-produced goods in the Home and Foreign countries, respectively:

$$S_t = P_{Ft} / P_{Ht}, \quad S_t^* = P_{Ht}^* / P_{Ft}^*.$$

Equations (2) and (3) make use of the consumer demand functions. Each Home household spends a constant share  $\frac{\nu}{2}$  on Home goods and  $\frac{2-\nu}{2}$  on Foreign goods, for example.

We also have the familiar condition that arises in open-economy models with a complete set of state-contingent claims when PPP does not hold:

$$(4) \quad \left( \frac{C_t}{C_t^*} \right)^\sigma = \frac{E_t P_t^*}{P_t} = \frac{E_t P_{Ht}^*}{P_{Ht}} (S_t^*)^{-\nu/2} S_t^{(\nu/2)-1}.$$

Total employment is determined by output in each industry in the Home country:

$$(5) \quad N_t = \int_0^1 N_t(f) df = A_t^{-1} \int_0^1 Y_t(f) df = A_t^{-1} (C_{Ht} V_{Ht} + C_{Ht}^* V_{Ht}^*).$$

where

$$V_{Ht} \equiv \int_0^1 \left( \frac{P_{Ht}(f)}{P_{Ht}} \right)^{-\xi} df, \quad \text{and} \quad V_{Ht}^* \equiv \int_0^1 \left( \frac{P_{Ht}^*(f)}{P_{Ht}^*} \right)^{-\xi} df.$$

Again, analogous equations hold for the Foreign country.

### 3. Log-linearized Model

In this section, we present some log-linear approximations to the model presented above. The full set of log-linearized equations appears in the Appendix. Our approach to the optimal policy decision is to consider a second-order approximation of the welfare function around the efficient non-stochastic steady state. The derivation of the loss function requires a second-order approximation of the utility function, but in the course of the derivation will actually require second-order approximations to some of the equations of the model. However, for many purposes, the first-order approximations are useful: the constraints in the optimization problem need only be approximated to the first order; the optimality conditions for monetary policy – the “target criteria” – are linear; and, we can analyze the dynamics under the optimal policy in the linearized model.

In our notation, lower case letters refer to the deviation of the log of the corresponding upper case from steady state.

We define the “currency misalignment” as:

$$(6) \quad m_t \equiv \frac{1}{2} (e_t + p_{Ht}^* - p_{Ht} + e_t + p_{Ft}^* - p_{Ft}).$$

$m_t$  is the average deviation of consumer prices in the Foreign country from consumer prices in the Home country. It is not the difference in consumer purchasing powers, which is the real exchange rate. Because of the symmetry in the model, the weights that each price deviation receives are the same, equal to one half. More generally, we could define the currency misalignment as a weighted average of price deviations, where the weights are the average of the Home and Foreign CPI weights.

If the price deviations for the two goods are not equal ( $e_t + p_{Ht}^* - p_{Ht} \neq e_t + p_{Ft}^* - p_{Ft}$ ), relative prices will be different in the Home and Foreign consumer markets ( $s_t \neq -s_t^*$ ). It is useful to use  $z_t$  to denote these relative price differences:

$$z_t \equiv \frac{1}{2}(p_{Ft} - p_{Ht} - (p_{Ft}^* - p_{Ht}^*)).$$

The market-clearing conditions, (2) and (3) are approximated as:

$$(7) \quad y_t = \frac{\nu}{2}c_t + \frac{2-\nu}{2}c_t^* + \frac{\nu}{2}\left(\frac{2-\nu}{2}\right)s_t - \frac{\nu}{2}\left(\frac{2-\nu}{2}\right)s_t^*,$$

$$(8) \quad y_t^* = \frac{\nu}{2}c_t^* + \frac{2-\nu}{2}c_t - \frac{\nu}{2}\left(\frac{2-\nu}{2}\right)s_t + \frac{\nu}{2}\left(\frac{2-\nu}{2}\right)s_t^*.$$

The condition arising from complete markets that equates the marginal utility of nominal wealth for Home and Foreign households, equation (4), is given by:

$$(9) \quad \sigma c_t - \sigma c_t^* = m_t + \frac{\nu-1}{2}s_t - \left(\frac{\nu-1}{2}\right)s_t^*.$$

We define “relative” and “world” values for any variables  $\zeta_t$  and  $\zeta_t^*$  as:  $\zeta_t^R \equiv \frac{1}{2}(\zeta_t - \zeta_t^*)$ , and  $\zeta_t^W \equiv \frac{1}{2}(\zeta_t + \zeta_t^*)$ . We can use equations (7)-(9) to express  $c_t$ ,  $c_t^*$ ,  $s_t$ , and  $s_t^*$  in terms of  $y_t$  and  $y_t^*$  and the price deviations,  $m_t$  and  $z_t$ .

$$(10) \quad c_t^R = \frac{\nu-1}{D}y_t^R + \frac{\nu(2-\nu)}{2D}m_t$$

$$(11) \quad c_t^W = y_t^W.$$

$$(12) \quad s_t = \frac{2\sigma}{D}y_t^R + z_t - \frac{(\nu-1)}{D}m_t,$$

$$(13) \quad s_t^* = -\frac{2\sigma}{D}y_t^R + z_t + \frac{(\nu-1)}{D}m_t.$$

where  $D \equiv \sigma\nu(2-\nu) + (\nu-1)^2$ .

The labor market clearing conditions given by (5) can be approximated as  $n_t = y_t - a_t$ , and  $n_t^* = y_t^* - a_t^*$ .

#### 4. Loss Functions and Optimal Policy

We derive the loss function for the cooperative monetary policy problem. The loss function is derived from a second-order approximation to households' utility functions.

The policymaker wishes to minimize

$$E_t \sum_{j=0}^{\infty} \beta^j X_{t+j}.$$

This loss function is derived from household's utility, given in Equation (1). The period loss,  $X_{t+j}$ , represents the difference between the maximum utility achievable under efficient allocations and the utility of the market-determined levels of consumption and leisure.

We derive the loss function for the cooperative policymaker which is the relevant criterion for evaluating world welfare. We aim to highlight the global inefficiency that arises from currency misalignments. We set aside the difficult issues involved with deriving the loss function for a non-cooperative policymaker and defining a non-cooperative policy game.

The Appendix shows the steps for deriving the loss function when there are no currency misalignments, but with home bias in preferences. We use a superscript  $\sim$  to denote the deviation of a variable from the value it would take under an efficient allocation given the sequence of exogenous shocks that have hit the economy. For example,  $\tilde{y}_t$  is the Home output gap (in logs). It is worth pointing out one aspect of the derivation. In closed economy models with no investment or government, consumption equals output. That is an exact relationship, and therefore the deviation of consumption from the efficient level equals the deviation of output from the efficient level to any order of approximation:  $\tilde{c}_t = \tilde{y}_t$ . In the open economy, the relationship is not as simple. When preferences of Home and Foreign agents are identical, and markets are complete, then the consumption aggregates in Home and Foreign are always equal (up to a constant of proportionality equal to relative wealth.) But that is not true when preferences are not the same. Equation (4) shows that we do not have  $C_t = C_t^*$  under complete markets, even if the law of one price holds for both goods. Because of this, we do not have  $\tilde{c}_t + \tilde{c}_t^*$  equal to  $\tilde{y}_t + \tilde{y}_t^*$ , except to a first-order approximation. Since we are using a second-order approximation of the utility function, we need to account for the effect of different preferences (or the effects of the terms of trade) in translating consumption gaps into output gaps.

We find when the law of one price holds for all goods:

$$(14) \quad X_t = \left[ \frac{\sigma}{D} + \phi \right] (\tilde{y}_t^R)^2 + (\sigma + \phi) (\tilde{y}_t^W)^2 + \frac{\xi}{2} (\sigma_{p_{Ht}}^2 + \sigma_{p_{F^*t}}^2).$$

The terms  $\sigma_{p_{Ht}}^2$  and  $\sigma_{p_{F^*t}}^2$  represent the cross-sectional variance of prices of Home goods and Foreign goods, respectively. (Recall  $D \equiv \sigma\nu(2-\nu) + (\nu-1)^2$ .)

The Appendix also shows the derivation of the loss function in the more general case in which currency misalignments are possible. As is standard in this class of models, price dispersion leads to inefficient use of labor. To a second-order approximation, this loss depends on the cross-section variances of  $p_{Ht}$ ,  $p_{Ht}^*$ ,  $p_{Ft}$ , and  $p_{Ft}^*$ .

The loss function is given by:

$$(15) \quad X_t = \left[ \frac{\sigma}{D} + \phi \right] (\tilde{y}_t^R)^2 + (\sigma + \phi) (\tilde{y}_t^W)^2 + \frac{\nu(2-\nu)}{4D} m_t^2 + \frac{\nu(2-\nu)}{4} z_t^2 \\ + \frac{\xi}{2} \left[ \frac{\nu}{2} \sigma_{p_{Ht}}^2 + \frac{2-\nu}{2} \sigma_{p_{H^*t}}^2 + \frac{\nu}{2} \sigma_{p_{Ft}}^2 + \frac{2-\nu}{2} \sigma_{p_{F^*t}}^2 \right].$$

This loss function, as well as the previous one (equation (14)), does not depend on how prices are set – indeed whether prices are sticky or not. The loss function here generalizes (14) to the case in which there are deviations from the law of one price, so that  $m_t \neq 0$  and  $z_t \neq 0$ . This can be seen by directly comparing (14) to (15). In (15),  $\sigma_{p_{Ht}}^2$  is the cross-sectional variance of Home goods prices in the Home country,  $\sigma_{p_{H^*t}}^2$  is the cross-sectional variance of Home goods prices in the Foreign country, etc. If the law of one price holds,  $p_{Ht}(f) = p_{Ht}^*(f) + e_t$  and  $p_{Ft}(f) = p_{Ft}^*(f) + e_t$  for each firm  $f$ . In that case,  $\sigma_{p_{Ht}}^2 = \sigma_{p_{H^*t}}^2$  and  $\sigma_{p_{Ft}}^2 = \sigma_{p_{F^*t}}^2$  because the exchange rate does not affect the cross-sectional variance of prices. If we have  $m_t = z_t = 0$ ,  $\sigma_{p_{Ht}}^2 = \sigma_{p_{H^*t}}^2$ , and  $\sigma_{p_{Ft}}^2 = \sigma_{p_{F^*t}}^2$ , then (15) reduces to (14).<sup>16</sup>

The loss is found to depend on both the square of the average currency misalignment,  $m_t^2$  and the squared difference in relative prices,  $z_t^2$ . Both of these terms arise because of deviations from equal prices for the same goods across countries.  $m_t$  is the average overpricing of consumer goods in the Foreign country relative to the Home. Even if the currency misalignment is zero ( $m_t = 0$ ), so the average of prices is the same in Home and Foreign, there can be a difference in relative prices ( $z_t \neq 0$ ) which also leads to a loss in welfare. In the local-currency pricing model we examine below,  $z_t = 0$  to a first-order approximation. Note that  $z_t$  can be written as the average deviation of consumer prices of exported

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<sup>16</sup> The loss function of equation (15) is also a more general case of the one in Devereux-Engel. Theirs would be the special case in which all the cross-sectional price dispersion terms are zero,  $z$  is zero,  $\nu = 1$ ,  $D = \sigma$ , and  $\phi = 0$ .

goods relative to locally-produced goods:  $z_t \equiv \frac{1}{2}(p_{Ft} + p_{Ht}^* - (p_{Ft}^* + p_{Ht}))$ . In an economy in which price-discriminating exporters charge different prices abroad than at home, there may be a distortion arising from  $z_t \neq 0$  even if there is no currency misalignment.

Why do these terms appear in the loss function? If both Home and Foreign output gaps are zero, and all inflation rates are zero, what problem do price deviations cause? From equations (12) and (13), if  $m_t \neq 0$  or  $z_t \neq 0$ , then internal relative prices ( $s_t$  and  $s_t^*$ ) must also differ from their efficient level if the output gap is zero. For example, suppose  $m_t > 0$  and  $z_t = 0$ , which from (12) and (13) implies we must have  $\tilde{s}_t < 0$  and  $\tilde{s}_t^* > 0$  if both output gaps are eliminated. On the one hand,  $m_t > 0$  tends to lead to overall consumption in Home to be high relative to Foreign consumption (equation (10).) That occurs because financial markets pay off to Home residents when their currency is weak. But Home residents have a home bias for Home goods. That would lead to overproduction in the Home country, were it not for relative price adjustments – which is why  $\tilde{s}_t < 0$ . If output gaps and inflation distortions were eliminated, there would still be a misallocation of consumption between Home and Foreign unless  $m_t = 0$  and  $z_t = 0$ .

The loss function in general depends on the dispersion of consumer prices. All Home firms, for example, have identical production functions, pay the same wage to all types of labor, and their output enters utility functions symmetrically with all other Home goods. If prices are to achieve an efficient allocation, the firms should charge the same price to all households. If consumer prices for these goods differ, then relatively more resources will be devoted toward goods with lower prices, which is a wasteful allocation of resources. Under the PCP model, only the variance of producer prices appears in the loss function, but that is due to the special assumption that the law of one price holds for all goods. In that case, the dispersion of prices of different types of Home goods, for example, will be the same in the Home and Foreign countries. While it is the prices faced by consumers that matters, the PCP model implies that consumer price dispersion in both countries can be eliminated if producer price dispersion of Home and Foreign goods is eliminated. Under LCP, there is no such relationship – the cross-sectional variance of each type of good can be different in the two countries, so each variance appears in the loss function.

We highlight the fact that the loss functions are derived without specific assumptions about price setting not to give a false patina of generality to the result, but to emphasize that the loss in welfare arises not specifically from price stickiness but from prices that do not deliver the efficient allocations. Of course it is our specific assumptions of nominal price and wage setting that give rise to the internal and external price misalignments in this model, and indeed monetary policy would be ineffective if there were

no nominal price or wage stickiness. But one could imagine especially a number of mechanisms that give rise to deviations from the law of one price, because the literature has produced a number of models based both on nominal stickiness and real factors. In the next section, we modify the CGG model in the simplest way – allowing local-currency pricing instead of producer-currency pricing – to examine further the implications of currency misalignments.

## 5. Price and Wage Setting

We follow CGG in assuming wages are set flexibly by monopolistic suppliers of labor, but goods prices are sticky. Wages adjust continuously, but households exploit their monopoly power by setting a wage that incorporates a mark-up over their utility cost of work.

Government is assumed to have only limited fiscal instruments. The government can set a constant output subsidy rate for monopolistic firms, which will achieve an efficient allocation in the non-stochastic steady state. But unfortunately, the mark-up charged by workers is time-varying because the elasticity of demand for their labor services is assumed to follow a stochastic process. These shocks are sometimes labeled “cost-push” shocks, and give rise to the well-known tradeoff in CGG’s work between controlling inflation and achieving a zero output gap.

Households are monopolistic suppliers of their unique form of labor services. Household  $h$  faces an elastic demand for its labor services, with the elasticity of demand given by  $\eta_t$ , the elasticity of substitution among different types of labor in the firm’s production function.

Using the first-order condition for household  $h$ ’s choice of labor supply, and because all households are identical (so  $W_t = W_t(h)$  and  $N_t = N_t(h)$ ), we have:

$$W_t / P_{Ht} = (1 + \mu_t^W) C_t^\sigma N_t^\phi S_t^{1-(\nu/2)}, \text{ where } \mu_t^W \equiv \frac{1}{\eta_t - 1} .$$

The optimal wage set by the household is a time-varying mark-up over the marginal disutility of work (expressed in consumption units.)

Under PCP, as in CGG, firms set prices in their own country’s currency and face a Calvo pricing technology. Under LCP, when firms are allowed to change prices according to the Calvo pricing rule, they set a price in their own currency for sales in their own country and a price in the other country’s currency for exports.

### *PCP*

We assume a standard Calvo pricing technology. A given firm may reset its prices with probability  $1 - \theta$  each period. When the firm resets its price, it will be able to reset its prices for sales in

both markets. We assume the PCP firm sets a single price in its own currency, so the law of one price holds.

The firm's objective is to maximize its value, which is equal to the value at state-contingent prices of its entire stream of dividends. Given equation (4), it is apparent that the firm that selects its price at time  $t$ , chooses its reset prices,  $P_{Ht}^0(f)$ , to maximize

$$E_t \sum_{j=0}^{\infty} \theta^j Q_{t,t+j} \left[ P_{Ht}^0(f) (C_{Ht+j}(f) + C_{Ht+j}^*(f)) - (1 - \tau_t) W_{t+j} N_{t+j}(f) \right],$$

subject to the sequence of demand curves from Home and Foreign households. In this equation, we define

$$Q_{t,t+j} \equiv \beta^j \left( C_{t+j} / C_t \right)^{-\sigma} (P_t / P_{t+j})$$
 as the stochastic discount factor.

Under the Calvo price setting mechanism, a fraction  $\theta$  of prices remain unchanged from the previous period. We can write:

$$P_{Ht} = \left[ \theta (P_{Ht-1})^{1-\xi} + (1-\theta) (P_{Ht}^0)^{1-\xi} \right]^{1/(1-\xi)}.$$

Following the usual derivations, we obtain a log-linearized New Keynesian Phillips curve for an open economy:

$$\pi_{Ht} = \delta (\tilde{w}_t - \tilde{p}_{Ht}) + \beta E_t \pi_{Ht+1},$$

or:

$$(16) \quad \pi_{Ht} = \delta \left[ \left( \frac{\sigma}{D} + \phi \right) \tilde{y}_t^R + (\sigma + \phi) \tilde{y}_t^W \right] + \beta E_t \pi_{Ht+1} + u_t,$$

where  $\delta \equiv (1-\theta)(1-\beta\theta)/\theta$ , and  $u_t \equiv \delta \mu_t^W$ .

Similarly for foreign producer-price inflation, we have:

$$(17) \quad \pi_{Ft}^* = \delta \left[ - \left( \frac{\sigma}{D} + \phi \right) \tilde{y}_t^R + (\sigma + \phi) \tilde{y}_t^W \right] + \beta E_t \pi_{Ft+1}^* + u_t^*.$$

### LCP

The same environment as the PCP case holds, except that the firm sets its price for export in the importer's currency rather than its own currency when it is allowed to reset prices. The Home firm, for example, sets  $P_{Ht}^*(f)$  in Foreign currency. The firm that can reset its price at time  $t$  chooses its reset prices,  $P_{Ht}^0(f)$  and  $P_{Ht}^{0*}(f)$ , to maximize

$$E_t \sum_{j=0}^{\infty} \theta^j Q_{t,t+j} \left[ P_{Ht}^0(z) C_{Ht+j}(f) + E_t P_{Ht}^{0*} C_{Ht+j}^*(f) - (1 - \tau_t) W_{t+j} N_{t+j}(f) \right].$$

As in the PCP case, a fraction  $\theta$  of prices remain unchanged from the previous period. The evolution of  $P_{Ht}$  and  $P_{Ht}^*$  is determined in the standard way in Calvo pricing models. Foreign prices are set analogously to Home prices. We can derive the following Phillips curves for the prices of locally produced goods:

$$(18) \quad \pi_{Ht} = \delta \left[ \left( \frac{\sigma}{D} + \phi \right) \tilde{y}_t^R + (\sigma + \phi) \tilde{y}_t^W + \frac{D - (\nu - 1)}{2D} m_t + \frac{2 - \nu}{2} z_t \right] + \beta E_t \pi_{Ht+1} + u_t,$$

$$(19) \quad \pi_{Ft}^* = \delta \left[ - \left( \frac{\sigma}{D} + \phi \right) \tilde{y}_t^R + (\sigma + \phi) \tilde{y}_t^W - \left( \frac{D - (\nu - 1)}{2D} \right) m_t + \frac{2 - \nu}{2} z_t \right] + \beta E_t \pi_{Ft+1}^* + u_t^*.$$

There are also price adjustment equations for the local prices of imported goods:

$$(20) \quad \pi_{Ht}^* = \delta \left[ \left( \frac{\sigma}{D} + \phi \right) \tilde{y}_t^R + (\sigma + \phi) \tilde{y}_t^W - \left( \frac{D + \nu - 1}{2D} \right) m_t - \frac{\nu}{2} z_t \right] + \beta E_t \pi_{Ht+1}^* + u_t.$$

$$(21) \quad \pi_{Ft} = \delta \left[ - \left( \frac{\sigma}{D} + \phi \right) \tilde{y}_t^R + (\sigma + \phi) \tilde{y}_t^W + \frac{D + \nu - 1}{2D} m_t - \frac{\nu}{2} z_t \right] + \beta E_t \pi_{Ft+1} + u_t^*.$$

Note from equations (18)-(21), if  $z_t = 0$  in these equations, then  $\pi_{Ft} - \pi_{Ht} = \pi_{Ft}^* - \pi_{Ht}^*$ . As the Appendix shows, if the initial condition  $z_0 = 0$  holds, it follows that  $z_t = 0$  in all periods in the LCP model, or, in other words,  $s_t^* = -s_t$ . So equations (18)-(21) can be simplified by setting  $z_t = 0$ . Under our symmetric model with Calvo pricing, relative prices  $p_{Ft} - p_{Ht}$  and  $p_{Ft}^* - p_{Ht}^*$  are always equal in the Home and Foreign country even though the law of one price does not hold. The  $z_t$  term in the loss function is not pertinent, and the only loss arising from deviations from the law of one price comes through the currency misalignment term,  $m_t$ .

### *Subsidies and definitions*

As in CGG, we assume that there is a constant subsidy that would be optimal in the non-stochastic steady state. We have a subsidy rate  $\tau$  that satisfies,  $(1 - \tau)(1 + \mu^P)(1 + \mu^W) = 1$ , where,  $\mu^W$  is the steady-state level of  $\mu_t^W$ .

The policymaker has home and foreign nominal interest rates as instruments. As is standard in the literature, we can model the policymaker as directly choosing output gaps, inflation levels, and (in the LCP case) deviations from the law of one price, subject to constraints. Log-linearized versions of the Home and Foreign households' Euler equations are given by:

$$(22) \quad i_t - E_t \pi_{t+1} = \sigma (E_t c_{t+1} - c_t),$$

$$(23) \quad i_t^* - E_t \pi_{t+1}^* = \sigma (E_t c_{t+1}^* - c_t^*).$$

In these equations,  $\pi_t$  and  $\pi_t^*$  refer to Home and Foreign consumer price inflation, respectively:

$$(24) \quad \pi_t = \frac{\nu}{2}\pi_{Ht} + \frac{2-\nu}{2}\pi_{Ft},$$

$$(25) \quad \pi_t^* = \frac{\nu}{2}\pi_{Ft}^* + \frac{2-\nu}{2}\pi_{Ht}^*.$$

## 6. Optimal Policy under PCP

Under Calvo price adjustment, the loss function can be rewritten in the form:

$$E_t \sum_{j=0}^{\infty} \beta^j X_{t+j} = E_t \sum_{j=0}^{\infty} \beta^j \Psi_{t+j}$$

where:

$$(26) \quad \Psi_t \propto \left[ \frac{\sigma}{D} + \phi \right] (\tilde{y}_t^R)^2 + (\sigma + \phi) (\tilde{y}_t^W)^2 + \frac{\xi}{2\delta} \left( (\pi_{Ht})^2 + (\pi_{Ft}^*)^2 \right).$$

This loss function extends the one derived in CGG to the case of home bias in preferences (i.e.,  $\nu \geq 1$  rather than  $\nu = 1$ .)

The policymaker chooses values for  $\tilde{y}_t$ ,  $\tilde{y}_t^*$ ,  $\pi_{Ht}$  and  $\pi_{Ft}^*$  to minimize the loss, subject to the sequence of Phillips curves (16) and (17). With full credibility, the policymaker can commit to the sequence of inflation rates and output gaps that solve this optimization problem. As in the closed economy, we find in the PCP model that the optimal tradeoff under commitment has the flavor of price-level targeting. When policymakers commit to a rule at time 0, the first-order conditions or “target criteria” at all dates are given by:

$$(27) \quad \tilde{y}_t + \xi(p_{Ht} - p_{H-1}) = 0, \quad \text{and} \quad \tilde{y}_t^* + \xi(p_{Ft}^* - p_{F-1}^*) = 0.$$

Policy under commitment trades off the output gap with the deviation of the current producer price and its level the period before the policy commitment was made,  $p_{Ht} - p_{H-1}$  or  $p_{Ft}^* - p_{F-1}^*$ .

Under discretion, the policymaker takes past values of  $\tilde{y}_t$ ,  $\tilde{y}_t^*$ ,  $\pi_{Ht}$  and  $\pi_{Ft}^*$  as given, and also does not make plans for future values of these variables understanding that future incarnations of the policymaker can alter any given plan. The policymaker at time  $t$  cannot influence  $E_t \pi_{Ht+1}$  and  $E_t \pi_{Ft+1}^*$  because future inflation levels are chosen by future policymakers and there are no endogenous state variables that can limit the paths of future inflation levels. Hence, the policymaker’s problem is essentially a static one – to maximize (26) subject to (16) and (17), taking  $E_t \pi_{Ht+1}$  and  $E_t \pi_{Ft+1}^*$  as given.

The optimal policy rules are given by:

$$(28) \quad \tilde{y}_t + \xi \pi_{Ht} = 0, \quad \text{and} \quad \tilde{y}_t^* + \xi \pi_{Ft}^* = 0.$$

The criteria given in (28) are identical to those that arise in the closed-economy version of this model. There is a tradeoff between the goals of eliminating the output gap and driving inflation to zero, and the elasticity of substitution among goods produced in the country determines the weights given to output gaps and inflation.

Note that home bias in preferences plays no role in the optimal policy rules in the PCP model. Indeed, the target criteria under commitment, (27), and under discretion, (28), are the same as in CGG's model that has no home bias.

It is worth emphasizing that the optimal policy entails a tradeoff between the output gap and the *producer price* inflation level. In a closed economy with no intermediate goods, there is no distinction between producer and consumer prices. But in an open economy there is an important distinction. The policies described in (27) or (28) imply that policymakers should not give any weight to inflation of imported goods. In conjunction with the Phillips curves, (16) and (17), these equations allows us to solve for the Home and Foreign output gaps and  $\pi_{Ht}$  and  $\pi_{Ft}^*$  as functions of current and expected future cost-push shocks,  $u_t$  and  $u_t^*$ . With the output gap determined by optimal policy, the terms of trade must adjust to insure goods market clearing. But the terms of trade adjust freely in the PCP world because nominal exchange rate changes translate directly into import price changes. In essence, the import sector is like a flexible-price sector, so policymakers can ignore inflation in that sector, as in Aoki (2001).

## 7. Optimal Policy under LCP

The transformed loss function,  $\Psi_t$ , can be written as:

$$(29) \quad \Psi_t \propto \left[ \frac{\sigma}{D} + \phi \right] (\tilde{y}_t^R)^2 + (\sigma + \phi) (\tilde{y}_t^W)^2 + \frac{\nu(2-\nu)}{4D} m_t^2 + \frac{\xi}{2\delta} \left( \frac{\nu}{2} (\pi_{Ht})^2 + \frac{2-\nu}{2} (\pi_{Ft})^2 + \frac{\nu}{2} (\pi_{Ft}^*)^2 + \frac{2-\nu}{2} (\pi_{Ht}^*)^2 \right).$$

The loss function is similar to the one under PCP. The main point to highlight is that squared deviations from the law of one price matter for welfare, as well as output gaps and inflation rates. Deviations from the law of one price are distortionary and are a separate source of loss in the LCP model. Comparing (29) to the more general loss function given by (15), recall that relative prices are equal (to a first-order approximation) within each country in our symmetric LCP model, so  $s_t = -s_t^*$ , and therefore  $z_t = 0$ .

The policymaker under discretion seeks to minimize the loss subject to the constraints of the sequence of Phillips curves, (18)-(21). There is an additional constraint in the LCP model. The relative price  $s_t$  cannot adjust freely under LCP as it can under PCP, so  $\pi_{Ft} - \pi_{Ht}$  and  $\pi_{Ht}^* - \pi_{Ft}^*$  are constrained by equations (12) and (13), which can now be consolidated and written as:

$$(30) \quad s_t = \frac{2\sigma}{D} y_t^R - \frac{(\nu-1)}{D} m_t.$$

Equation (30) puts a backward-looking element into the determination of relative prices in the LCP model. Because  $\pi_{Ft} - \pi_{Ht} = s_t - s_{t-1}$ , equation (30) implies that  $y_{t-1}^R$  and  $m_{t-1}$  matter for the determination of  $\pi_{Ft} - \pi_{Ht}$ . This contrasts with the PCP model. In that model, Home-currency inflation of Home goods and Foreign-currency inflation of Foreign goods are entirely forward-looking. Internal relative prices, such as  $\pi_{Ft} - \pi_{Ht}$ , are also forward looking because the local-currency price of imports adjusts instantaneously with changes in the exchange rate.

Our analysis of the LCP model is simplified considerably and much more transparent if we consider only the special case in which preferences are linear in leisure, so that  $\phi = 0$ . This simplification makes the evolution of relative prices,  $s_t$ , independent of policy. Policy analysis is then more similar to the PCP model, in that each period's choice of output levels does not constrain the choices for the subsequent period. We note that the parameter  $\phi$  does not appear in the optimal policy rules for the PCP model, so comparison of the optimal rules under LCP with  $\phi = 0$  does not entail looking at a special case of the PCP model. We also note that the previous literature examining LCP models – Devereux and Engel (2003) and Corsetti and Pesenti (2005) – have also assumed  $\phi = 0$ . Under this assumption, we can use equations (18), (21), and (30) to write:

$$(31) \quad s_t - s_{t-1} = -\delta \tilde{s}_t + \beta E_t(s_{t+1} - s_t) + u_t^* - u_t.$$

In the efficient steady state, the log of the relative price of imports is determined by the log of the productivity differential, so  $\tilde{s}_t = s_t - (a_t - a_t^*)$ . Substitute this into equation (31) and we have a second-order expectational difference equation that determines  $s_t$  independently of policy.

Using (31), the loss function can be simplified to:

$$\Psi_t \propto \frac{\sigma}{D} (\tilde{y}_t^R)^2 + \sigma (\tilde{y}_t^W)^2 + \frac{\nu(2-\nu)}{4D} m_t^2 + \frac{\xi}{\delta} \left( (\pi_t^R)^2 + (\pi_t^W)^2 + \frac{\nu(2-\nu)}{4} (s_t - s_{t-1})^2 \right)$$

Since  $s_t - s_{t-1}$  is independent of policy, we can express the policymaker's problem as choosing relative and world output gaps,  $\tilde{y}_t^R$  and  $\tilde{y}_t^W$ , relative and world CPI inflation rates,  $\pi_t^R$  and  $\pi_t^W$ , and the currency misalignment,  $m_t$  to minimize the expected present discounted value of the loss subject to the “gap” version of equation (30) and the linear combination of the Phillips curves that give us equations for CPI inflation in each country (which are derived here under the assumption that  $\phi = 0$ .) We use the definitions of CPI inflation given in equations (24) and (25).

$$(32) \quad \tilde{s}_t = \frac{2\sigma}{D} \tilde{y}_t^R - \frac{(\nu-1)}{D} m_t$$

$$(33) \quad \pi_t^R = \delta \left[ \frac{\sigma(\nu-1)}{D} \tilde{y}_t^R + \frac{\sigma\nu(2-\nu)}{2D} m_t \right] + \beta E_t \pi_{t+1}^R + (\nu-1)u_t^R$$

$$(34) \quad \pi_t^W = \delta \sigma \tilde{y}_t^W + \beta E_t \pi_{t+1}^W + u_t^W.$$

When policymakers are able to commit to a rule at time 0, target criteria at all dates are:

$$(35) \quad \tilde{y}_t^W + \xi(p_t^W - p_{-1}^W) = 0,$$

$$(36) \quad \frac{\nu-1}{D} \tilde{y}_t^R + \frac{\nu(2-\nu)}{2D} m_t + \xi(p_t^R - p_{-1}^R) = 0.$$

If policymakers operate under discretion, the optimal choices are:

$$(37) \quad \tilde{y}_t^W + \xi \pi_t^W = 0.$$

$$(38) \quad \frac{\nu-1}{D} \tilde{y}_t^R + \frac{\nu(2-\nu)}{2D} m_t + \xi \pi_t^R = 0.$$

#### *Optimal Policy under PCP vs. LCP*

It is helpful to compare the target criteria under PCP and LCP. The tradeoffs between world inflation and world output can be seen to be identical in the two models. Under PCP, producer price inflation appears in the tradeoff. However, world producer price inflation is equal to world consumer price inflation under PCP. To see this,

$$\pi_t^W = \frac{1}{2} \left( \frac{\nu}{2} \pi_{Ht} + \frac{2-\nu}{2} \pi_{Ft} + \frac{\nu}{2} \pi_{Ft}^* + \frac{2-\nu}{2} \pi_{Ht}^* \right) = \frac{1}{2} (\pi_{Ht} + \pi_{Ft}^*).$$

The second equality holds because the relative prices are equal in Home and Foreign under PCP, so  $\pi_{Ht}^* = \pi_{Ft}^* + \pi_{Ht} - \pi_{Ft}$ . By averaging equations (27) which give the optimal policy under commitment in the PCP model, and then taking the first difference, we can write:

$$(39) \quad \tilde{y}_t^W - \tilde{y}_{t-1}^W + \xi \pi_t^W = 0.$$

The policymaker trades off the growth in the world output gap with world inflation. Taking the first-difference of (35) shows that this policy criterion is identical under LCP. This tradeoff is the exact analogy to the closed economy tradeoff between the output gap and inflation, and the intuition of that tradeoff is well understood. On the one hand, with asynchronized price setting, inflation leads to misalignment of relative prices, so any non-zero level of inflation is distortionary. On the other hand, because the monopoly power of labor is time-varying due to the time-varying elasticity of labor demand, output levels can be inefficiently low or high even when inflation is zero. Condition (39) describes the terms of that tradeoff. Inflation is more costly the higher is the elasticity of substitution among varieties of goods,  $\xi$ , because a higher elasticity will imply greater resource misallocation when there is inflation.

Taking the difference in the two policy criteria under PCP from equation (27), dividing by two and taking first-differences, we find:

$$(40) \quad \tilde{y}_t^R - \tilde{y}_{t-1}^R + \xi(\pi_{Ht} - \pi_{Ft}) = 0.$$

In contrast, taking first differences of (36) in the LCP model, we find:

$$(41) \quad \frac{\nu-1}{D}(\tilde{y}_t^R - \tilde{y}_{t-1}^R) + \frac{\nu(2-\nu)}{2D}(m_t - m_{t-1}) + \xi\pi_t^R = 0$$

First, when the two economies are closed, so that  $\nu=2$ , equation (41) reduces to  $\tilde{y}_t^R - \tilde{y}_{t-1}^R + \xi\pi_t^R = 0$ . Of course, when  $\nu=2$ , there is no difference between PPI and CPI inflation, and so in this special case the optimal policies under LCP and PCP are identical. That is nothing more than reassuring, since the distinction between PCP and LCP should not matter when the economies are closed.

When  $\nu \neq 2$ , understanding these optimality conditions is more subtle. It helps to consider the case of no home bias in preferences, so  $\nu=1$ . Suppose inflation rates are zero, so that there is no misallocation of labor within each country. Further, imagine that the world output gap is zero. There are still two possible distortions. First, relative Home to Foreign output may not be at the efficient level. Second, even if output levels are efficient, the allocation of output to Home and Foreign households may be inefficient if there are currency misalignments.

When  $\nu=1$ , it follows from equation (32) that relative output gaps are determined only by the terms of trade:  $2\tilde{y}_t^R = \tilde{s}_t$ . On the other hand, from equation (10), when  $\nu=1$ , relative consumption is misaligned when there are currency misalignments,  $2\sigma\tilde{c}_t^R = m_t$ .

Under PCP, the law of one price holds continuously, so there are no currency misalignments. In that case,  $m_t = 0$ , and relative Home to Foreign consumption is efficient. Policy can influence the terms of trade in order to achieve the optimal tradeoff between relative output gaps and relative inflation, as expressed in equation (40). Policy can affect the terms of trade under PCP because the terms of trade can adjust instantaneously and completely through nominal exchange-rate adjustment. Recall,  $s_t = p_{Ft} - p_{Ht} = e_t + p_{Ft}^* - p_{Ht}$ . While  $p_{Ft}^*$  and  $p_{Ht}$  do not adjust freely, the nominal exchange rate  $e_t$  is not sticky, so the terms of trade adjust freely.

Under LCP, the nominal exchange rate does not directly influence the consumer prices of Home to Foreign goods in either country. For example, in the Home country,  $s_t = p_{Ft} - p_{Ht}$ . Because prices are set in local currencies, neither  $p_{Ft}$  and  $p_{Ht}$  adjust freely to shocks. In fact, as we have seen, when  $\phi=0$ , monetary policy has no control over the internal relative prices.

But under LCP, there are currency misalignments, and monetary policy can control those. From (6), and using the fact that  $s_t = -s_t^*$  in the LCP model,  $m_t \equiv e_t + p_{Ht}^* - p_{Ht} = e_t + p_{Ft}^* - p_{Ft}$ , so the currency

misalignment adjusts instantaneously with nominal exchange rate movements. Because policy cannot influence the relative output distortion (when  $\nu = 1$ ) but can influence the relative consumption distortion, the optimal policy puts full weight on the currency misalignment. When  $\nu = 1$ , we can write (41) as  $m_t - m_{t-1} + 2\sigma\xi\pi_t^R = 0$ . When  $m_t - m_{t-1} > 0$ , so that the Home currency is becoming more undervalued, and  $\pi_t - \pi_t^* > 0$ , the implications for policy are obvious. Home monetary policy must tighten relative to Foreign. But the more interesting case to consider is when Home inflation is running high, so that  $\pi_t - \pi_t^* > 0$ , but the currency is becoming less undervalued, so that  $m_t - m_{t-1} < 0$ . Then equation (41) tells us that the goals of maintaining low inflation and a correctly aligned currency are in conflict. Policies that improve the inflation situation may exacerbate the currency misalignment. Equation (41) parameterizes the tradeoff.

Finally, it is instructive to rewrite condition (41) using  $q_t$ , the consumption real exchange rate, defined as  $q_t = e_t + p_t^* - p_t$ .  $\tilde{q}_t$  is the deviation of the real exchange rate from its efficient level, and we can make use of the relationship  $D\tilde{q}_t = 2\sigma(\nu - 1)\tilde{y}_t^R + \sigma\nu(2 - \nu)m_t$  to write (41) as:

$$(42) \quad \tilde{q}_t - \tilde{q}_{t-1} + 2\sigma\xi\pi_t^R = 0$$

Equation (42) represents the second of the target criteria as a tradeoff between deviations of the real exchange rate from its efficient level and relative CPI inflation rates.

The optimal policy is not successful in eliminating the currency misalignment. Nor does policy drive inflation to zero or eliminate the output gap. Monetary policymakers do not have sufficient control over the economy to achieve the efficient outcome.

An important difference between the policy rules under LCP compared to PCP is that the LCP rules involve consumer price inflation, but the PCP rules involve producer price inflation. The distortion that inflation imposes in a closed-economy model arises because inflation leads to price dispersion of goods that, for efficiency reasons, should have the same price. By the same logic, under LCP, it is desirable *ceteris paribus* to eliminate all inflation, because that would eliminate the price dispersion of Home and Foreign goods within each country. In the PCP model, the law of one price holds for each good produced in each country. Eliminating producer price inflation will eliminate any price dispersion. Moreover, in that setting, it is not the case that we want to eliminate consumer price inflation. There can be inflation of the Foreign currency price of Home goods, for example, for two reasons: because of inflation in the Home currency prices and because of depreciation of the Foreign currency. Eliminating the former, under staggered pricing, eliminates price dispersion. But the latter affects all Foreign currency prices of Home goods equally, so does not contribute to price dispersion of Home goods in the Foreign country. Moreover, the exchange rate change can lead to beneficial changes in the price of Home

goods relative to Foreign goods within the Foreign country. Relative prices under PCP can adjust flexibly when exchange rates are flexible, and that is desirable from an efficiency standpoint, so it is desirable to allow for non-zero changes in the Foreign currency price of Home goods (and, symmetrically, the Home currency price of Foreign goods.)

### 8. No Mark-up Shocks

It is well known that in the PCP model, if there are no mark-up shocks and the optimal steady-state subsidies are in place, the efficient allocation is obtainable under optimal monetary policy. A policy that sets inflation of Home and Foreign prices to zero eliminates all distortions. The flexibility of the exchange rate allows for optimal terms of trade response to shocks. This holds true both under commitment and under discretion.

Under LCP, interestingly, optimal policy sets Home and Foreign CPI inflation ( $\pi_t$  and  $\pi_t^*$ ) to zero in this case, but does not deliver the efficient outcome. It is easy to see that CPI inflation is zero in both countries under the optimal policies. Substitute the target criteria under commitment, (35) and (36), into the Phillips curves, (33) and (34), assuming mark-up shocks are always zero ( $u_t^R = u_t^W = 0$ .) The solutions to the expectational difference equations are  $\pi_t^R = \pi_t^W = 0$ , which imply  $\pi_t = \pi_t^* = 0$ . The same conclusion is reached applying the optimal policies under discretion, (37) and (38).

If CPI inflation rates are zero, it follows from the optimal rules that  $\tilde{y}_t^W = 0$  and  $\tilde{q}_t = 0$ : world output is at efficient levels, as is the real exchange rate. But this still does not imply that allocations are efficient. Using (32), we conclude  $m_t = -(\nu - 1)\tilde{s}_t$  and  $\tilde{y}_t^R = \frac{\nu(2-\nu)}{2}\tilde{s}_t$ . The evolution of  $\tilde{s}_t$  is governed by (31). When there is pricing to market, the relative price of Foreign to Home goods in either market is not set efficiently. Even though policymakers can achieve an efficient level of world output when there are not mark-up shocks, they cannot ensure that the mix of Home and Foreign output is optimal. The price signals under LCP do not lead to efficient allocations.

### 9. Instrument Rules

Gali (2008) derives optimal instrument rules or Taylor rules in a model that has the following structure:

$$\text{Phillips curve: } \pi_t = \beta\pi_{t+1} + \kappa x_t + u_t$$

$$\text{Stochastic process for cost-push shocks: } u_t = \rho_u u_{t-1} + \varepsilon_t$$

$$\text{Euler equation: } i_t - E_t \pi_{t+1} - r_t^e = \sigma(E_t x_{t+1} - x_t)$$

Optimal rule under discretion:  $x_t = -\frac{\kappa}{\alpha_x} \pi_t$

Optimal rule under commitment:  $x_t = -\frac{\kappa}{\alpha_x} (p_t - p_{-1})$ ,

where  $r_t^e$  is the real interest rate under efficient allocations (the ‘‘Wicksellian real rate’’).

Gali shows for this model that under discretion, the optimal rule can be implemented with a Taylor rule of the form:

$$(43) \quad i_t = r_t^e + \Theta u_t + \phi_\pi \pi_t,$$

where  $\Theta \equiv [\kappa\sigma(1 - \rho_u) - \alpha_x(\phi_\pi - \rho_u)] / [\kappa^2 + \alpha_x(1 - \beta\rho_u)]$ , and  $\phi_\pi$  is an arbitrary coefficient satisfying  $\phi_\pi > 1$ . Under commitment, the optimal allocation can be achieved with an instrument rule of the form:

$$(44) \quad i_t = r_t^e - \left[ \phi_p + (1 - \tilde{\lambda}) \left( 1 - \frac{\sigma\kappa}{\alpha_x} \right) \right] \sum_{k=0}^t \tilde{\lambda}^{k+1} u_{t-k} + \phi_p (p_t - p_{-1}),$$

where  $\tilde{\lambda} \equiv [1 - \sqrt{1 - 4\beta a^2}] / 2a\beta$ ,  $a = \alpha_x / [\alpha_x(1 + \beta) + \kappa^2]$ , and  $\phi_p$  is arbitrary so long as  $\phi_p > 0$ .<sup>17</sup>

The dynamics of ‘‘world’’ and ‘‘relative’’ variables in the LCP model can be written as isomorphisms to this model of Gali’s.<sup>18</sup> The set of equations that determine the world variables and the set of equations that determine the relative variables in the LCP model are each isomorphic to Gali’s model of a closed economy.

For world variables, we have the following mapping from variables in the LCP model to variables in the Gali model:  $\pi_t^W \rightarrow \pi_t$ ,  $p_t^W \rightarrow p_t$ ,  $\tilde{y}_t^W \rightarrow x_t$ ,  $i_t^W \rightarrow i_t$ , and  $u_t^W \rightarrow u_t$ . The parameters map as:  $\xi \rightarrow \kappa / \alpha_x$ ,  $\delta\sigma \rightarrow \kappa$ ,  $\sigma \rightarrow \sigma$ .

For relative variables, the following mapping from the LCP model to the Gali model defines the isomorphism:  $\pi_t^R \rightarrow \pi_t$ ,  $p_t^R \rightarrow p_t$ ,  $q_t / \sigma \rightarrow x_t$ ,  $i_t^R \rightarrow i_t$ , and  $(v - 1)u_t^R \rightarrow u_t$ . The parameters map in the same way as for world variables.

The implication is that there are versions of equation (43) and (44) that hold for both world variables and relative variables. For example, the world version of equation (43) is a Taylor rule that relates the world interest rate to the world output gap and the world mark-up shock, and the relative version of equation (43) relates relative interest rates to the relative output gap and the relative mark-up shocks.

<sup>17</sup> The optimal rule under commitment is derived assuming  $\rho_u = 0$ .

<sup>18</sup> The dynamics of ‘‘relative’’ and ‘‘world’’ variables in the PCP model are also isomorphic, where relative inflation here is understood to be producer price inflation.

The sum of world and relative variables equals the value of those variables in the Home country. The parameters in equations (43) and (44) are the same for both the world and relative versions, with the possible exception of the serial correlation of the mark-up shocks,  $\rho_u$ . If we assume that the serial correlation of world mark-up shocks equals the serial correlation of relative mark-up shocks (which would happen if Home and Foreign mark-up shocks had the same serial correlation), then we can simply add the world and relative versions of equations (43) and (44) to get Taylor rules that relate the Home interest rate to Home inflation and Home mark-up shocks.<sup>19</sup> By subtracting the relative version of these equations from the world version, we get the analogous equations for the Foreign country.

So, under the assumption that Home and Foreign mark-ups are first-order autoregressions with the same serial correlation, we can derive Taylor rules that relate the nominal interest rate in each country only to CPI inflation in that country, the efficient real interest rate, and mark-up shocks.

In the PCP model, we can derive similar instrument rules, but the relevant inflation rate in each equation is the PPI inflation rate, as in CGG.

The surprising thing about the instrument rules in all of these models – the closed-economy model, LCP, and PCP – is that they do not explicitly include the activity variables: output gaps and real exchange rates. This leads to the conclusion that policymakers can use Taylor rules that include only inflation rates (and the appropriate function of mark-up shocks and the efficient level of the real interest rate.)

However, there are several important caveats to that conclusion. First, as CGG and Gali (2008) note, the form of these instrument rules is not unique even under the set of assumptions used to derive them. Because the “targeting rules” are linear functions that relate inflation to the output gaps and, in the case of the LCP model, the real exchange rate or the currency misalignment, there are equivalent Taylor rules that do include these activity variables.

Our previous discussion characterized optimal policy in terms of target criteria or targeting rules rather than as monetary rules expressed as interest-rate reaction functions (or instrument rules.) Those rules hold under more general assumptions than equations (43) and (44). The instrument rules that support these optimal choices by policymakers depend on the stochastic processes of the exogenous productivity and cost-push shocks. As Svensson (1999, 2002) has emphasized, targeting rules such as the ones derived here are not dependent on the processes for underlying shocks while instrument rules are. The stochastic process for the shocks can be defined very broadly – it might, for example, be a multivariate process for the shock itself and signals about the shock received by the policymaker. If policymakers receive a signal, the instrument rule will change but the targeting rule will not. In general,

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<sup>19</sup> Note the optimal rule under commitment, (44), assumes  $\rho_u = 0$  in both countries.

if the stochastic process for the exogenous variables were to change, the instrument rule would change but not the targeting rule.

We derive policy rules under commitment without describing the device that the policymaker uses to achieve the credibility needed to commit to a rule. Svensson (1999, 2002) argues that it is more credible to commit to a targeting rule than an instrument rule. The targeting rule does not change even when there is a change in the stochastic process of the exogenous variables, while the optimal instrument rule might. Markets might be more convinced that the targeting rule is being enforced when there are regime changes such as this, because it is invariant to the regime change.

## 10. Conclusions

Clarida, Gali, and Gertler (2002) implies that the key tradeoffs in an open economy are the same as in a closed economy. Policymakers should target a linear combination of inflation and the output gap. This paper shows that in fact, when our model is rich enough to allow for currency misalignments, the tradeoffs should involve not only inflation and the output gap but also the exchange rate misalignment. Moreover, in contrast to CGG, we show that policymakers should target consumer price inflation in the open economy.

The paper derives the policymaker's loss function when there is home bias in consumption and deviations from the law of one price. The loss function does depend on the structure of the model, of course, but not on the specific nature of price setting. Currency misalignments may arise in some approaches for reasons other than local-currency-pricing. For example, there may be nominal wage stickiness but imperfect pass-through that arises from strategic behavior by firms as in the models of Atkeson and Burstein (2007, 2008) or Corsetti, Dedola, and Leduc (2010). Future work can draw on the results derived here – the demonstration of the global welfare loss from pricing to market.

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## **Appendix: Not for Publication**

Currency Misalignments and Optimal Monetary Policy:  
A Reexamination

## Appendix A

### Model Equations

#### A1.a. Households

The representative household in the home country maximizes

$$(A1) \quad U_t(h) = E_t \left\{ \sum_{j=0}^{\infty} \beta^j \left[ \frac{1}{1-\sigma} C_{t+j}(h)^{1-\sigma} - \frac{1}{1+\phi} N_{t+j}(h)^{1+\phi} \right] \right\}, \quad \sigma > 0, \phi \geq 0$$

$C_t(h)$  is the consumption aggregate. We assume Cobb-Douglas preferences:

$$(A2) \quad C_t(h) = (C_{Ht}(h))^{\frac{\nu}{2}} (C_{Ft}(h))^{1-\frac{\nu}{2}}, \quad 0 \leq \nu \leq 2.$$

In turn,  $C_{Ht}(h)$  and  $C_{Ft}(h)$  are CES aggregates over a continuum of goods produced in each country:

$$(A3) \quad C_{Ht}(h) = \left( \int_0^1 C_{Ht}(h, f)^{\frac{\xi-1}{\xi}} df \right)^{\frac{\xi}{\xi-1}} \quad \text{and} \quad C_{Ft}(h) = \left( \int_0^1 C_{Ft}(h, f)^{\frac{\xi-1}{\xi}} df \right)^{\frac{\xi}{\xi-1}}.$$

$N_t(h)$  is an aggregate of the labor services that the household sells to each of a continuum of firms located in the home country:

$$(A4) \quad N_t(h) = \int_0^1 N_t(h, f) df.$$

Households receive wage income,  $W_t(h)N_t(h)$ , aggregate profits from home firms,  $\Gamma_t$ . They pay lump-sum taxes each period,  $T_t$ . Each household can trade in a complete market in contingent claims (arbitrarily) denominated in the home currency. The budget constraint is given by:

$$(A5) \quad P_t C_t(h) + \sum_{\nabla^{t+1} \in \Omega_{t+1}} Z(\nabla^{t+1} | \nabla^t) D(h, \nabla^{t+1}) = W_t(h)N_t(h) + \Gamma_t - T_t + D(h, \nabla^t),$$

where  $D(h, \nabla^t)$  represents household  $h$ 's payoffs on state-contingent claims for state  $\nabla^t$ .  $Z(\nabla^{t+1} | \nabla^t)$  is the price of a claim that pays one dollar in state  $\nabla^{t+1}$ , conditional on state  $\nabla^t$  occurring at time  $t$ .

In this equation,  $P_t$  is the exact price index for consumption, given by:

$$(A6) \quad P_t = k^{-1} P_{Ht}^{\nu/2} P_{Ft}^{1-(\nu/2)}, \quad k = (1 - (\nu/2))^{1-(\nu/2)} (\nu/2)^{\nu/2}.$$

$P_{Ht}$  is the Home-currency price of the Home aggregate good and  $P_{Ft}$  is the Home currency price of the Foreign aggregate good. Equation (A6) follows from cost minimization. Also, from cost minimization,  $P_{Ht}$  and  $P_{Ft}$  are the usual CES aggregates over prices of individual varieties,  $f$ :

$$(A7) \quad P_{Ht} = \left( \int_0^1 P_{Ht}(f)^{1-\xi} df \right)^{\frac{1}{1-\xi}}, \quad \text{and} \quad P_{Ft} = \left( \int_0^1 P_{Ft}(f)^{1-\xi} df \right)^{\frac{1}{1-\xi}}.$$

Foreign households have analogous preferences and face an analogous budget constraint.

Because all Home households are identical, we can drop the index for the household and use the fact that aggregate per capita consumption of each good is equal to the consumption of each good by each household. The first-order conditions for consumption are given by:

$$(A8) \quad P_{Ht} C_{Ht} = \frac{\nu}{2} P_t C_t,$$

$$(A9) \quad P_{Ft} C_{Ft} = \left( 1 - \frac{\nu}{2} \right) P_t C_t,$$

$$(A10) \quad C_{Ht}(f) = \left( \frac{P_{Ht}(f)}{P_{Ht}} \right)^{-\xi} C_{Ht} \quad \text{and} \quad C_{Ft}(f) = \left( \frac{P_{Ft}(f)}{P_{Ft}} \right)^{-\xi} C_{Ft},$$

$$(A11) \quad \beta \left( C(\nabla^{t+1}) / C(\nabla^t) \right)^{-\sigma} (P_t / P_{t+1}) = \ddot{Z}(\nabla^{t+1} | \nabla^t).$$

In equation (A11), we explicitly use an index for the state at time  $t$  for the purpose of clarity.  $\ddot{Z}(\nabla^{t+1} | \nabla^t)$  is the normalized price of the state contingent claim. That is, it is defined as  $Z(\nabla^{t+1} | \nabla^t)$  divided by the probability of state  $\nabla^{t+1}$  conditional on state  $\nabla^t$ .

Note that the sum of  $Z(\nabla^{t+1} | \nabla^t)$  across all possible states at time  $t + 1$  must equal  $1/R_t$ , where  $R_t$  denotes the gross nominal yield on a one-period non-state-contingent bond. Therefore, taking a probability-weighted sum across all states of equation (A11), we have the familiar Euler equation:

$$(A12) \quad \beta R_t E_t \left[ \left( C(\nabla^{t+1}) / C(\nabla^t) \right)^{-\sigma} (P_t / P_{t+1}) \right] = 1.$$

Analogous equations hold for Foreign households. Since contingent claims are (arbitrarily) denominated in Home currency, the first-order condition for Foreign households that is analogous to equation (A11) is:

$$(A13) \quad \beta \left( C^*(\nabla^{t+1}) / C^*(\nabla^t) \right)^{-\sigma} (E_t P_t^* / E_{t+1} P_{t+1}^*) = \ddot{Z}(\nabla^{t+1} | \nabla^t).$$

Here,  $E_t$  refers to the home currency price of foreign currency exchange rate.<sup>1</sup>

As noted above, we will assume at this stage that labor input of all households is the same, so  $N_t = N_t(h)$ .

#### A1.b. Firms

Each Home good,  $Y_t(f)$  is made according to a production function that is linear in the labor input. These are given by:

$$(A14) \quad Y_t(f) = A_t N_t(f).$$

Note that the productivity shock,  $A_t$ , is common to all firms in the Home country.  $N_t(f)$  is a CES composite of individual home-country household labor, given by:

$$(A15) \quad N_t(f) = \left( \int_0^1 N_t(h, f)^{\frac{\eta_t-1}{\eta_t}} dh \right)^{\frac{\eta_t}{\eta_t-1}},$$

where the technology parameter,  $\eta_t$ , is stochastic and common to all Home firms.

Profits are given by:

$$(A16) \quad \Gamma_t(f) = P_{Ht}(f) C_{Ht}(f) + E_t P_{Ht}^*(f) C_{Ht}^*(f) - (1 - \tau_t) W_t N_t(f).$$

In this equation,  $P_{Ht}(f)$  is the home-currency price of the good when it is sold in the Home country.  $P_{Ht}^*(f)$  is the foreign-currency price of the good when it is sold in the Foreign country.  $C_{Ht}(f)$  is aggregate sales of the good in the home country:

$$(A17) \quad C_{Ht}(f) = \int_0^1 C_{Ht}(h, f) dh.$$

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<sup>1</sup> We offer an apology to the reader here. We want to stick to CCG's notation, who use  $e_t$  for the log of the nominal exchange rate. Consistency requires us to use  $E_t$  to refer to the level of the nominal exchange rate, so we have used the distinct but similar notation  $E_t$  to be the conditional expectation operator.

$C_{Ht}^*(f)$  is defined analogously. It follows that  $Y_t(f) = C_{Ht}(f) + C_{Ht}^*(f)$ .

There are analogous equations for  $Y_t^*(f)$ , with the foreign productivity shock given by  $A_t^*$ , the foreign technology parameter shock given by  $\eta_t^*$ , and foreign subsidy given by  $\tau_t^*$ .

### A1.c Equilibrium

Goods market clearing conditions in the Home and Foreign country are given by:

$$(A18) \quad Y_t = C_{Ht} + C_{Ht}^* = \frac{\nu}{2} \frac{P_t C_t}{P_{Ht}} + \left(1 - \frac{\nu}{2}\right) \frac{P_t^* C_t^*}{P_{Ht}^*} = k^{-1} \left( \frac{\nu}{2} S_t^{1-(\nu/2)} C_t + \left(1 - \frac{\nu}{2}\right) (S_t^*)^{-\nu/2} C_t^* \right),$$

$$(A19) \quad Y_t^* = C_{Ft} + C_{Ft}^* = \left(1 - \frac{\nu}{2}\right) \frac{P_t C_t}{P_{Ft}} + \frac{\nu}{2} \frac{P_t^* C_t^*}{P_{Ft}^*} = k^{-1} \left( \frac{\nu}{2} (S_t^*)^{1-(\nu/2)} C_t^* + \left(1 - \frac{\nu}{2}\right) S_t^{-\nu/2} C_t \right).$$

We have used  $S_t$  and  $S_t^*$  to represent the price of imported to locally-produced goods in the Home and Foreign countries, respectively:

$$(A20) \quad S_t = P_{Ft} / P_{Ht},$$

$$(A21) \quad S_t^* = P_{Ht}^* / P_{Ft}^*.$$

Equations (A11) and (A13) give us the familiar condition that arises in open-economy models with a complete set of state-contingent claims when PPP does not hold:

$$(A22) \quad \left( \frac{C_t}{C_t^*} \right)^\sigma = \frac{E_t P_t^*}{P_t} = \frac{E_t P_{Ht}^*}{P_{Ht}} (S_t^*)^{-\nu/2} S_t^{(\nu/2)-1}.$$

Total employment is determined by output in each industry:

$$(A23) \quad N_t = \int_0^1 N_t(f) df = A_t^{-1} \int_0^1 Y_t(f) df = A_t^{-1} (C_{Ht} V_{Ht} + C_{Ht}^* V_{Ht}^*),$$

where

$$(A24) \quad V_{Ht} \equiv \int_0^1 \left( \frac{P_{Ht}(f)}{P_{Ht}} \right)^{-\xi} df, \quad \text{and} \quad V_{Ht}^* \equiv \int_0^1 \left( \frac{P_{Ht}^*(f)}{P_{Ht}^*} \right)^{-\xi} df.$$

### A2 Price and Wage Setting

Households are monopolistic suppliers of their unique form of labor services. Household  $h$  faces demand for its labor services given by:

$$(A25) \quad N_t(h) = \left( \frac{W_t(h)}{W_t} \right)^{-\eta_t} N_t,$$

where

$$(A26) \quad W_t = \left( \int_0^1 W_t(h)^{1-\eta_t} dh \right)^{\frac{1}{1-\eta_t}}.$$

The first-order condition for household  $h$ 's choice of labor supply is given by:

$$(A27) \quad \frac{W_t(h)}{P_t} = (1 + \mu_t^W) (C_t(h))^\sigma (N_t(h))^\phi, \quad \text{where} \quad \mu_t^W \equiv \frac{1}{\eta_t - 1}.$$

The optimal wage set by the household is a time-varying mark-up over the marginal disutility of work (expressed in consumption units.)

Because all households are identical, we have  $W_t = W_t(h)$  and  $N_t = N_t(h)$ .

Since all households are identical, we have from equation (A27):

$$(A28) \quad W_t / P_{Ht} = (1 + \mu_t^W) C_t^\sigma N_t^\phi S_t^{1-(\nu/2)}.$$

We adopt the following notation. For any variable  $K_t$ :

$\dot{K}_t$  is the value under flexible prices.

$\bar{K}_t$  is the value of variables under globally efficient allocations. In other words, this is the value for variables if prices were flexible, and optimal subsidies to monopolistic suppliers of labor and monopolistic producers of goods were in place. This includes a time-varying subsidy to suppliers of labor to offset the time-varying mark-up in wages in equation (A27).

### Flexible Prices

Home firms maximize profits given by equation (A16), subject to the demand curve (A10). They optimally set prices as a mark-up over marginal cost:

$$(A29) \quad \dot{P}_{Ht}(f) = \dot{E}_t \dot{P}_{Ht}^*(f) = (1 - \tau_t)(1 + \mu^P) \dot{W}_t / A_t, \text{ where } \mu^P \equiv \frac{1}{\xi - 1}.$$

When optimal subsidies are in place:

$$(A30) \quad \bar{P}_{Ht}(f) = \bar{E}_t \bar{P}_{Ht}^*(f) = \bar{W}_t / A_t.$$

From (A27), (A29), and (A30), it is apparent that the optimal subsidy satisfies

$$(A31) \quad (1 - \tau_t)(1 + \mu^P)(1 + \mu_t^W) = 1.$$

Note from (A29) that all flexible price firms are identical and set the same price. Because the demand functions of Foreign residents have the same elasticity of demand for Home goods as Home residents, firms set the same price for sale abroad:

$$(A32) \quad E_t \dot{P}_{Ht}^* = \dot{P}_{Ht} \text{ and } E_t \bar{P}_{Ht}^* = \bar{P}_{Ht}$$

From (A31), using (A28), we have:

$$(A33) \quad \dot{P}_{Ht} = \dot{E}_t \dot{P}_{Ht}^* = \dot{W}_t / ((1 + \mu^W) A_t) \text{ and } \dot{P}_{Ft}^* = \dot{E}_t^{-1} \dot{P}_{Ft} = \dot{W}_t^* / ((1 + \mu^{*W}) A_t^*).$$

We can conclude:

$$(A34) \quad \dot{S}_t^{*-1} = \dot{S}_t.$$

Because  $\dot{P}_{Ht}(f)$  is identical for all firms, (A23) collapses to

$$(A35) \quad \dot{Y}_t = A_t \dot{N}_t.$$

### PCP

We assume a standard Calvo pricing technology. A given firm may reset its prices with probability  $1 - \theta$  each period. We assume that when the firm resets its price, it will be able to reset its prices for sales in both markets. We assume the PCP firm sets a single price in its own currency, so the law of one price holds.

The firm's objective is to maximize its value, which is equal to the value at state-contingent prices of its entire stream of dividends. Given equation (A11), it is apparent that the firm that selects its price at time  $t$ , chooses its reset price,  $P_{Ht}^0(f)$ , to maximize

$$(A36) \quad E_t \sum_{j=0}^{\infty} \theta^j Q_{t,t+j} \left[ P_{Ht}^0(f) (C_{Ht+j}(f) + C_{Ht+j}^*(f)) - (1 - \tau_t) W_{t+j} N_{t+j}(f) \right],$$

subject to the sequence of demand curves given by equation (A10) and the corresponding Foreign demand equation for Home goods. In this equation, we define

$$(A37) \quad Q_{t,t+j} \equiv \beta^j \left( C_{t+j} / C_t \right)^{-\sigma} (P_t / P_{t+j}).$$

The solution for the optimal price for the Home firm for sale in the Home country is given by:

$$(A38) \quad P_{Ht}^0(z) = \frac{\xi}{\xi - 1} \frac{E_t \sum_{j=0}^{\infty} \theta^j Q_{t,t+j} (1 - \tau_t) W_{t+j} P_{Ht+j}^\xi (C_{Ht+j} + C_{Ht+j}^*) / A_{t+j}}{E_t \sum_{j=0}^{\infty} \theta^j Q_{t,t+j} P_{Ht+j}^\xi (C_{Ht+j} + C_{Ht+j}^*)}.$$

Under the Calvo price setting mechanism, a fraction  $\theta$  of prices remain unchanged from the previous period. From equation (A7), we can write:

$$(A39) \quad P_{Ht} = \left[ \theta (P_{Ht-1})^{1-\xi} + (1-\theta) (P_{Ht}^0)^{1-\xi} \right]^{1/(1-\xi)},$$

### LCP

The same environment as the PCP case holds, with the sole exception that the firm sets its price for export in the importer's currency rather than its own currency when it is allowed to reset prices. The Home firm, for example, sets  $P_{Ht}^*(f)$  in Foreign currency. The firm that can reset its price at time  $t$  chooses its reset prices,  $P_{Ht}^0(f)$  and  $P_{Ht}^{0*}(f)$ , to maximize

$$(A40) \quad E_t \sum_{j=0}^{\infty} \theta^j Q_{t,t+j} \left[ P_{Ht}^0(z) C_{Ht+j}(f) + E_t P_{Ht}^{0*} C_{Ht+j}^*(f) - (1-\tau_t) W_{t+j} N_{t+j}(f) \right].$$

The solution for  $P_{Ht}^0(z)$  is given by:

$$(A41) \quad P_{Ht}^0(z) = \frac{\xi}{\xi-1} \frac{E_t \sum_{j=0}^{\infty} \theta^j Q_{t,t+j} (1-\tau_t) W_{t+j} P_{Ht+j}^{\xi} C_{Ht+j} / A_{t+j}}{E_t \sum_{j=0}^{\infty} \theta^j Q_{t,t+j} P_{Ht+j}^{\xi} C_{Ht+j}}$$

We find for export prices,

$$(A42) \quad P_{Ht}^{0*}(z) = \frac{\xi}{\xi-1} \frac{E_t \sum_{j=0}^{\infty} \theta^j Q_{t,t+j} (1-\tau_t) W_{t+j} (P_{Ht+j}^*)^{\xi} C_{Ht+j}^* / A_{t+j}}{E_t \sum_{j=0}^{\infty} \theta^j Q_{t,t+j} E_{t+j} (P_{Ht+j}^*)^{\xi} C_{Ht+j}^*}.$$

Equation (A39) holds in the LCP case as well. However, the law of one price does not hold. For export prices, we have:

$$(A43) \quad P_{Ht}^* = \left[ \theta (P_{Ht-1}^*)^{1-\xi} + (1-\theta) (P_{Ht}^{0*})^{1-\xi} \right]^{1/(1-\xi)}$$

### Subsidies

As in CGG, we will assume that subsidies to monopolists are not set at their optimal level except in steady-state. That is, instead of the efficient subsidy given in equation (A31), we have:

$$(A44) \quad (1-\tau)(1+\mu^P)(1+\mu^W) = 1.$$

Here,  $\mu^W$  is the steady-state level of  $\mu_t^W$ . We have dropped the time subscript on the subsidy rate  $\tau_t$  because it is not time-varying.

## Appendix B

### Log-linearized Model

In this section, we present log-linear approximations to the models presented above.

In this section, we present all of the equations of the log-linearized model, but we separate out those that are used in the derivation of the loss function (which do not involve price setting or wage setting) and those that are not.

#### *Equations used for derivation of loss functions*

We define the log of the currency misalignment as the average of the difference between Foreign and Home prices:

$$(B1) \quad m_t \equiv \frac{1}{2}(e_t + p_{Ht}^* - p_{Ht} + e_t + p_{Ft}^* - p_{Ft}).$$

In the flexible-price and PCP models,  $m_t = 0$ .

We also define the export premium as the average by which consumer prices of imported goods exceeds the average of locally produced goods:

$$(B2) \quad z_t \equiv \frac{1}{2}(p_{Ht}^* + p_{Ft} - p_{Ft}^* - p_{Ht})$$

In all three models, to a first order,  $\ln(V_{Ht}) = \ln(V_{Ht}^*) = \ln(V_{Ft}) = \ln(V_{Ft}^*) = 0$ . That allows us to approximate equation (A23) and its foreign counterpart as:

$$(B3) \quad n_t = y_t - a_t, \quad \text{and}$$

$$(B4) \quad n_t^* = y_t^* - a_t^*.$$

The market-clearing conditions, (A18) and (A19) are approximated as:

$$(B5) \quad y_t = \frac{\nu}{2}c_t + \frac{2-\nu}{2}c_t^* + \frac{\nu}{2}\left(\frac{2-\nu}{2}\right)s_t - \frac{\nu}{2}\left(\frac{2-\nu}{2}\right)s_t^*,$$

$$(B6) \quad y_t^* = \frac{\nu}{2}c_t^* + \frac{2-\nu}{2}c_t - \frac{\nu}{2}\left(\frac{2-\nu}{2}\right)s_t + \frac{\nu}{2}\left(\frac{2-\nu}{2}\right)s_t^*.$$

The condition arising from complete markets that equates the marginal utility of nominal wealth for Home and Foreign households, equation (A22), is given by:

$$(B7) \quad \sigma c_t - \sigma c_t^* = m_t + \frac{\nu-1}{2}s_t - \left(\frac{\nu-1}{2}\right)s_t^*.$$

We define “relative” and “world” log output by:

$$(B8) \quad y_t^R \equiv \frac{1}{2}(y_t - y_t^*)$$

$$(B9) \quad y_t^W \equiv \frac{1}{2}(y_t + y_t^*).$$

For use later, it is helpful to use equations (B5)-(B7) to express  $c_t$ ,  $c_t^*$ ,  $s_t$ , in terms of  $y_t$  and  $y_t^*$  and the price deviations,  $m_t$  and  $z_t$ .

$$(B10) \quad c_t = \frac{\nu-1}{D}y_t^R + y_t^W + \frac{\nu(2-\nu)}{2D}m_t$$

$$(B11) \quad c_t^* = -\left(\frac{\nu-1}{D}\right)y_t^R + y_t^W - \frac{\nu(2-\nu)}{2D}m_t,$$

where  $D \equiv \sigma\nu(2-\nu) + (\nu-1)^2$ .

We can further simplify these by defining:

$$(B12) \quad c_t^R \equiv \frac{1}{2}(c_t - c_t^*)$$

$$(B13) \quad c_t^W \equiv \frac{1}{2}(c_t + c_t^*)$$

Then

$$(B14) \quad c_t^R = \frac{\nu-1}{D} y_t^R + \frac{\nu(2-\nu)}{2D} m_t$$

$$(B15) \quad c_t^W = y_t^W.$$

And, solving for the terms of trade, we find:

$$(B16) \quad s_t = \frac{2\sigma}{D} y_t^R + z_t - \frac{(\nu-1)}{D} m_t,$$

$$(B17) \quad s_t^* = -\frac{2\sigma}{D} y_t^R + z_t + \frac{(\nu-1)}{D} m_t.$$

Under a globally efficient allocation, the marginal rate of substitution between leisure and aggregate consumption should equal the marginal product of labor times the price of output relative to consumption prices. To see the derivation more cleanly, we insert the shadow real wages in the efficient allocation,  $\bar{w}_t - \bar{p}_{Ht}$  and  $\bar{w}_t^* - \bar{p}_{Ft}^*$  into equations (B18) and (B19) below. So, the efficient allocation would be achieved in a model with flexible wages and optimal subsidies. These equations then can be understood intuitively by looking at the wage setting equations below ((B22)-(B23), and (B24)-(B25)) assuming the optimal subsidy is in place. But, to emphasize, they do not depend on a particular model of wage setting, and are just the standard efficiency condition equating the marginal rate of substitution between leisure and aggregate consumption to the marginal rate of transformation.

$$(B18) \quad a_t = \bar{w}_t - \bar{p}_{Ht} = \left( \frac{\sigma}{D} + \phi \right) \bar{y}_t^R + (\sigma + \phi) \bar{y}_t^W - \phi a_t,$$

$$(B19) \quad a_t^* = \bar{w}_t^* - \bar{p}_{Ft}^* = -\left( \frac{\sigma}{D} + \phi \right) \bar{y}_t^R + (\sigma + \phi) \bar{y}_t^W - \phi a_t^*.$$

#### *Equations of wage and price setting*

The real Home and Foreign product wages, from equation (A28), are given by:

$$(B20) \quad w_t - p_{Ht} = \sigma c_t + \phi n_t + \frac{2-\nu}{2} s_t + \mu_t^W,$$

$$(B21) \quad w_t^* - p_{Ft}^* = \sigma c_t^* + \phi n_t^* + \left( \frac{2-\nu}{2} \right) s_t^* + \mu_t^{*W}.$$

We can express  $w_t - p_{Ht}$ , and  $w_t^* - p_{Ft}^*$  in terms of  $y_t$  and  $y_t^*$  and the exogenous disturbances,  $a_t$ ,  $a_t^*$ ,  $\mu_t^W$ , and  $\mu_t^{*W}$ :

$$(B22) \quad w_t - p_{Ht} = \left( \frac{\sigma}{D} + \phi \right) y_t^R + (\sigma + \phi) y_t^W + \frac{D-(\nu-1)}{2D} m_t + \frac{2-\nu}{2} z_t - \phi a_t + \mu_t^W,$$

$$(B23) \quad w_t^* - p_{Ft}^* = -\left( \frac{\sigma}{D} + \phi \right) y_t^R + (\sigma + \phi) y_t^W - \left( \frac{D-(\nu-1)}{2D} \right) m_t + \frac{2-\nu}{2} z_t - \phi a_t^* + \mu_t^{*W},$$

#### Flexible Prices

We can solve for the values of all the real variables under flexible prices by using equations (B3), (B4), (B10), (B11), (B16), (B22) and (B23), as well as the price-setting conditions, from (A33):

$$(B24) \quad \dot{w}_t - \dot{p}_{Ht} = a_t,$$

$$(B25) \quad \dot{w}_t^* - \dot{p}_{Ft}^* = a_t^*.$$

### PCP

Log-linearization of equations (A38) and (A39) gives us the familiar New Keynesian Phillips curve for an open economy:

$$(B26) \quad \pi_{Ht} = \delta(w_t - p_{Ht} - a_t) + \beta E_t \pi_{Ht+1},$$

where  $\delta = (1-\theta)(1-\beta\theta)/\theta$ .

We can rewrite this equation as:

$$\pi_{Ht} = \delta(\tilde{w}_t - \tilde{p}_{Ht}) + \beta E_t \pi_{Ht+1},$$

or, using (B22) and (B18):

$$(B27) \quad \pi_{Ht} = \delta \left[ \left( \frac{\sigma}{D} + \phi \right) \tilde{y}_t^R + (\sigma + \phi) \tilde{y}_t^W \right] + \beta E_t \pi_{Ht+1} + u_t,$$

where  $u_t = \delta \mu_t^W$ .

Similarly for foreign producer-price inflation, we have:

$$(B28) \quad \pi_{Ft}^* = \delta \left[ - \left( \frac{\sigma}{D} + \phi \right) \tilde{y}_t^R + (\sigma + \phi) \tilde{y}_t^W \right] + \beta E_t \pi_{Ft+1}^* + u_t^*.$$

### LCP

Equation (B26) holds in the LCP model as well. But in the LCP model, the law of one price deviation is not zero. We have:

$$(B29) \quad \pi_{Ht} = \delta \left[ \left( \frac{\sigma}{D} + \phi \right) \tilde{y}_t^R + (\sigma + \phi) \tilde{y}_t^W + \frac{D-(v-1)}{2D} m_t + \frac{2-v}{2} z_t \right] + \beta E_t \pi_{Ht+1} + u_t,$$

$$(B30) \quad \pi_{Ft}^* = \delta \left[ - \left( \frac{\sigma}{D} + \phi \right) \tilde{y}_t^R + (\sigma + \phi) \tilde{y}_t^W - \left( \frac{D-(v-1)}{2D} \right) m_t + \frac{2-v}{2} z_t \right] + \beta E_t \pi_{Ft+1}^* + u_t^*.$$

In addition, from (A42) and (A43), we derive:

$$(B31) \quad \pi_{Ht}^* = \delta(w_t - p_{Ht} - e_t - a_t) + \beta E_t \pi_{Ht+1}^* = \delta(w_t - p_{Ht} - m_t - z_t - a_t) + \beta E_t \pi_{Ht+1}^*$$

We can rewrite this as

$$(B32) \quad \pi_{Ht}^* = \delta \left[ \left( \frac{\sigma}{D} + \phi \right) \tilde{y}_t^R + (\sigma + \phi) \tilde{y}_t^W - \left( \frac{D+v-1}{2D} \right) m_t - \frac{v}{2} z_t \right] + \beta E_t \pi_{Ht+1}^* + u_t.$$

Similarly, we can derive:

$$(B33) \quad \pi_{Ft}^* = \delta \left[ - \left( \frac{\sigma}{D} + \phi \right) \tilde{y}_t^R + (\sigma + \phi) \tilde{y}_t^W + \frac{D+v-1}{2D} m_t - \frac{v}{2} z_t \right] + \beta E_t \pi_{Ft+1}^* + u_t^*.$$

Consider equations (B29)-(B33). If  $z_t = 0$  in these equations, then  $\pi_{Ft} - \pi_{Ht} = \pi_{Ft}^* - \pi_{Ht}^*$ . This in turn implies  $z_{t+1} = 0$ . By induction, if the initial condition  $z_0 = 0$  holds, it follows that  $z_t = 0$  in all periods in the LCP model, or, in other words,  $s_t^* = -s_t$ . That is, the relative price of Foreign to Home goods is the same in both countries. We emphasize that this is true for a first-order approximation in the LCP model.

So we can simplify equations (B29)-(B33):

$$(B34) \quad \pi_{Ht} = \delta \left[ \left( \frac{\sigma}{D} + \phi \right) \tilde{y}_t^R + (\sigma + \phi) \tilde{y}_t^W + \frac{D-(v-1)}{2D} m_t \right] + \beta E_t \pi_{Ht+1} + u_t,$$

$$(B35) \quad \pi_{Ft}^* = \delta \left[ - \left( \frac{\sigma}{D} + \phi \right) \tilde{y}_t^R + (\sigma + \phi) \tilde{y}_t^W - \left( \frac{D-(v-1)}{2D} \right) m_t \right] + \beta E_t \pi_{Ft+1}^* + u_t^*.$$

$$(B36) \quad \pi_{Ht}^* = \delta \left[ \left( \frac{\sigma}{D} + \phi \right) \tilde{y}_t^R + (\sigma + \phi) \tilde{y}_t^W - \left( \frac{D + \nu - 1}{2D} \right) m_t \right] + \beta E_t \pi_{Ht+1}^* + u_t.$$

$$(B37) \quad \pi_{Ft} = \delta \left[ - \left( \frac{\sigma}{D} + \phi \right) \tilde{y}_t^R + (\sigma + \phi) \tilde{y}_t^W + \frac{D + \nu - 1}{2D} m_t \right] + \beta E_t \pi_{Ft+1} + u_t^*.$$

## Appendix C

### C.I Derivation of Welfare Function in Clarida-Gali-Gertler model with Home Bias in preferences

The object is to rewrite the welfare function, which is defined in terms of home and foreign consumption and labor effort into terms of the squared output gap and squared inflation. We derive the joint welfare function of home and foreign households, since we will be examining cooperative monetary policy.

Most of the derivation requires only 1<sup>st</sup>-order approximations of the equations of the model, but in a few places, 2<sup>nd</sup>-order approximations are needed. If the approximation is 1<sup>st</sup>-order, I'll use the notation “ $+o(\|a^2\|)$ ” to indicate that there are 2<sup>nd</sup>-order and higher terms left out, and if the approximation is 2<sup>nd</sup>-order, I will use “ $+o(\|a^3\|)$ ”. ( $a$  is notation for the log of the productivity shock)

From equation (A1), the period utility of the planner is given by:

$$(C1) \quad v_t \equiv \frac{1}{1-\sigma} (C_t^{1-\sigma} + C_t^{*1-\sigma}) - \frac{1}{1+\phi} (N_t^{1+\phi} + N_t^{*1+\phi}).$$

Take a second-order log approximation around the non-stochastic steady state. We assume allocations are efficient in steady state, so we have  $C^{1-\sigma} = C^{*1-\sigma} = N^{1+\phi} = N^{*1+\phi}$ . The fact that  $C^{1-\sigma} = N^{1+\phi}$  follows from the fact that in steady state,  $C = N$  from market clearing and symmetry, and  $C^{-\sigma} = N^\phi$  from the condition that the marginal rate of substitution between leisure and consumption equals one in an efficient non-stochastic steady state.

We get:

$$(C2) \quad v_t = 2 \left( \frac{1}{1-\sigma} - \frac{1}{1+\phi} \right) C^{1-\sigma} + C^{1-\sigma} (c_t + c_t^*) + \frac{1-\sigma}{2} C^{1-\sigma} ((c_t)^2 + (c_t^*)^2) \\ - C^{1-\sigma} (n_t + n_t^*) - \frac{1+\phi}{2} C^{1-\sigma} ((n_t)^2 + (n_t^*)^2) + o(\|a^3\|)$$

Since we can equivalently maximize an affine transformation of (C2), it is convenient to simplify that equation to get:

$$(C3) \quad v_t = c_t + c_t^* - n_t - n_t^* + \frac{1-\sigma}{2} (c_t^2 + c_t^{*2}) - \frac{1+\phi}{2} (n_t^2 + n_t^{*2}) + o(\|a^3\|).$$

Utility is maximized when consumption and employment take on their efficient values:

$$(C4) \quad v_t^{\max} = \bar{c}_t + \bar{c}_t^* - \bar{n}_t - \bar{n}_t^* + \frac{1-\sigma}{2} (\bar{c}_t^2 + \bar{c}_t^{*2}) - \frac{1+\phi}{2} (\bar{n}_t^2 + \bar{n}_t^{*2}) + o(\|a^3\|).$$

In general, this maximum may not be attainable because of distortions. We can write  $x_t = \bar{x}_t + \tilde{x}_t$ , where  $\tilde{x}_t \equiv x_t - \bar{x}_t$ . So, we have:

$$(C5) \quad v_t = \left[ \bar{c}_t + \bar{c}_t^* - \bar{n}_t - \bar{n}_t^* + \frac{1-\sigma}{2} (\bar{c}_t^2 + \bar{c}_t^{*2}) - \frac{1+\phi}{2} (\bar{n}_t^2 + \bar{n}_t^{*2}) \right] \\ + \tilde{c}_t + \tilde{c}_t^* - \tilde{n}_t - \tilde{n}_t^* + \frac{1-\sigma}{2} (\tilde{c}_t^2 + \tilde{c}_t^{*2} + 2\bar{c}_t \tilde{c}_t + 2\bar{c}_t^* \tilde{c}_t^*) - \frac{1+\phi}{2} (\tilde{n}_t^2 + \tilde{n}_t^{*2} + 2\bar{n}_t \tilde{n}_t + 2\bar{n}_t^* \tilde{n}_t^*) \\ + o(\|a^3\|)$$

or,

$$(C6) \quad \begin{aligned} v_t - v_t^{\max} &= \tilde{c}_t + \tilde{c}_t^* - \tilde{n}_t - \tilde{n}_t^* + \frac{1-\sigma}{2}(\tilde{c}_t^2 + \tilde{c}_t^{*2}) - \frac{1+\phi}{2}(\tilde{n}_t^2 + \tilde{n}_t^{*2}) \\ &\quad + (1-\sigma)(\bar{c}_t \tilde{c}_t + \bar{c}_t^* \tilde{c}_t^*) - (1+\phi)(\bar{n}_t \tilde{n}_t + \bar{n}_t^* \tilde{n}_t^*) + o(\|a^3\|) \end{aligned}$$

We can rewrite this as:

$$(C7) \quad \begin{aligned} v_t - v_t^{\max} &= 2\tilde{c}_t^W - 2\tilde{n}_t^W + (1-\sigma)((\tilde{c}_t^R)^2 + (\tilde{c}_t^W)^2) - (1+\phi)((\tilde{n}_t^R)^2 + (\tilde{n}_t^W)^2) \\ &\quad + 2(1-\sigma)(\bar{c}_t^R \tilde{c}_t^R + \bar{c}_t^W \tilde{c}_t^W) - 2(1+\phi)(\bar{n}_t^R \tilde{n}_t^R + \bar{n}_t^W \tilde{n}_t^W) + o(\|a^3\|) \end{aligned}$$

The object is to write (C7) as a function of squared output gaps and squared inflation if possible. We need a second-order approximation of  $2\tilde{c}_t^W - 2\tilde{n}_t^W$ . But because the rest of the terms are squares and products, the 1<sup>st</sup>-order approximations that have already been derived will be sufficient.

Recalling that  $m_t = 0$  in the PCP model, we can write equations (B14)-(B15) as:

$$(C8) \quad c_t^R = \frac{\nu-1}{D} y_t^R + o(\|a^2\|),$$

$$(C9) \quad c_t^W = y_t^W + o(\|a^2\|).$$

It follows from (C8) and (C9) that:

$$(C10) \quad \bar{c}_t^R = \frac{\nu-1}{D} \bar{y}_t^R + o(\|a^2\|),$$

$$(C11) \quad \bar{c}_t^W = \bar{y}_t^W + o(\|a^2\|),$$

$$(C12) \quad \tilde{c}_t^R = \frac{\nu-1}{D} \tilde{y}_t^R + o(\|a^2\|),$$

$$(C13) \quad \tilde{c}_t^W = \tilde{y}_t^W + o(\|a^2\|).$$

Next, we can easily derive:

$$(C14) \quad \tilde{n}_t^R = \tilde{y}_t^R + o(\|a^2\|), \text{ and}$$

$$(C15) \quad \tilde{n}_t^W = \tilde{y}_t^W + o(\|a^2\|).$$

These follow as in (B3)-(B4) because  $n_t = y_t - a_t + o(\|a^2\|)$  and  $\bar{n}_t = \bar{y}_t - a_t$  (and similarly in the Foreign country.)

We need expressions for  $\bar{n}_t^R$  and  $\bar{n}_t^W$ . We have, using (B18)-(B19):

$$a_t^R = \left( \frac{\sigma}{D} + \phi \right) \bar{y}_t^R - \phi a_t^R + o(\|a^2\|),$$

$$a_t^W = (\sigma + \phi) \bar{y}_t^W - \phi a_t^W + o(\|a^2\|).$$

Using  $a_t^R = \bar{y}_t^R - \bar{n}_t^R$  and  $a_t^W = \bar{y}_t^W - \bar{n}_t^W$ , we can write these as

$$(C16) \quad \bar{n}_t^R = \frac{1-\sigma}{1+\phi} \left( \frac{(1-\nu)^2}{D} \right) \bar{y}_t^R + o(\|a^2\|), \text{ and}$$

$$(C17) \quad \bar{n}_t^W = \frac{1-\sigma}{1+\phi} \bar{y}_t^W + o(\|a^2\|).$$

Turning attention back to the loss function in equation (C7), we focus first on the terms  $(1-\sigma)((\tilde{c}_t^R)^2 + (\tilde{c}_t^W)^2) - (1+\phi)((\tilde{n}_t^R)^2 + (\tilde{n}_t^W)^2) + 2(1-\sigma)(\bar{c}_t^R \tilde{c}_t^R + \bar{c}_t^W \tilde{c}_t^W) - 2(1+\phi)(\bar{n}_t^R \tilde{n}_t^R + \bar{n}_t^W \tilde{n}_t^W)$ .

These involve only squares and cross-products of  $\tilde{c}_t^R, \tilde{c}_t^W, \bar{c}_t^R, \bar{c}_t^W, \tilde{n}_t^R, \tilde{n}_t^W, \bar{n}_t^R,$  and  $\bar{n}_t^W$ . We can substitute from equations (C10)-(C17) into this expression. It is useful provide a few lines of algebra since it is a bit messy:

$$(C18) \quad (1-\sigma)\left((\tilde{c}_t^R)^2 + (\tilde{c}_t^W)^2\right) - (1+\phi)\left((\tilde{n}_t^R)^2 + (\tilde{n}_t^W)^2\right) \\ + 2(1-\sigma)\left(\bar{c}_t^R \tilde{c}_t^R + \bar{c}_t^W \tilde{c}_t^W\right) - 2(1+\phi)\left(\bar{n}_t^R \tilde{n}_t^R + \bar{n}_t^W \tilde{n}_t^W\right) \\ = \left[ (1-\sigma)\left(\frac{\nu-1}{D}\right)^2 - (1+\phi) \right] (\tilde{y}_t^R)^2 - (\sigma+\phi)(\tilde{y}_t^W)^2 + 2\nu(2-\nu)\left(\frac{(1-\sigma)(\nu-1)}{D}\right)^2 \bar{y}_t^R \tilde{y}_t^R$$

Now return to the  $2\tilde{c}_t^W - 2\tilde{n}_t^W$  term in equation (C7) and do a 2<sup>nd</sup>-order approximation. Start with equation (A18), dropping the  $k^{-1}$  term because it will not affect the approximation, and noting that in the PCP model,  $S_t^* = S_t^{-1}$ :

$$(C19) \quad Y_t = \frac{\nu}{2} S_t^{(2-\nu)/2} C_t + \left(\frac{2-\nu}{2}\right) S_t^{\nu/2} C_t^*$$

Then use equation (A22), but using the fact that  $S_t^* = S_t^{-1}$  and there are no deviations from the law of one price:

$$(C20) \quad C_t^* = C_t S_t^{\frac{1-\nu}{\sigma}}.$$

Substitute in to get:

$$(C21) \quad Y_t = \frac{\nu}{2} S_t^{\frac{2-\nu}{2}} C_t + \left(\frac{2-\nu}{2}\right) S_t^{\frac{\nu}{2} + \frac{1-\nu}{\sigma}} C_t.$$

Solve for  $C_t$ :

$$(C22) \quad C_t = Y_t \left( \frac{\nu}{2} S_t^{\frac{2-\nu}{2}} + \left(\frac{2-\nu}{2}\right) S_t^{\frac{\nu}{2} + \frac{1-\nu}{\sigma}} \right)^{-1}.$$

Then we get this 2<sup>nd</sup>-order approximation:

$$(C23) \quad c_t = y_t - \left(\frac{2-\nu}{2}\right) \left(\nu + \frac{1-\nu}{\sigma}\right) s_t - \frac{1}{2} \left(\frac{2-\nu}{2}\right) \frac{\nu}{2} (\nu-1)^2 \left(\frac{\sigma-1}{\sigma}\right)^2 s_t^2 + o(\|a^3\|).$$

Symmetrically,

$$(C24) \quad c_t^* = y_t^* + \left(\frac{2-\nu}{2}\right) \left(\nu + \frac{1-\nu}{\sigma}\right) s_t - \frac{1}{2} \left(\frac{2-\nu}{2}\right) \frac{\nu}{2} (\nu-1)^2 \left(\frac{\sigma-1}{\sigma}\right)^2 s_t^2 + o(\|a^3\|).$$

Averaging these two equations, we get:

$$(C25) \quad c_t^W = y_t^W - \frac{1}{2} \left(\frac{2-\nu}{2}\right) \frac{\nu}{2} (\nu-1)^2 \left(\frac{\sigma-1}{\sigma}\right)^2 s_t^2 + o(\|a^3\|).$$

Now we can take a 1<sup>st</sup>-order approximation for  $s_t$  to substitute out for  $s_t^2$ . From equation (B16), setting  $m_t = 0$  and  $z_t = 0$ , we have:

$$(C26) \quad s_t^2 = \frac{4\sigma^2}{D^2} (y_t^R)^2 + o(\|a^3\|).$$

Substituting into equation (C26), we can write:

$$(C27) \quad c_t^W = y_t^W - \frac{\nu(2-\nu)}{2} \left(\frac{(\nu-1)(\sigma-1)}{D}\right)^2 (y_t^R)^2 + o(\|a^3\|).$$

Evaluating (C27) at flexible prices, we have:

$$(C28) \quad \bar{c}_t^W = \bar{y}_t^W - \frac{\nu(2-\nu)}{2} \left( \frac{(\nu-1)(\sigma-1)}{D} \right)^2 (\bar{y}_t^R)^2 + o(\|a^3\|).$$

It follows from the fact that  $\tilde{c}_t^W = c_t^W - \bar{c}_t^W$  that

$$(C29) \quad \tilde{c}_t^W = \tilde{y}_t^W - \frac{\nu(2-\nu)}{2} \left( \frac{(\nu-1)(\sigma-1)}{D} \right)^2 ((\tilde{y}_t^R)^2 + 2\bar{y}_t^R \tilde{y}_t^R) + o(\|a^3\|).$$

See section C.3 below for the second-order approximations for  $\tilde{n}_t$  and  $\tilde{n}_t^*$ :

$$(C30) \quad \tilde{n}_t = \tilde{y}_t + \frac{\xi}{2} \sigma_{p_{H^t}}^2 + o(\|a^3\|)$$

$$(C31) \quad \tilde{n}_t^* = \tilde{y}_t^* + \frac{\xi}{2} \sigma_{p_{F^*t}}^2 + o(\|a^3\|).$$

Adding these two equations together gives us:

$$(C32) \quad 2\tilde{n}_t^W = 2\tilde{y}_t^W + \frac{\xi}{2} \sigma_{p_{H^t}}^2 + \frac{\xi}{2} \sigma_{p_{F^*t}}^2 + o(\|a^3\|)$$

Substitute expressions (C29)-(C31) along with (C18) into the loss function (C7):

$$(C33) \quad \begin{aligned} v_t - v_t^{\max} &= -\nu(2-\nu) \left( \frac{(\nu-1)(\sigma-1)}{D} \right)^2 ((\tilde{y}_t^R)^2 + 2\bar{y}_t^R \tilde{y}_t^R) - \frac{\xi}{2} (\sigma_{p_{H^t}}^2 + \sigma_{p_{F^*t}}^2) \\ &+ \left[ (1-\sigma) \left( \frac{\nu-1}{D} \right)^2 - (1+\phi) \right] (\tilde{y}_t^R)^2 - (\sigma+\phi) (\tilde{y}_t^W)^2 + 2\nu(2-\nu) \left( \frac{(1-\sigma)(\nu-1)}{D} \right)^2 \bar{y}_t^R \tilde{y}_t^R \\ &= - \left[ \frac{\sigma}{D} + \phi \right] (\tilde{y}_t^R)^2 - (\sigma+\phi) (\tilde{y}_t^W)^2 - \frac{\xi}{2} (\sigma_{p_{H^t}}^2 + \sigma_{p_{F^*t}}^2) \end{aligned}$$

This expression reduces to CGG's when there is no home bias ( $\nu = 1$ ). To see this from their expression at the top of p. 903, multiply their utility by 2 (since they take average utility), and set their  $\gamma$  equal to  $1/2$  (so their country sizes are equal.)

## C.2 Derivation of Welfare Function under LCP with Home Bias in Preferences

The second-order approximation to welfare in terms of logs of consumption and employment of course does not change, so equation (C6) still holds. As before, we break down the derivation into two parts. We use first-order approximations to structural equations to derive an approximation to the quadratic term

$$(1-\sigma) \left( (\tilde{c}_t^R)^2 + (\tilde{c}_t^W)^2 \right) - (1+\phi) \left( (\tilde{n}_t^R)^2 + (\tilde{n}_t^W)^2 \right) + 2(1-\sigma) \left( \bar{c}_t^R \tilde{c}_t^R + \bar{c}_t^W \tilde{c}_t^W \right) - 2(1+\phi) \left( \bar{n}_t^R \tilde{n}_t^R + \bar{n}_t^W \tilde{n}_t^W \right).$$

Then we use second order approximations to the structural equations to derive an expression for  $2\tilde{c}_t^W - 2\tilde{n}_t^W$ .

The quadratic term involves squares and cross-products of  $\tilde{c}_t^R$ ,  $\tilde{c}_t^W$ ,  $\bar{c}_t^R$ ,  $\bar{c}_t^W$ ,  $\tilde{n}_t^R$ ,  $\tilde{n}_t^W$ ,  $\bar{n}_t^R$ , and  $\bar{n}_t^W$ . Expressions (C10)-(C11) still gives us first-order approximations for  $\bar{c}_t^R$  and  $\bar{c}_t^W$ ; equations (C14)-(C15) are first-order approximations for  $\tilde{n}_t^R$  and  $\tilde{n}_t^W$ ; and, (C16)-(C17) are first-order approximations for  $\bar{n}_t^R$  and  $\bar{n}_t^W$ . But we need to use equations (B14)-(B15) and (C10)-(C11) to derive:

$$(C34) \quad \tilde{c}_t^R = \frac{\nu-1}{D} \tilde{y}_t^R + \frac{\nu(2-\nu)}{2D} m_t + o(\|a^2\|),$$

$$(C35) \quad \tilde{c}_t^W = \tilde{y}_t^W + o(\|a^2\|).$$

With these equations, we can follow the derivation as in equation (C18). After tedious algebra, we arrive at the same result, with the addition of the terms  $\frac{(1-\sigma)\nu^2(2-\nu)^2}{4D^2}m_t^2$  and  $\frac{(1-\sigma)\nu(2-\nu)(\nu-1)}{D^2}m_t y_t^R$ . Note that the last term involves output levels, not output gaps. That is, we have:

$$\begin{aligned}
& (1-\sigma)\left((\tilde{c}_t^R)^2 + (\tilde{c}_t^W)^2\right) - (1+\phi)\left((\tilde{n}_t^R)^2 + (\tilde{n}_t^W)^2\right) \\
& + 2(1-\sigma)\left(\bar{c}_t^R \tilde{c}_t^R + \bar{c}_t^W \tilde{c}_t^W\right) - 2(1+\phi)\left(\bar{n}_t^R \tilde{n}_t^R + \bar{n}_t^W \tilde{n}_t^W\right) \\
\text{(C36)} \quad & = \left[ (1-\sigma)\left(\frac{\nu-1}{D}\right)^2 - (1+\phi) \right] (\tilde{y}_t^R)^2 - (\sigma+\phi)(\tilde{y}_t^W)^2 + 2\nu(2-\nu)\left(\frac{(1-\sigma)(\nu-1)}{D}\right)^2 \bar{y}_t^R \tilde{y}_t^R \\
& + \frac{(1-\sigma)\nu^2(2-\nu)^2}{4D^2}m_t^2 + \frac{(1-\sigma)\nu(2-\nu)(\nu-1)}{D^2}m_t y_t^R
\end{aligned}$$

The derivation of  $2\tilde{c}_t^W - 2\tilde{n}_t^W$  is similar to the PCP model. However, one tedious aspect of the derivation is that we cannot make use of the equality  $S_t^* = S_t^{-1}$  that holds under PCP and flexible prices. We write out the equilibrium conditions for home output, and its foreign equivalent, from equations (A18) and (A19):

$$\text{(C37)} \quad Y_t = \frac{\nu}{2}S_t^{1-(\nu/2)}C_t + \left(1 - \frac{\nu}{2}\right)(S_t^*)^{-\nu/2}C_t^*,$$

$$\text{(C38)} \quad Y_t^* = \frac{\nu}{2}(S_t^*)^{1-(\nu/2)}C_t^* + \left(1 - \frac{\nu}{2}\right)S_t^{-\nu/2}C_t.$$

We directly take second-order approximations of these equations around the efficient non-stochastic steady state:

$$\begin{aligned}
& y_t + \frac{1}{2}y_t^2 = \frac{\nu}{2}c_t + \left(\frac{2-\nu}{2}\right)c_t^* + \frac{\nu}{2}\left(\frac{2-\nu}{2}\right)s_t - \frac{\nu}{2}\left(\frac{2-\nu}{2}\right)s_t^* \\
\text{(C39)} \quad & + \frac{1}{2}\left\{ \frac{\nu}{2}c_t^2 + \left(\frac{2-\nu}{2}\right)c_t^{*2} + \frac{\nu}{2}\left(\frac{2-\nu}{2}\right)^2 s_t^2 + \left(\frac{\nu}{2}\right)^2 \left(\frac{2-\nu}{2}\right)s_t^{*2} \right\} \\
& + \frac{\nu}{2}\left(\frac{2-\nu}{2}\right)s_t c_t - \frac{\nu}{2}\left(\frac{2-\nu}{2}\right)s_t^* c_t^* + o(\|a^3\|) \\
& y_t^* + \frac{1}{2}y_t^{*2} = \frac{\nu}{2}c_t^* + \left(\frac{2-\nu}{2}\right)c_t + \frac{\nu}{2}\left(\frac{2-\nu}{2}\right)s_t^* - \frac{\nu}{2}\left(\frac{2-\nu}{2}\right)s_t \\
\text{(C40)} \quad & + \frac{1}{2}\left\{ \frac{\nu}{2}c_t^{*2} + \left(\frac{2-\nu}{2}\right)c_t^2 + \frac{\nu}{2}\left(\frac{2-\nu}{2}\right)^2 s_t^{*2} + \left(\frac{\nu}{2}\right)^2 \left(\frac{2-\nu}{2}\right)s_t^2 \right\} \\
& + \frac{\nu}{2}\left(\frac{2-\nu}{2}\right)s_t^* c_t^* - \frac{\nu}{2}\left(\frac{2-\nu}{2}\right)s_t c_t + o(\|a^3\|)
\end{aligned}$$

Averaging (C39) and (C40), we find:

$$\text{(C41)} \quad y_t^W + \frac{1}{2}\left((y_t^R)^2 + (y_t^W)^2\right) = c_t^W + \frac{1}{2}\left((c_t^R)^2 + (c_t^W)^2\right) + \frac{\nu(2-\nu)}{16}(s_t^2 + s_t^{*2}) + o(\|a^3\|).$$

Next, we can use equations (B14)-(B17) to get approximations for  $(c_t^R)^2$ ,  $(c_t^W)^2$ ,  $s_t^2$ , and  $s_t^{*2}$ . These equations are linear approximations for  $c_t^R$ ,  $c_t^W$ ,  $s_t$ , and  $s_t^*$ , but since we are looking to approximate the squares of these variables, that is sufficient. With some algebra, we find:

$$(C42) \quad c_t^W = y_t^W - \frac{\nu(2-\nu)}{2} \left( \frac{(\nu-1)(1-\sigma)}{D} \right)^2 (y_t^R)^2 - \frac{\nu(2-\nu)}{8D^2} m_t^2 - \frac{(1-\sigma)\nu(2-\nu)(\nu-1)}{2D^2} m_t y_t^R - \frac{\nu(2-\nu)}{8} z_t^2 + o(\|a^3\|)$$

Note that if set  $m_t = 0$  and  $z_t = 0$  in (C42), we would arrive at the second-order approximation for  $c_t^W$  from the PCP model.

Then following the derivations as in the PCP model derivation of (C29), we can write:

$$(C43) \quad \tilde{c}_t^W = \tilde{y}_t^W - \frac{\nu(2-\nu)}{2} \left( \frac{(\nu-1)(\sigma-1)}{D} \right)^2 \left( (\tilde{y}_t^R)^2 + 2\bar{y}_t^R \tilde{y}_t^R \right) - \frac{\nu(2-\nu)}{8D^2} m_t^2 - \frac{(1-\sigma)\nu(2-\nu)(\nu-1)}{2D^2} m_t y_t^R - \frac{\nu(2-\nu)}{8} z_t^2 + o(\|a^3\|)$$

As shown in section B.3, we can make the following second-order approximation:

$$(C44) \quad 2\tilde{n}_t^W = 2\tilde{y}_t^W + \frac{\xi}{2} \left[ \frac{\nu}{2} \sigma_{p_{Ht}}^2 + \frac{2-\nu}{2} \sigma_{p_{H^*t}}^2 + \frac{\nu}{2} \sigma_{p_{F^*t}}^2 + \frac{2-\nu}{2} \sigma_{p_{Ft}}^2 \right] + o(\|a^3\|).$$

We then can substitute (C43), and (C44), along with (C36) into the loss function (C7).

We find:

$$(C45) \quad \begin{aligned} v_t - v_t^{\max} &= 2\tilde{y}_t^W - \nu(2-\nu) \left( \frac{(\nu-1)(\sigma-1)^2}{D} \right) \left( (\tilde{y}_t^R)^2 + 2\bar{y}_t^R \tilde{y}_t^R \right) - \frac{\nu(2-\nu)}{4D} m_t^2 \\ &\quad - \frac{(1-\sigma)\nu(2-\nu)(\nu-1)}{D^2} m_t y_t^R - \frac{\nu(2-\nu)}{4} z_t^2 \\ &\quad - 2\tilde{y}_t^W - \frac{\xi}{2} \left[ \frac{\nu}{2} \sigma_{p_{Ht}}^2 + \frac{2-\nu}{2} \sigma_{p_{H^*t}}^2 + \frac{\nu}{2} \sigma_{p_{F^*t}}^2 + \frac{2-\nu}{2} \sigma_{p_{Ft}}^2 \right] \\ &\quad + \left[ (1-\sigma) \left( \frac{\nu-1}{D} \right)^2 - (1+\phi) \right] (\tilde{y}_t^R)^2 - (\sigma+\phi) (\tilde{y}_t^W)^2 + 2\nu(2-\nu) \left( \frac{(1-\sigma)(\nu-1)}{D} \right)^2 \bar{y}_t^R \tilde{y}_t^R \\ &\quad + \frac{(1-\sigma)\nu^2(2-\nu)^2}{4D^2} m_t^2 + \frac{(1-\sigma)\nu(2-\nu)(\nu-1)}{D^2} m_t y_t^R \\ &= - \left[ \frac{\sigma}{D} + \phi \right] (\tilde{y}_t^R)^2 - (\sigma+\phi) (\tilde{y}_t^W)^2 - \frac{\nu(2-\nu)}{4D} m_t^2 - \frac{\nu(2-\nu)}{4} z_t^2 \\ &\quad - \frac{\xi}{2} \left[ \frac{\nu}{2} \sigma_{p_{Ht}}^2 + \frac{2-\nu}{2} \sigma_{p_{H^*t}}^2 + \frac{\nu}{2} \sigma_{p_{F^*t}}^2 + \frac{2-\nu}{2} \sigma_{p_{Ft}}^2 \right] + o(\|a^3\|) \end{aligned}$$

### C.3 Derivations of Price Dispersion Terms in Loss Functions

In the PCP case, we can write

$$(C46) \quad A_t N_t = A_t \int_0^1 N_t(f) df = Y_t \int_0^1 \left( \frac{P_{Ht}(f)}{P_{Ht}} \right)^{-\xi} df = Y_t V_t,$$

where  $V_t \equiv \int_0^1 \left( \frac{P_{Ht}(f)}{P_{Ht}} \right)^{-\xi} df$ . Taking logs, we can write:

$$(C47) \quad a_t + n_t = y_t + v_t.$$

We have  $v_t \equiv \ln \left( \int_0^1 e^{-\xi \hat{p}_m(f)} df \right)$ ,

where we define

$$(C48) \quad \hat{p}_{H_t}(f) \equiv p_{H_t}(f) - p_{H_t}.$$

Following Gali (2008), we note

$$(C49) \quad e^{(1-\xi)\hat{p}_{H_t}(f)} = 1 + (1-\xi)\hat{p}_{H_t}(f) + \frac{(1-\xi)^2}{2}\hat{p}_{H_t}(f)^2 + o(\|a^3\|).$$

By the definition of the price index  $P_{H_t}$ , we have  $\int_0^1 e^{(1-\xi)\hat{p}_{H_t}(f)} df = 1$ . Hence, from (C49),

$$(C50) \quad \int_0^1 \hat{p}_{H_t}(f) df = \frac{\xi-1}{2} \int_0^1 \hat{p}_{H_t}(f)^2 df + o(\|a^3\|).$$

We also have

$$(C51) \quad e^{-\xi\hat{p}_{H_t}(f)} = 1 - \xi\hat{p}_{H_t}(f) + \frac{\xi^2}{2}\hat{p}_{H_t}(f)^2 + o(\|a^3\|)$$

It follows, using (C50):

$$(C52) \quad \int_0^1 e^{-\xi\hat{p}_{H_t}(f)} df = 1 - \xi \int_0^1 \hat{p}_{H_t}(f) df + \frac{\xi^2}{2} \int_0^1 \hat{p}_{H_t}(f)^2 df + o(\|a^3\|) = 1 + \frac{\xi}{2} \int_0^1 \hat{p}_{H_t}(f)^2 df + o(\|a^3\|)$$

Note the following relationship:

$$(C53) \quad \int_0^1 \hat{p}_{H_t}(f)^2 df = \int_0^1 (p_{H_t}(f) - E_f(p_{H_t}(f)))^2 df + o(\|a^3\|) = \text{var}(p_{H_t}) + o(\|a^3\|).$$

Using our notation for variances,  $\sigma_{p_{H_t}}^2 \equiv \text{var}(p_{H_t})$ , and taking the log of (C52) we arrive at

$$(C54) \quad v_t = \frac{\xi}{2} \sigma_{p_{H_t}}^2 + o(\|a^3\|).$$

Substituting this into equation (C47), and recalling that  $\bar{y}_t = \bar{n}_t + a_t$ , we arrive at equation (C30).

The derivation of (C31) for the Foreign country proceeds identically.

For the LCP model, we will make use of the following second-order approximation to the equation  $Y_t = C_{H_t} + C_{H_t}^*$ :

$$(C55) \quad y_t = \frac{\nu}{2} c_{H_t} + \left(\frac{2-\nu}{2}\right) c_{H_t}^* + \frac{1}{2} \left(\frac{\nu}{2}\right) \left(\frac{2-\nu}{2}\right) (c_{H_t}^2 + 2c_{H_t} c_{H_t}^* + c_{H_t}^{*2}) + o(\|a^3\|).$$

In the LCP model, we can write:

$$(C56) \quad A_t N_t = A_t \int_0^1 N_t(f) df = C_{H_t} \int_0^1 \left(\frac{P_{H_t}(f)}{P_{H_t}}\right)^{-\xi} df + C_{H_t}^* \int_0^1 \left(\frac{P_{H_t}^*(f)}{P_{H_t}^*}\right)^{-\xi} df = C_{H_t} V_{H_t} + C_{H_t}^* V_{H_t}^*,$$

where the definitions of  $V_{H_t}$  and  $V_{H_t}^*$  are analogous to that of  $V_t$  in the PCP model. Taking a second-order log approximation to (C56), we have:

$$(C57) \quad \begin{aligned} a_t + n_t &= \frac{\nu}{2} (c_{H_t} + v_{H_t}) + \left(\frac{2-\nu}{2}\right) (c_{H_t}^* + v_{H_t}^*) \\ &+ \frac{1}{2} \left(\frac{\nu}{2}\right) \left(\frac{2-\nu}{2}\right) ((c_{H_t} + v_{H_t})^2 + 2(c_{H_t} + v_{H_t})(c_{H_t}^* + v_{H_t}^*) + (c_{H_t}^* + v_{H_t}^*)^2) + o(\|a^3\|) \end{aligned}$$

We can follow the same steps as in the PCP model to conclude:

$$(C58) \quad v_{H_t} = \frac{\xi}{2} \sigma_{p_{H_t}}^2 + o(\|a^3\|)$$

$$(C59) \quad v_{H_t}^* = \frac{\xi}{2} \sigma_{p_{H_t}^*}^2 + o(\|a^3\|)$$

Substituting these expressions into (C57) and cancelling higher order terms, we find:

(C60)

$$a_t + n_t = \frac{\nu}{2}(c_{Ht} + \frac{\xi}{2}\sigma_{p_{Ht}}^2) + \left(\frac{2-\nu}{2}\right)(c_{Ht}^* + \frac{\xi}{2}\sigma_{p_{H^*t}}^2) + \frac{1}{2}\left(\frac{\nu}{2}\right)\left(\frac{2-\nu}{2}\right)(c_{Ht}^2 + 2c_{Ht}c_{Ht}^* + c_{Ht}^{*2}) + o(\|a^3\|)$$

Then using equation (C55), we can write:

$$(C61) \quad a_t + n_t = y_t + \frac{\xi}{2}\left(\frac{\nu}{2}\sigma_{p_{Ht}}^2 + \left(\frac{2-\nu}{2}\right)\frac{\xi}{2}\sigma_{p_{H^*t}}^2\right) + o(\|a^3\|).$$

Keeping in mind that  $\bar{y}_t = \bar{n}_t + a_t$ , we can write:

$$(C62) \quad \tilde{n}_t = \tilde{y}_t + \frac{\xi}{2}\left(\frac{\nu}{2}\sigma_{p_{Ht}}^2 + \left(\frac{2-\nu}{2}\right)\frac{\xi}{2}\sigma_{p_{H^*t}}^2\right) + o(\|a^3\|).$$

Following analogous steps for the Foreign country,

$$(C63) \quad \tilde{n}_t^* = \tilde{y}_t^* + \frac{\xi}{2}\left(\frac{\nu}{2}\sigma_{p_{F^*t}}^2 + \left(\frac{2-\nu}{2}\right)\frac{\xi}{2}\sigma_{p_{Ft}}^2\right) + o(\|a^3\|).$$

Adding (C62) and (C63) gives us equation (C44).

Finally, to derive the loss functions for policymakers, we note that the loss function is the present expected discounted value of the period loss functions derived here (equation (C33) for the PCP model and (C45) for the LCP model.) That is, the policymaker seeks to minimize

$$-E_t \sum_{j=0}^{\infty} \beta^j (u_{t+j} - u_{t+j}^{\max}).$$

Following Woodford (2003, chapter 6), we can see that, in the PCP model, if prices are adjusted according to the Calvo price mechanism given by equation (A39) for  $P_{Ht}$  that

$$(C64) \quad \sum_{j=0}^{\infty} \beta^j \sigma_{p_{Ht+j}}^2 = \frac{\theta}{(1-\beta\theta)(1-\theta)} \sum_{j=0}^{\infty} \beta^j \pi_{Ht+j}^2.$$

Analogous relationships hold for  $P_{Ft}^*$  in the PCP model, and for  $P_{Ht}$ ,  $P_{Ft}^*$ ,  $P_{Ft}$ , and  $P_{Ht}^*$  in the LCP model. We can then substitute this relationship into the present value loss function,

$$-E_t \sum_{j=0}^{\infty} \beta^j (u_{t+j} - u_{t+j}^{\max}),$$

to derive the loss functions of the two models presented in the text.

The text writes the loss function as:

$$(C65) \quad \Psi_t \propto \left[ \frac{\sigma}{D} + \phi \right] (\tilde{y}_t^R)^2 + (\sigma + \phi) (\tilde{y}_t^W)^2 + \frac{\nu(2-\nu)}{4D} m_t^2 + \frac{\xi}{2\delta} \left( \frac{\nu}{2} (\pi_{Ht})^2 + \frac{2-\nu}{2} (\pi_{Ft})^2 + \frac{\nu}{2} (\pi_{Ft}^*)^2 + \frac{2-\nu}{2} (\pi_{Ht}^*)^2 \right)$$

We can use the fact that if  $\frac{\nu}{2}\pi_{Ht} + \frac{2-\nu}{2}\pi_{Ft} = \pi_t$ , then

$$\frac{\nu}{2}(\pi_{Ht})^2 + \frac{2-\nu}{2}(\pi_{Ft})^2 = \pi_t^2 + \frac{\nu}{2}\left(\frac{2-\nu}{2}\right)(\pi_{Ft} - \pi_{Ht})^2.$$

This follows because for any  $a$ ,  $x$ , and  $y$ , if  $ax + (1-a)y = z$ , then  $ax^2 + (1-a)y^2 = z^2 + a(1-a)(x-y)^2$ .

Likewise,

$$\frac{\nu}{2}(\pi_{Ft}^*)^2 + \frac{2-\nu}{2}(\pi_{Ht}^*)^2 = (\pi_t^*)^2 + \frac{\nu}{2}\left(\frac{2-\nu}{2}\right)(\pi_{Ht}^* - \pi_{Ft}^*)^2.$$

It follows that

$$\begin{aligned}
& \frac{\nu}{2}(\pi_{Ht})^2 + \frac{2-\nu}{2}(\pi_{Ft})^2 + \frac{\nu}{2}(\pi_{Ft}^*)^2 + \frac{2-\nu}{2}(\pi_{Ht}^*)^2 \\
&= \pi_t^2 + \frac{\nu}{2}\left(\frac{2-\nu}{2}\right)(\pi_{Ft} - \pi_{Ht})^2 + (\pi_t^*)^2 + \frac{\nu}{2}\left(\frac{2-\nu}{2}\right)(\pi_{Ht}^* - \pi_{Ft}^*)^2 \\
\text{(C66)} \quad &= \pi_t^2 + (\pi_t^*)^2 + \left(\frac{\nu(2-\nu)}{2}\right)(s_t - s_{t-1}) \\
&= 2(\pi_t^R)^2 + 2(\pi_t^W)^2 + \left(\frac{\nu(2-\nu)}{2}\right)(s_t - s_{t-1})
\end{aligned}$$

The second equality follows because, under Calvo pricing, to a first order  $\pi_{Ft} - \pi_{Ht} = \pi_{Ft}^* - \pi_{Ht}^* = s_t - s_{t-1}$ . The third equality follows because  $(\pi_t^R)^2 + (\pi_t^W)^2 = \frac{(\pi_t - \pi_t^*)^2 + (\pi_t - \pi_t^*)^2}{4} = \frac{\pi_t^2 + (\pi_t^*)^2}{2}$ . Then substitute (C66) into (C65) to get the simplified loss function presented in the paper.