New product introduction and capacity investment by incumbents: effects of size on strategy

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September 2009

Abstract

The paper addresses the question how production capacities on an established market influence the innovativeness of firms. We analyze the strategic interactions in an oligopoly setting where firms have the option to introduce a new product in addition to the established one.

We show that the firm with a smaller capacity on the established market has a higher incentive to innovate and reaches a larger market share on the market for the new product. Furthermore, changing capacities on the established market has qualitatively different impacts on the smaller and the larger firm. The larger firm can use capacity expansion as an instrument to prevent its competitor from innovating, whereas the smaller firm cannot prevent innovation of its competitor in this way. Overall, total payoffs depend non-monotonously on the capacities on the established market and the firm with smaller capacity on the established market might outperform the larger firm with respect to total payoffs.

1 Introduction

In 2007 PC maker Asus introduced the EEE PC, a low-priced mini-notebook (so-called netbook) with reduced weight and performance. At this point Asus had a market share of only about 4% on the notebook market. Other PC makers followed suit, and in 2008 almost all major players in the notebook market were offering netbooks. Currently, Asus still has a market share of about 30% on the netbook market, whereas the leaders on the notebook markets are rather minor

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players there: for example, in 2008 HP had 20% on the notebook market and 6 % on the netbook market. For Dell these figures are 15% and 3% respectively[1].

A recent innovation in the banking sector, which spread faster in developing countries than in Europe and the United States is mobile phone payment. From a technological perspective banks have the option to introduce payment by mobile phone as a new product competing with well established means like credit cards. Mobile phone payment would be cheaper for the customer, but the banks would loose revenue from their credit card portfolio. The Economist (February 17th, 2007, pp. 67-70) argues that "some of the smaller banks, which do not have payment-card portfolios to protect, might have the most to gain from offering customers a way to use their mobile phones to pay for items directly from their accounts", and, consequently, the higher incentives to add this new product to their product portfolio.

In a similar fashion (The Economist, April 1st, 2006, pp.55-56), Microsoft, currently producing fabulously profitable products like Windows and Office, has been slow in introducing the online components "Windows Live" and "Office Live". Although this could undermine the market position of Windows and Office, Microsoft now still goes ahead with this to answer competitive threats resulting from online applications produced by firms like Salesforce.com who only have minor stakes in the standard software market.

The common features of these examples are that (i) new product variants are added to an established oligopolistic market, (ii) the new product is offered by incumbents, (iii) market leaders on the new market have minor market shares on the established market, and (iv) the new product is not replacing the old product, but is offered in addition. The observation that on many occasions new products are introduced by incumbents is reinforced by empirical studies like Chandy and Tellis (2000). Their paper also highlights the negative relationship between firm size and the propensity to introduce (radically) new products. Similar to other influential contributions (see e.g. Christensen (1997)) the authors explain this by the bureaucratic inertia and organizational structure in large firms. Although these aspects are certainly valid, this paper points out that in markets characterized by the four features described above, pure strategic considerations induce a similar relationship between firm size, innovativeness and new capacity investments. In that respect we address the following research questions:

1. What is the effect of the size of a firm's production capacity on the established market on its incentive to enter the new market?

2. What is the effect of the size of the competitors' production capacity on the established market on the firm's incentive to enter the new market?

3. If a firm enters the new market, how is the optimal production capac-

[1] The mentioned market share numbers can be found at http://www.guardian.co.uk/technology/blog/2008/dec/10/acer-asus-netbooks (netbooks) and http://community.winsupersite.com/blogs/paul/archive/2008/06/14/notebook-computer-market-share-q1-2008.aspx (notebooks).

ity on the new market influenced by capacity levels of the firm and its competitors on the established market?

4. Does a higher capacity on the established market always lead to relative advantages with respect to profit?

In order to address these questions we consider a stylized duopolistic setup. Both firms are operating in an established homogenous product market on which an 'old' product is sold. The firms have different production capacities so that we can distinguish between a small and a large firm. They both have the opportunity to introduce a differentiated 'new' product, which competes with the old product. A firm that enters the new market does not only provide extra competition for the rival but also cannibalizes its own sales of the old product.

The considered sequence of actions is that first, both firms have to decide whether or not to launch the new product, which would require incurring sunk costs. In case either one of them has decided to introduce the new product, a heterogenous product market arises. The firms that enter the new market then have to determine the production capacity of the new product. Finally both firms have to fix the price of all goods they are offering. Both firms maximize payoffs consisting of profits on both markets net of possible sunk costs.

We focus on the influence of capacities on the established market on the outcome of strategic firm interaction. It turns out that the interplay of four mechanisms drives the effects of old capacities on the behavior of rational firms in such a setting: *the cannibalization effect, the size effect*, *the strategic effect* and *the indirect effect.* The *cannibalization effect* results from the observation that a firm with a larger capacity on the old market is more reluctant to reduce the price of the old product. This lowers the incentive to introduce the new product and to increase capacity on the new market. Furthermore, the *size effect* refers to the fact that if the capacities of a firm on the established market goes up, the price level for the new product goes down. Therefore, the marginal returns of building capacity on the new market decreases which makes the new market less attractive for the firm. The *strategic effect* captures that if a firm's capacity for the old product increases and this firm does not enter the new market, then this induces a smaller competitor's capacity for the new product. This negative effect on the competitor's capacity on the new market does not arise if the firm enters the new market. In such a case the increase in old market capacity is counterbalanced by the induced reduction of the new market capacity of the same firm in such a way that the capacity investment incentives of the competitor stay unchanged. Therefore, according to the strategic effect large capacities on the old market have a negative effect on incentives to enter the new market. Finally, the impact of on increase of the competitor's old market capacity on the incentives of a firm to enter the new market is also influenced by *the indirect effect.* If the old market capacities of the competitor go up this reduces his investment in capacities on the new market, which positively affects the incentives of the focal firm to innovate.

Considering the interplay of these effects we demonstrate that incumbents with a high capacity on the established market have lower incentives to introduce a new product. And, if they decide to enter the new market, they invest less in corresponding production capacities compared to smaller competitors. Consequently, profits from producing the new product will be higher for the small firm, and this could even result in higher overall profits. Furthermore, changing capacities on the established market has qualitatively different impacts on the smaller and the larger firm. Overall, total payoffs depend non-monotonously on the capacities on the established market.

The obtained insights have relevant implications for managing firms operating in oligopolistic environments.

1.) For the larger firm on the old market it might be optimal not to move into all new markets the smaller competitors (and new competitors) enter. According to our analysis, equilibrium constellations where only the small firm innovates are prominently present. In such scenarios the lack of innovativeness of the large firm is a consequence of strategic considerations rather than an implication of the organizational structure.

2.) Capacity expansion on the old market should be done with care since it reduces the own innovation activities. For a small firm even more caution is needed because capacity expansion might also increase the innovation activities of the competitor, which has detrimental effects on profits of the small firm.

3.) In industries where frequently new product designs are added to the existing range of designs, having a relatively small capacity for production of the existing designs might be a good starting point to attain profit leadership. The advantage of the small firm results from the fact that even if its optimal for the large firm to refrain from innovating, the small firm can still gain substantial additional profits with the new design. Thus, our analysis also provides a new explanation for the phenomenon of leapfrogging in markets.

4.) The larger firm can use capacity expansion as an instrument to prevent its competitor from innovating, whereas the smaller firm cannot prevent innovation by its competitor in this way. Preventing the competitor from innovating can be seen as a strategy closely related to entry deterrence. Straightforward intuition derived from the entry deterrence literature suggests that expanding capacity can be used as an instrument to keep the competitor out of the new market. A new insight gained in this paper is that relative firm size determines whether such a strategy can be succesfully applied.

The paper is organized as follows. Section 2 gives an overview over the related literature, section 3 presents the model, while the results of our analysis are presented in section 4. Section 5 summarizes the main findings and Section 6 concludes. The formal analysis and the proofs are given in the Appendix.

2 Literature Review

Our work is related to several streams of literature. With respect to the main research questions we are posing, previous work on the relationship between firm size and innovative activities is relevant. A second stream of related work deals with the optimal choice of product portfolios in oligopolistic markets with differentiated products. Finally, work exploring price- and capacity-choices in imperfect markets is a third stream of literature that is relevant to this paper.

The relationship between firm size and innovation has been extensively discussed in the economics of innovation literature. The vast majority of the contributions to this literature is based on empirical analysis and findings are strongly affected by the particular industry under consideration (see Evangelista and Mastrostefano (2006)). Generally speaking there seems to be evidence that R&D effort is positively related to firm size but that the number of innovations relative to R&D expenditures decreases with firm size. Put differently, small firms account for a disproportional number of innovations (see Cohen and Klepper (1996)). Also from a managerial perpsective the relationship between firm size and innovative activities has been studied. Zenger (1994) uses survey data to argue that diseconomies of scale in R&D are to some extend due to the fact that small firms are able to attract and retain engineers with higher ability and skill. The literature discussing the so called Innovator's Dilemma (see Christensen (1997)) points out that differences with respect to internal organization and project selection procedures between small and large firms are responsible for the observation that large incumbents might fail to react appropriately to the emergence of new technologies that are disruptive. These studies, although providing some explanation for different innovation behavior of firms of different size, do not take into account strategic considerations of these firms. The analysis of asymmetric innovation incentives between competitors in oligopolistic markets has been largely carried out in the framework of incumbent-entrant competitions or of patent-races in single markets where innovations refer to the introduction of cost reducing new technologies (see e.g. Reinganum (1983), Vickers (1986)). Our focus is different in that we consider markets with several incumbents and (potential) co-exsitence of several products. Coexistence of established and new products in oligpolistic settings occurs in Scmidt and Porteus (2000, 2007) where the single incumbent offers an existing product and both incumbent and entrant have the option to introduce a new product. Neither in this literature nor in the work on patent races there is an explicit consideration of the effect of firm size on innovative activities.

Our paper is also related to a second stream of literature which focuses on the optimal design of product lines. The issue which has been addressed in the context of firms with multiple horizontally and vertically differentiated products is that introducing a new product may not only mean encroaching on the market segment of the competitors' products, but also to cannibalize the sales of its own existing products. In such situations it is important to analyze the optimal design of the product line or to determine the optimal sequence of launching new products (high-quality or low-quality first). In what follows, we do not give a comprehensive overview of the literature, but instead choose representative contributions and point out the differences with respect to our paper. Moorthy and Png (1992) address the issue of cannibalization by analyzing the effects of sequential or simultanous introduction of vertically differentiated products to market segments with different valuations. They demonstrate that sequential introduction is advantageous if cannibalization is a problem and customers are relatively more impatient than the firm. Krishnan and Zhu (2006) consider DIPs (development-intensive products) for which development fixed costs are much higher than variable costs and show that product-line design for DIPs differs from the recommendations derived for other classes of products. We also consider the introduction of a new vertically and horizontally differentiated product, but in contrast to our paper, the above contributions consider the product line design problem of a monopolist, whereas we are mainly interested in firm choices in imperfectly competitive markets.

The existing literature on product line design in oligopolistic frameworks (e.g. Brander and Eaton (1984), Johnson and Myatt (2006)) does neither explicitly deal with firm size issues nor cannibalization and therefore is only remotely related to our work. However, from an empirical perspective cannibalization in a competitive environment has been studied. Figueiredo et al (2007) consider a model of industry evolution, where the leading firm opens new market segments which are sufficiently distant to avoid cannibalization. They provide an empirical assessment of their prediction by looking at the printer industry. In their empirical study Chandy and Tellis (1998) point out that the willingness of a firm to cannibalize the actual or potential value of its past investments determines the propensity to innovate.

Our paper contributes to this line of research in several ways. First, it clarifies the role of firm size with respect to the magnitude of the cannibalization effect. Second, it provides a formal analysis of cannibalization within a multi-product oligopoly framework. Third, it points out that in cases where product line extension is due to innovation, a number of effects matter that are distinct from the pure cannibalization effect.

Papers in the third stream of related literature deal with capacity- and price-setting games. In their seminal paper Kreps and Scheinkman (1983) demonstrate that the Cournot outcome can be considered as the result of a two-stage game where firms first select their production capacities and then select the prices of the individual products. The basic Kreps-Sheinkman setup has been extended to the case where the two firms offer differentiated products in Yin and Ng (1997). In an operations management context, Van Mieghem and Dada (1999) study postponement strategies in a two-stage decision model capturing capacity, production, and price choices under demand uncertainty. They provide results on the economic and operational value of price postponement and production postponement strategies where their focus is primarily on monopolistic firms (although they also briefly study extensions to oligopoly). Anupindi and Jiang (2008) develop this model further and consider flexible firms (who can postpone production decisions until the demand curve is known) and inflexible firms. For flexible firms they show that for a symmetric duopoly the Kreps-Sheinkman result holds, i.e. price and quantity competition are strategically equivalent, illustrating the importance of flexibility in environments with demand uncertainty. Goyal and Netessine (2007) study the value of flexibility of production technology in a duopoly framework with demand uncertainty, where each firm manufactures two products and engages in competition with the other firm in both markets. Although the research questions underlying this paper are quite distinct from ours, their model setup is closely related. Like the present paper, they consider competition on several related markets under capacity contraints, which to our knowledge has not been dealt with in the previous literature.

An important difference between our setup and the multi-product capacity choice models reviewed above is that although we also let the firms select their capacities and prices for the new product, the capacity for the old product is fixed. In this way we can capture the effect of firm size on the incentive to introduce new products. In addition an innovative contribution to oligopoly theory is that we establish existence and properties of price-setting equilibria in a setting where multiproduct firms compete on linked product markets under capacity constraints.

3 Model

Consider a duopoly where both firms, denoted by Firm 1 and 2, have capacities K_i^o $(i = 1, 2)$ available for production of an established product. Firm 1 is the larger firm, so that $K_1^o > K_2^o$. Both firms have the option to add a new differentiated product to their product portfolio. The firms maximize profits and compete in prices. Strategic interaction between the firms happens in three stages.

Innovation Stage:
Firm 1 and firm 2 simultaneously have to make the decision whether to innovate and introduce the new product. If firm i decides to do so $(D_i = Y)$, it incurs set-up costs equal to I. If firm i does not enter the new market, we denote this by $D_i = N$.

Capacity Investment Stage:
All firms that decided to launch the new product have to decide on their corresponding production capacity level K_i^n. The investment cost is equal to cK_i^n with $c > 0$.

Pricing Stage:
Both firms select their prices for the old product p_i^o and, in case they entered the new market, also their new product's price p_i^n. Variable production costs are normalized to zero. All prices are chosen simultaneously and quantities are given by the minimum of individual demand for firm i and its corresponding capacity.

In case none of the firms has introduced the new product, demand for the old product is given by

$$Q^o(p^o) = 1 - p^o$$

whenever this expression in non-negative, where p^o is the price at which the consumer can purchase the old good. Consumers in first instance buy from the firm that charges the lowest price for a particular product. When this firm meets its capacity constraints, the remaining consumers buy from the firm with the highest price. In case prices are equal, we assume that demand is shared equally among the two firms.

If at least one of the firms introduces the new product, the homogenous product market from before turns into a heterogenous product market. The new product is, as in Dawid et al. (2006), vertically and horizontally differentiated from the old one. The degree of vertical and horizontal differentiation is parametrized in a way that the inverse demand system reads

$$p^o = 1 - Q^o - \eta Q^n, \tag{1}$$
$$p^n = 1 - \eta Q^o - \gamma Q^n. \tag{2}$$

This type of inverse demand system can be derived from a representative consumer model with a quality augmented version of the standard quadratic utility function (see Symeonidis (2003), Vives (1999)). For the parameter γ we impose that $0 < \gamma \leq 1$. If the latter inequality holds in a strict sense, it indicates that the new product is vertically differentiated from the old product. The parameter η determines the degree of horizontal differentiation. Markets for the two products are separated for $\eta = 0$, while the products are homogenous for $\gamma = \eta = 1$. In what follows we impose that

$$0 \leq \eta < \gamma \leq 1. \tag{3}$$

Obviously, this demand system implies that the price of the new product cannot exceed one and therefore we only consider unit capacity costs in the range

$$0 < c < 1. \tag{4}$$

The range of parameters γ, η and c for which (3) and (4) are satisfied, will be referred to as the admissible set of parameters.

Based on this demand system and the allocation rule, actual quantities of both firms in both markets are functions of all capacities and all prices. In particular, if capacities on one market are binding for at least one firm the determination of market clearing quantities is not completely straight forward. Details concerning the resulting quantities $q_i^h \;\; i = 1, 2, \;\; h \in \{o, n\}$ for given capacities and prices are given in Appendix A.2. The payoff of firm i is given by

$$\pi_i = \begin{cases} p_i^o q_i^o + p_i^n q_i^n - cK_i^n - I, & \text{if } i \text{ enters the new market} \\ p_i^o q_i^o. & \text{else} \end{cases} \tag{5}$$

4 Economic Analysis

Our economic analysis is based on the characterization of subgame-perfect equilibria of our three-stage game. Our discussion here summarizes the main findings of the formal detailed analysis which is provided in Appendix A.4. We focus on the most interesting case where a pure strategy equilibrium at the pricing stage with positive output prices on both markets exists. In Appendix A.4 we show that this requires sufficiently low production capacities on both markets (see Propositions 9, 10 and 13). In fact, since capacity choices on the new market are endogenous, the existence conditions for equilibria can be expressed solely in terms of the capacities on the old market. More precisely, regardless of the market entry decisions of both firms, equilibria with positive prices exist whenever (see Propsition 14 in Appendix A.4)

$$K_1^o \leq \frac{1-c}{3\eta}, \tag{6}$$

$$2K_1^o + K_2^o \leq \frac{\gamma - \eta(1-c)}{\gamma - \eta^2}. \tag{7}$$

$$K_1^o + K_2^o < 1. \tag{8}$$

The following proposition states that both firms charge identical prices for identical products, always fully employ their production capacities, and presents the equilibrium capacity levels on the new market.

Proposition 1 *Assume that (6) - (8) hold. Then both firms fully use their capacities and charge identical positive prices for the old product.*

- *If firm i enters the new market but firm j does not, then firm i chooses capacity*

$$K_{i,YN}^{n*} = \begin{cases} \frac{1-c-\eta(2K_i^o+K_j^o)}{2\gamma} & 2K_i^o + K_j^o \geq \frac{\gamma-\eta}{\gamma-\eta^2} - \frac{1}{\eta}c \\ K^* & 2K_i^o + K_j^o < \frac{\gamma-\eta}{\gamma-\eta^2} - \frac{1}{\eta}c, \end{cases} \tag{9}$$

  where the capacity

$$K^* = \frac{1-\eta}{2(\gamma - \eta^2)}$$

  is the optimal output quantity of the new product of a firm that is a monopolist on both markets. Firm i charges $p_i^n > 0$.

- *If both firms enter the new market they choose capacity*

$$K_{i,YY}^{n*} = \frac{1 - c - 3\eta K_i^o}{3\gamma}, \; i = 1, 2. \tag{10}$$

  Furthermore, they charge prices $p_i^n = p^n > 0, i = 1, 2$.

The value of K^* corresponds to the quantity a firm would choose on the new market if it were monopolist on both markets. A firm who is the sole supplier on the new market should never invest in capacities exceeding that level because optimal price choice would then imply that some of the generated capacities on the new market stay idle. The reason is that if the competitor is only present on the old market it will always fully use its capacity on that market and therefore the firm producing on both markets is essentially solving the monopoly problem on the two connected markets, where the market size of the old market has been reduced by the competitor's capacity. It turns out that the monopolist output level on the new market is independent from the competitor's capacity on the old market and therefore K^* is independent from the capacities on the old market. The proposition above states that K^* is the optimal capacity level of a single new market entrant under the condition that the old market is sufficiently small. Hence, it is only optimal for the single entrant to choose this K^*, if (i) sales on the new market do not have a too negative effect on the old market revenue, and (ii) the quantity on the old market does not reduce the unit output price of the new market too much. In case the old market is of considerable size, the entrant adjusts by setting a capacity lower than K^*.

Note that in case both firms have entered the new market, the fact that all capacities are binding implies by (1) and (2) that market prices are given by

$$p^o = 1 - (K_1^o + K_2^o) - \eta(K_1^{n*} + K_2^{n*}), \tag{11}$$

$$p^n = 1 - \eta(K_1^o + K_2^o) - \gamma(K_1^{n*} + K_2^{n*}). \tag{12}$$

In case only firm i enters, K_j^{n*} is set to zero. Expressions (11) and (12) highlight the direct and indirect (through its impact on K_i^{n*}) effects of the level of the old market capacities on the market prices. It is easy to see that no general statement concerning the relative size of prices on the two markets are possible. If capacities on the old market are sufficiently low we have $p^o > p^n$, whereas for large capacities on the old market the opposite inequality holds.

An important result (see (9), (10)) is that capacity investment in the new market is decreasing in the firm's capacity on the old market. The explanation is as follows. From (5) and the facts that both firms fully use their capacities and charge identical prices for both products (see Proposition 1), we obtain that profit excluding the set-up costs (which are sunk at the capacity investment stage) can be expressed as

$$\pi_i = p^o K_i^o + p^n K_i^n - cK_i^n.$$

Then, the investment incentive on the new market for any given K_i^n (under the Nash assumption that capacities K_j, n of the opponent are given) is determined by

$$\frac{\partial \pi_i}{\partial K_i^n} = K_i^o \underbrace{\frac{\partial p^o}{\partial K_i^n}}_{<0} + p^n + K_i^n \underbrace{\frac{\partial p^n}{\partial K_i^n}}_{<0} - c.$$

Considering how the different terms in this sum change if K_i^o increases, we can distinguish two different effects influencing the incentives for firm i to invest in new capacity:

- *Cannibalization Effect*: The first term in the sum decreases if K_i^o increases which is due to the cannibalization effect. If the capacity on the old market becomes larger, the firm is affected more strongly by the decrease in the price p^o induced by additional investment on the new market.

- *Size Effect*: The second term in the sum, p^n, decreases if K_i^o goes up which is due to the size effect. If the capacity on the old market becomes larger, the remaining market size of the new market goes down, which reduces for given capacities for the new product the prices of the new product and thereby reduces the incentives to build additional capacity on the new market. For given K_i^n the remaining two terms in the sum do not change if K_i^o is increased.

Both the cannibalization effect and the size effect decrease the incentives to invest in K_i^n and therefore an increase in K_i^o induces a decrease in the level of K_i^n if i is the only firm to enter the new market.

In case firm j also enters, then the implications of the size effect and cannibalization effect hold *for a given* K_j^n. However, also the incentive for firm j to invest in capacity on the new market decreases if K_i^o goes up for a given K_i^n. Obviously, for firm j there is no cannibalization effect, but the size effect is still at work. Hence, the (negatively sloped) reaction functions of both firms in the capacity investment stage shift inwards if K_i^o goes up, where the shift is more pronounced for firm i. As shown in proposition 1, if both firms are active the capacity choice of firm j in equilibrium is unchanged. Put differently, the negative size effect for firm j is counterbalanced by the positive effect of the decrease of K_i^n on the incentives to invest for firm j. Given that K_j^{n*} is not affected by the change in K_i^o, the implications of an increase in capacity of firm i on the old market for its incentives to invest in capacity on the new market is again fully captured by the size effect plus the cannibalization effect.

This discussion allows us to answer one of our main research questions: The larger the capacity of a firm on the old market the smaller is ceteris paribus the investment in capacity on the new market. The effects of a capacity increase of the competitor on the old market depend on whether the competitor also enters the new market. In case both firms enter the new market, the competitor's capacity on the old market does not influence the investment decision of a firm on the new market. In case the competitor stays out, an increase of its capacity on the old market either negatively affects the capacity investments of the considered firm on the new market or has no influence at all.

4.1 What Determines the Profitability of the New Market?

At the innovation stage the firms have to choose whether to innovate or not. The interaction of the firms can be represented by a 2×2 normal form game with the following payoff matrix, where firm 1 is the row player and firm 2 the column player:

$$\begin{array}{|c|c|c|} \hline & Y & N \\ \hline Y & \pi_{1,YY}-I, \pi_{2,YY}-I & \pi_{1,YN}-I, \pi_{2,YN} \\ \hline N & \pi_{1,NY}, \pi_{2,NY}-I & \pi_{1,NN}, \pi_{2,NN} \\ \hline \end{array}. \tag{13}$$

In fact, the entries in this matrix correspond to the net profits of the firms depending on the entry decisions of both competitors. In particular, $\pi_{1,YY}$ gives the sum of profits for firm 1 on both markets net of capacity investment costs if both firms enter the new market, $\pi_{1,YN}$ gives the sum of profits for firm 1 on both markets if only firm 1 enters, $\pi_{1,NY}$ are the profits of firm 1 on the old market if only firm 2 enters, and the profits of firm 1 on the old market if both refrain from entering the new market are denoted by $\pi_{1,NN}$. Analogous notation holds for firm 2. The incentives for entry thus depend on whether the rival enters or not and are given by

$$\Delta\pi_{1,Y} := \pi_{1,YY} - I - \pi_{1,NY} \tag{14}$$
$$\Delta\pi_{1,N} := \pi_{1,YN} - I - \pi_{1,NN} \tag{15}$$
$$\Delta\pi_{2,Y} := \pi_{2,YY} - I - \pi_{2,YN} \tag{16}$$
$$\Delta\pi_{2,N} := \pi_{2,NY} - I - \pi_{2,NN}, \tag{17}$$

where $\Delta\pi_{i,h},\ i \in \{1,2\}, h \in Y, N$ is the entry incentive for firm i given that firm j's entry decision is h.

To understand how the capacities on the old market influence the incentives to enter the new market we consider the following partition of the total derivatives of $\Delta\pi_{i,Y}$ with respect to K_i^o.

$$\begin{aligned} \frac{d\ \Delta\pi_{i,Y}}{d\ K_i^o} & \\ = & \left[\frac{\partial\pi_{i,YY}}{\partial K_i^o} - \frac{\partial\pi_{i,NY}}{\partial K_i^o}\right] + \underbrace{\frac{\partial\pi_{i,YY}}{\partial K_i^n}}_{=0}\frac{\partial K_{i,YY}^{n*}}{\partial K_i^o} + \frac{\partial\pi_{i,YY}}{\partial K_j^n}\underbrace{\frac{\partial K_{j,YY}^{n*}}{\partial K_i^o}} = 0 - \frac{\partial\pi_{i,NY}}{\partial K_j^n}\frac{\partial K_{j,NY}^{n*}}{\partial K_i^o} \\ = & \underbrace{\left[p_{i,YY}^o - p_{i,NY}^o\right]}_{\text{cannib. eff. } <0} + \underbrace{K_{i,YY}^{n*}\frac{\partial p_{i,YY}^n}{\partial K_i^o}}_{\text{size eff. } <0} \underbrace{-\frac{\partial\pi_{i,NY}}{\partial K_j^n}\frac{\partial K_{j,NY}^{n*}}{\partial K_i^o}}_{\text{strategic eff. } \leq 0} \\ & < 0. \end{aligned}$$

The first two terms are negative and capture the cannibalization and size effects of an increase of the capacity of firm i on the old market on the incentives to enter the new market. First, the larger K_i^o is, the stronger is firm i affected by the price decrease of the old product that results if it enters the new market. This is the cannibalization effect. Second, due to the size effect the price of the new product is decreased if K_i^o goes up, which reduces the attractiveness of that market. Finally, the incentive to enter for firm i is also influenced by an effect which so far has not been introduced:

- *Strategic effect*: If firm i does not enter the new market the capacity of its competitor on the new market is inversely related to the capacity of firm i on the old market, whereas, as explained in the previous section, no such influence occurs if firm i enters the new market. Since ceteris paribus a large capacity of the competitor on the new market negatively affects the profits of firm i, this implies that the incentives for firm i to stay out of the new market increase if K_i^o goes up.

Each of these effects implies that the incentives to enter the new market decrease if the capacity on the old market becomes larger and therefore the same holds for the total effect.

The situation is less clear cut if we consider the effect of an increase of the opponent's capacity K_j^o on the incentives of firm i to enter the new market. Proceeding as above we obtain

$$\frac{d\ \Delta\pi_{i,Y}}{d\ K_j^o} = \underbrace{K_{i,YY}^{n*} \frac{\partial p_{i,YY}^n}{\partial K_j^o}}_{\text{size eff. } <0} + \underbrace{\left[\underbrace{\frac{\partial \pi_{i,YY}}{\partial K_j^n}}_{<0} \underbrace{\frac{\partial K_{j,YY}^{n*}}{\partial K_j^o}}_{<0} - \underbrace{\frac{\partial \pi_{i,NY}}{\partial K_j^n}}_{<0} \underbrace{\frac{\partial K_{j,NY}^{n*}}{\partial K_j^o}}_{\leq 0} \right]}_{\text{indirect eff. } >0}.$$

First, note that no cannibalization effect arises, but there is still a negative size effect. Second, in this case there is also a positive effect of an increase of K_j^o.

- *Indirect effect:* If K_j^o goes up, then this has a non-increasing effect on the capacity of this firm on the new market, where the effect is always strictly negative if firm i decides to enter the new market. The marginal effect on the new market capacity of firm j is always at least as large in absolute terms if firm i enters the new market compared to when it stays out. Furthermore, the positive effect on firm i's profit of a decrease of the new market capacity of firm j is larger if firm i is active on both markets rather than only on the old one. Overall, the reduction of K_j^n induced by an increase of K_j^o has a positive effect on firm i profits and this positive effect is stronger if firm i enters the new market.

The intuition for the positive sign of the indirect effect is that both the prices in the old and the new market are positively affected by a decrease of K_j^n, and fim i benefits only from the price increase on the old market once it is not active on the new market. Hence, the indirect effect suggests that entering the new market becomes *more attractive* if the competitors capacity on the old market goes up. Generally speaking it is not clear whether the size effect or the indirect effect dominates and how the incentive to enter the new market depends on the

	Cannibalization	Size	Strategic	Indirect	Overall
$\frac{d\Delta\pi_{i,Y}}{dK_i^o}$	<0	<0	<0		<0
$\frac{d\Delta\pi_{i,Y}}{dK_j^o}$		<0		>0	≥ 0
$\frac{d\Delta\pi_{i,N}}{dK_i^o}$	<0	<0			<0
$\frac{d\Delta\pi_{i,N}}{dK_j^o}$		<0			<0

Table 1: Effects of an increase of capacities in the old market on the incentives of firm i to enter the new market.

opponent's capacity on the old market. In our specification it turns out that the indirect effect always (weakly) dominates. This is proven in Proposition 2.

Similar considerations can be made under the assumption that firm j does not innovate. Table 1 summarizes which of the different effects we identified emerge in the different settings and the impact it has on the incentive for firm i to enter the market. Proposition 2 formalizes the above discussion and also establishes that the incentive to innovate is larger for the smaller firm.

Proposition 2 *Assume that parameters satisfy the condition* $K_2^o < K_1^o \leq \frac{1-c}{3\eta}$, *then*

(i)

$$\frac{d\Delta\pi_{i,Y}}{dK_i^o} < 0, \quad \frac{d\Delta\pi_{i,Y}}{dK_j^o} \geq 0, \;\; i,j \in \{1,2\}, i \neq j$$

(ii)

$$\frac{d\Delta\pi_{i,N}}{dK_j^o} < 0, \;\; i,j \in \{1,2\}$$

(iii)

$$\Delta\pi_{1,Y} < \Delta\pi_{2,Y}, \;\; \Delta\pi_{1,N} < \Delta\pi_{2,N}.$$

Part (iii) of this proposition answers one of our main research questions. The explanation why a larger capacity on the old market reduces the incentives to innovate rests on the different effects summarized in Table 1. If old market capacities of both firms are identical, it is obvious that both have identical incentives to innovate. Now consider a situation where the old market capacity of firm 1 is increased implying $K_1^o > K_2^o$. If both firms expect the other firm to innovate then such a change of old market capacity decreases the innovation incentives of firm 1. According to the first line of Table 1 this is due to the combination of the cannibalization effect, the size effect and the strategic effect. However due to dominant role of the indirect effect innovation incentives of firm 2 are increased (see second line of Table 1). Therefore, the incentive is larger for firm 2 than for firm 1. If both firms expect that the opponent does not innovate, then lines 3 and 4 of Table 1 show that the incentives to innovate for both firms decline. However, whereas firm 2 is only affected by the size effect, firm 1 also also has to take the cannibalization effect into account. Accordingly, also in this case the innovation incentive of the smaller firm is larger. It should be noted however, that no statement about the relative size of the innovation incentive can be made if the two firms have different expectations about the innovation decision of the opponent. Therefore, these arguments do not rule out the existence of a subgame-perfect equilibrium of the game where the larger firm 1 innovates but the smaller firm 2 does not. However, the following proposition shows that for any parameter constellation where such an equilibrium exists, there must also exist an equilibrium where the smaller firm does innovate.

Proposition 3 *For any parameter constellation where a subgame perfect equilibrium exists in which (the large) firm 1 carries out the innovation project, there also exists a subgame perfect equilibrium in which (the small) firm 2 innovates. The reverse statement need not be true.*

The issue of existence of equilibria of the game is settled in Proposition 14 where it is shown that under conditions (6) - (8) at least one subgame-perfect equilibrium in pure strategies exists.

4.2 Effects of Size on Strategy and Profits

First consider the effects of changes of K_i^o, K_j^o on firm i's profit π_i under the assumption that this change of capacities does not alter the innovation decisions of any firm. In that benchmark case in general the effect of an increase of the old market capacity of a firm on its profit is non-monotonous. This comes from the facts that any available capacities are fully used in equilibrium and there is an interplay of the several effects discussed in the previous section.

However, monotonicity of payoffs with respect to the capacity of the competitor can be established.

Proposition 4 *Consider ranges of K_1^o, K_2^o where conditions (6) - (8) are satisfied. Then*

$$\frac{\partial \pi_{i,D_1 D_2}}{\partial K_j^o} < 0, \quad i, j = 1, 2, j \neq i, D_1, D_2 \in \{Y, N\}.$$

Interpreting this finding in light of the different effects identified in the previous section, we observe that in case the competitor is not active on the new market an increase of the competitor's capacity on the old market just results in a downward pressure on both prices and therefore has a negative effect on the firm's profit (size effect). If the competitor is active on the new market in addition to the size effects also the indirect effect becomes relevant. An increase in the old market capacity then results in a decrease of the capacity the competitor builds on the new market, which reduces the downward pressure on prices. However, as is shown in the proposition, as far as the dependency of payoffs from capacities is concerned, the indirect effect is dominated by the size effect. Hence the focal firm is negatively affected if the competitor increases its old market capacity.

The negative dependence of profits on the competitor's old market capacity, although very intuitive, hinges on the assumption that the innovation decisions of both firms stay unaffected by the change in old market capacity. As we show in the remainder of this section, a change in the capacities on the old market might however lead to modifications of the innovation decisions of the firm. We show that such modifications result in non straight-forward profit effects. In particular, profits might exhibit jumps and such jumps might induce additional non-monotonicities which are in stark contrast to the claims of Proposition 4. It turns out that the innovation decisions of the larger and the smaller firm are asymmetrically affected by changes in the old market capacities.

4.2.1 Switches of the Innovation Decisions of the Small Firm and their Profit Effects

Switches of the innovation strategy of the small firm induce jumps of the profit of the large firm. The following proposition shows that either an increase in the capacity of firm 1 or an increase in the capacity of firm 2 can induce a transition from an equilibrium where Firm 2 innovates to one where it does not. Such a transition always results in an upward jump of the profit of the large firm.

Proposition 5 *For all admissible values of η, γ, c there exists a range of investment costs I and capacities K_1^o, K_2^o satisfying (6) - (8)such that:*

(i) An increase in capacity K_1^o of the large firm leads to a switch from an equilibrium where only the small firm innovates to one where no firm innovates. The increase in K_1^o leads to an upward jump of the payoff of firm 1.

(ii) An increase in capacity K_2^o of the small firm leads to a switch from an equilibrium where only the small firm innovates to one where no firm innovates. The increase in K_2^o leads to an upward jump of the payoff of firm 1.

To illustrate this proposition we show in Figure 1 a generic representation of the different (equilibrium) combinations of firms' innovation strategies depending on the capacities on the old market. It is optimal for a firm to innovate when the introduction of the new product increases its profit. Translated to the figure this means that, given that firm 2 innovates, to the left of the line $\Delta\pi_{1,Y} = 0$ it is optimal for firm 1 to innovate, while to the right of this line it is not optimal to do so. Analogously it holds that, given that firm 2 does not innovate, to the left of the line $\Delta\pi_{1,N} = 0$ it is optimal for firm 1 to innovate, while to the right of the line firm 1 refrains from innovating. In the same way the optimal innovation policy of firm 2 can be directly derived from the lines

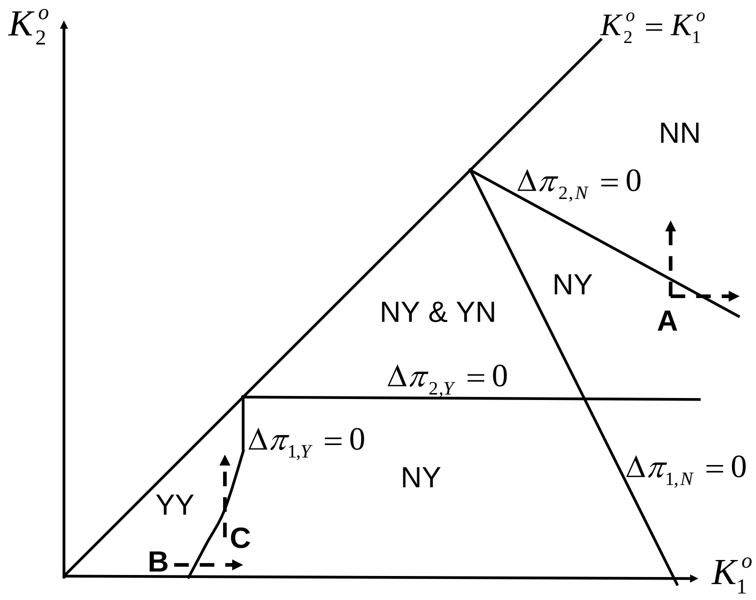


Figure 1: Equilibrium Combinations of Innovation Choices Depending on Capacities on the Old Market.

$\Delta\pi_{2,Y} = 0$ and $\Delta\pi_{2,N} = 0$, with the only difference that for firm 2 innovating is optimal below the corresponding line, while firm 2 should not innovate above this line.

To see the transitions described in Proposition 5 consider the combination of capacities indicated by label 'A'. For such capacities the small firm innovates but the large firm does not. As indicated by the dashed arrows, it follows from lines 3 and 4 in Table 1 that an increase in K_1^o as well as an increase in K_2^o implies a transition to the region where no firm innovates. The effects of such transitions on the payoffs are illustrated in Figure 2, where π_i^* denotes the equilibrium payoff of firm i. In both cases the profit of firm 1 jumps upwards, whereas the profit of firm 2 changes continuously. The upward jump of the payoff of the large firm is caused by the upward jump of the price for the old product resulting from the disappearance of the new product. This price jump also positively affects the profits of the small firm on the old market, but for that firm this positive effect is counterbalanced by the loss of profit resulting from the loss of sales of the new product.

This discussion shows that if the large firm does not innovate an increase in old market capacity might stop the small firm from innovating. As becomes obvious from Figure 1[2] in situations where the large firm does innovate a capacity extension never prevents the small firm from innovation. Put differently,

[2]More formally, the statement follows from Lemma 15 in Appendix B.

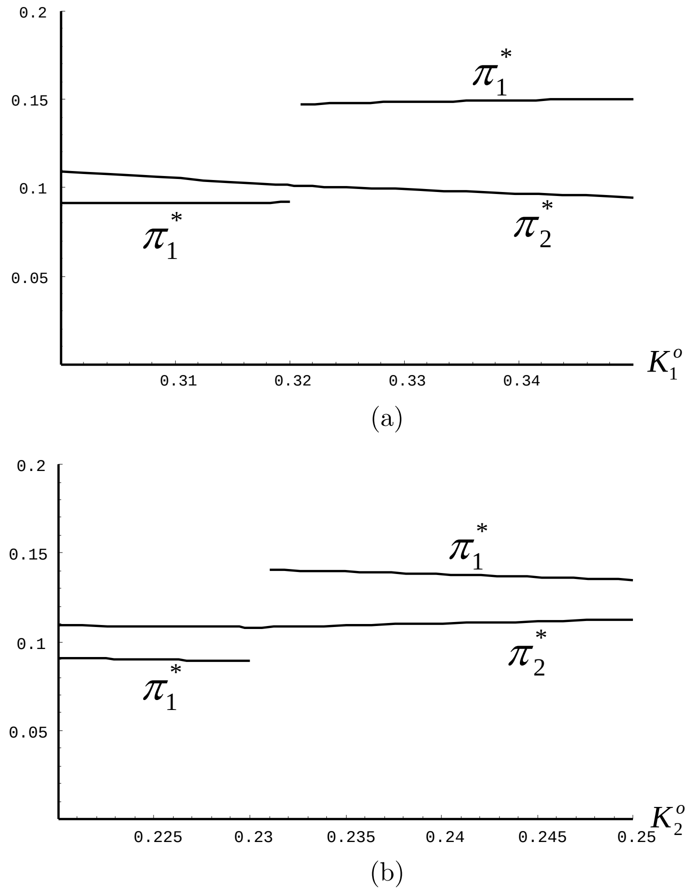


Figure 2: Equilibrium payoffs of both firms for (a) increasing capacity of firm 1, (b) increasing capacity of firm 2 starting with capacities $K_1^o = 0.3, K_2^o = 0.22$ ($\eta = 0.5, \gamma = 0.75, c = 0.1, I = 0.09$).

a transition from a YY-equilibrium to a YN-equilibrium cannot be triggered by either the increase in K_1^o or in K_2^o. This is an implication of the fact that the innovation incentive of the smaller firm is larger and therefore any capacity increase first deters the large firm from innovation.

4.2.2 Switches of the Innovation Decisions of the Large Firm and their Profit Effects

Based on the insight that innovation incentives are smaller for the larger firm it is intuitive that a capacity increase in a YY-equilibrium situation eventually leads to a transistion to a NY-equilibrium where the large firm no longer innovates. The following proposition verifies this intuition for an increase in the capacity of the large firm and shows that the induced strategy switch of firm 1 results in an upward profit jump of firm 2.

Proposition 6 *For all admissible values of η, γ, c there exists a range of investment costs I and capacities K_1^o, K_2^o satisfying (6) - (8)such that an increase in capacity K_1^o of the large firm leads to a switch from an equilibrium where both firms innovate to one where only the small firm innovates. The increase in K_1^o leads to an upward jump of the payoff of firm 2.*

The transition described in this proposition is illustrated in Figure 1 by the dashed arrow originating at the capacity combination labelled by 'B'. It should be noted that contrary to the switch from innovating to not innovating of the small firm, which was induced by capacity increases of any firm, the large firm only switches to not innovating if the own capacity increases. Such a switch can never be triggered by an increase of the old market capacity of the small firm, because given that the competitor innovates, the innovation incentives of firm 1 go up if firm 2 increases its capacity (see line 2 of Table 1). Quite on the contrary, an increase of the capacity of firm 2 can trigger a change of firm 1's innovation strategy from not innovating to innovating (such a transition is illustrated by the dashed line originating at point 'C' in Figure 1). The following proposition provides a sufficient condition for such a transition to occur.

Proposition 7 *For all values of η, γ, c satisfying*

$$c < \frac{\eta(\gamma - \eta)}{\gamma - \eta^2} \tag{18}$$

there exists a range of investment costs I and capacities K_1^o, K_2^o satisfying (6)-(8) such that an increase in capacity K_2^o of the small firm leads to a switch from an equilibrium where only the small firm innovates to one where both firms innovate. The increase in K_2^o leads to a downward jump of the payoff of firm 2.

To interpret condition (18), note that the right hand side of the inequality equals the derivative of the demand for the new product with respect to the price of the old product times $(\gamma - \eta)$. Accordingly, a transition from a NY-equilibrium to a YY-equilibrium occurs for a large range of capacity costs c if the new market is only weakly differentiated from the old market. Furthermore, the larger γ is, i.e. the weaker the degree of vertical differentiation of the new product is, the larger the range of capacity costs where the transition occurs. This is quite intuitive. Strong differentiation between the two markets implies a weak influence of the old capacity on the price of the new product and hence the incentives to innovate for firm 1 are becoming independent of K_2^o. Accordingly, an increase in the old market capacity of firm 2 in such a situation does not trigger a change in the innovation strategy of firm 1.

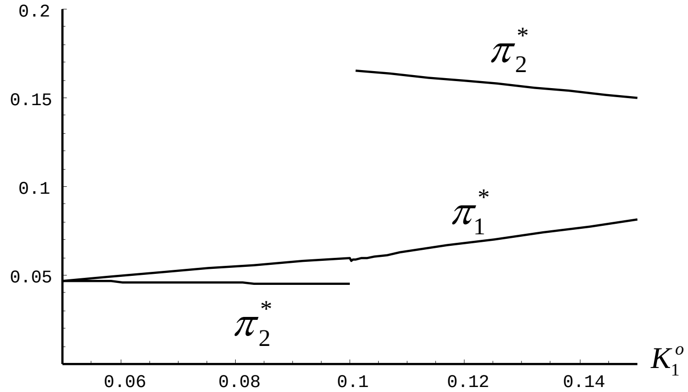
The profit effects of capacity changes that induce switches of the innovation strategy of the large firm are illustrated in Figure 3. In Figure 3a both firm 1 and firm 2 innovate for K_1^o small enough, while only firm 2 innovates for larger values of K_1^o. Increasing K_1^o makes the profit of firm 1 larger and also leads to an upward jump of the profit of firm 2 as firm 1 switches from innovating to not innovating. This switch reduces firm 1's quantity of the new product to zero, which raises prices of the old and the new product. This in turn causes the upward profit jump of firm 2. Except from the upward jump, the profit of firm 2 is decreasing in K_1^o as established in Proposition 4. The overall result is that the dependency of firm 2's profits on firm 1's old market production capacity is non-monotonic.

In Figure 3b an increase in the capacity of the firm 2 induces a transition from the NY-equilibrium to the YY-equilibrium. As discussed above, this transition is caused by the dominance of the indirect effect. The resulting downward jump of the profit of firm 2 is caused by the price decreases on both markets that are due the fact that firm 1's quantity on the new market jumps from zero to a positive level. Also, the profit of firm 1 is decreased. Putting together the two panels of this figure an additional interesting asymmetry between the effects of capacity increases of the large and the small firm can be observed. Whereas both competitors benefit from the increase of the capacity of firm 1, both are negatively affected by the increase of the capacity of firm 2.

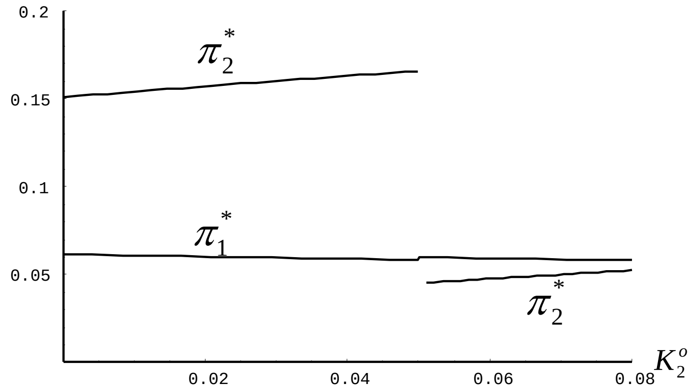
Finally, it should be noted that in all four panels of Figures 2 and 3 there is a range of capacities where the total profit of firm 2 is larger than the profit of firm 1. Due to the presence of an opportunity of product innovation, an advantage with respect to capacities on the old market does not necessarily translate to a relative profit advantage. A straightforward conclusion from this observation seems to be that firm 1 should scrap some of its old market capacity. However, as can be easily inferred from Figure 3a, such a conclusion would be false. If firm 1 reduces its capacity to the level of its smaller competitor (this capacity corresponds to the lower bound of the range depicted in the Figure 3a) it will reduce its absolute profit. It should be noted that such a cut would hurt the competitor even more hence equalizing the two profit levels.

5 Discussion of Main Findings and Conclusions

This paper is about the effect of firm size on the incentives to innovate in oligopolistic markets. The framework developed in the paper, where existing



(a)



(b)

Figure 3: Equilibrium payoffs of both firms for (a) increasing capacity of firm 1 starting with capacities $K_1^o = 0.05, K_2^o = 0.05$ and (b) increasing capacity of firm 2 starting with capacities $K_1^o = 0.1, K_2^o = 0$ ($\eta = 0.5, \gamma = 0.75, c = 0.1, I = 0.09$).

capacities on the established market are taken as proxies for firm size, allows us to explicitly focus on the role of differences in firm size rather than on related issues, like incumbent-entrant interactions or differences in production technology, that have been addressed in most of the literature. Our main focus is on the analysis of innovative activities in markets with several incumbents, but it should be noted that by setting old market capacities of one firm to zero the developed framework also applies to scenarios where an incumbent and a market entrant compete for consumers on the new market. Also, the four qualitatively different effects we identified as the determinants of the impact of firm size on the incentives to innovate are relevant beyond the exact framework considered in this paper.

We show that the interplay of the different effects imply that the incentive for the larger firm to innovate is below that of its smaller competitor, if both firms have identical expectations about the innovation strategy of the opponent. In particular this implies that for a cetain range of capacities on the old market the unique pair of equilibrium strategies induces that only the smaller firm introduces the new product. In such a scenario the asymmetry of the innovation behavior might imply that the firm with the smaller capacities on the old market ends up with larger profits than its larger competitor. In case both firms introduce the new product the asymmetry in innovation incentives is reflected in the choice of production capacities for the new product. The firm with the smaller capacities on the old market builds lager capacities for the new product and therefore gains the larger market share on the new market. A direct implication of the dependency of innovative activities from old market capacities is that a firm may influence the set of producers that enter the new market by expanding or contracting its capacities on the old market. We show in this paper that the way such an instrument can be used differs significanlty between the larger and the smaller firm in the market. Capacity expansion of the larger firm never increases the set of innovators. Rather, depending on the initial size of the two firms, a capacity expansion of the large firm might prevent this firm itself or its small competitor from entering the new market. Contrary to that an expansion of the small firm on the old market might imply that its competitor switches from not innovating to innovating. Switches in the innovation strategies of the competitor induce jumps of the profits of a firm and in particular we have shown that capacity expansion of the small firm on the old market might have negative implications for firm profits even if the investment costs needed for the expansion are not taken into account. For the large firm, an increase of the capacity on the old market always has a positive effect on profits. As has been discussed in the preview of our main findings in the Introduction, our findings have numerous managerial implications, both with respect to innovation strategies in dynamic oligopolistic markets and with respect to capacity building in such market environments.

At first sight our findings about the larger innovation incentives of small firms might seem to be at odds with empirical observations in many industries, where the majority of new products are introduced by the largest incumbents in the market. In that respect it is important to stress that our observations hold in oligopolistic settings where firm are completely identical with the exception of their size. In particular, we assume that there no differences with respect to production costs, innovative abilities or financial ressources between the firms. Obviously, such factors play a decisive role for actual innovations in many markets, but the agenda of this paper is to distill the role of firm size. Also, in this paper we restrict attention to non-drastic innovations, where the old product does not vanish from the market after the introduction of the new one. In scenarios where the demand on the old market disappears after the innovation the effect of the capacities on the old market in general deviates from the observations made in this paper.

The current set-up where firms innovate in a situation where they are active producers of some standard product, is highly relevant in today's economy, as the examples listed in the Introduction exemplify. Nevertheless, the model of this paper leaves room for various extensions. First, where in the current set-up the capacity for the original product is exogenous, we could also think of making it endogenous. Then the problem is to fix capacity of the original product while taking into account competition as well as future product innovations that require a different kind of capital stock.

Second, the current framework is deterministic, implying that it leaves out all kinds of uncertainties. The effect of uncertainty on investment is studied in real option theory (see, e.g., Dixit and Pindyck (1994)). Employing this theory may generate results concerning the effect of uncertainty on the options to innovate of the two competing firms.

Third, building up capacity is a dynamic process. Therefore, an interesting but mathematically complex project would be to develop and analyze a dynamic model with the aim to determine the strategic effects within a framework of two firms being active in building up capacity for both the standard and the innovative product. In Caulkins et al. (2008) capital accumulation for both a standard and an innovative product is considered, but in that paper this is done in a non-strategic framework, i.e. there is only one firm being a monopolist in both the standard and the innovative product market.

Appendix

A Additional Model Details and Results

A.1 Derivation of Demand

The market model is in the spirit of the Stackelberg-Spence-Dixit model, as it has been denoted in Tirole (1988). Before the eventual innovation phase a homogenous product market exists on which the standard product is traded. There are S identical consumers. As long as the innovative product does not exist, they all have the utility function

$$U^o\left(Q^o\right) = Q^o - \frac{1}{2}Q^{o2} + M_1, \tag{19}$$

in which Q^o is the quantity of the standard product and $M_1 = y - Q^o p^o$ (p^o being the unit market price of the standard product while y stands for consumer income) denotes the expenditure on outside goods. We assume that each consumer spends only a small part of her income on the industry's product. This implies that income effects on the industry under consideration can be ignored and partial equilibrium analysis can be applied (Symeonidis (2003)).

If at least one of the firms has innovated, existence of the new innovative product changes the consumer utility function in the following way (see, e.g., Vives (1999), p.145):

$$U^n\left(Q^o, Q^n\right) = Q^o + Q^n - \frac{1}{2}Q^{o2} - \frac{1}{2}\gamma Q^{n2} - \eta Q^o Q^n + M_2, \tag{20}$$

with all the parameters positive and $M_2 = y - p^o Q^o - p^n Q^n$ (p^n and Q^n being the unit price and the quantity of the innovative product, respectively) being the expenditure on outside goods. Again it is assumed that each consumer only spends a small part of her income on the industry's products implying that income effects can be ignored and partial equilibrium analysis can be applied. Maximization of utility with respect to both quantities yield the inverse demand system given in the text and the following direct demand system:

$$\tilde{Q}^n(p^o, p^n) = \frac{1-\eta}{\gamma-\eta^2} - \frac{1}{\gamma-\eta^2}p^n + \frac{\eta}{\gamma-\eta^2}p^o, \tag{21}$$

$$\tilde{Q}^o(p^o, p^n) = \frac{\gamma-\eta}{\gamma-\eta^2} + \frac{\eta}{\gamma-\eta^2}p^n - \frac{\gamma}{\gamma-\eta^2}p^o, \tag{22}$$

Concavity of the consumer utility function is ensured by our assumption $\eta \leq \gamma$ (see, e.g., Dixit (1979)).

A.2 Determination of Quantities under Capacity Constraints

If both producers set identical prices for a good, obviously we have $p^h = p_1^h = p_2^h, h \in \{o, n\}$. If no capacity constraints bind, actual quantities are given by the demands (21) and (22). In case prices differ, then if the capacity constraint(s) become(s) binding for the cheaper firm we have $p^h = \max[p_1^h, p_2^h], h \in \{o, n\}$. Otherwise, the cheaper firm can serve the entire market and $p^h = \min[p_1^h, p_2^h]$. Furthermore, if the unconstrained demand for one product at p^h exceeds the capacities there and consumers take these constraints into account, the demand for the other product changes in structure and capacity size rather than prices of the substitute good becomes relevant. In particular, if $\tilde{Q}^n(p^o, p^n) > K_1^n + K_2^n$ the demand for the old product reads

$$\tilde{\tilde{Q}}^o(p_o, K_1^n, K_2^n) = 1 - \eta(K_1^n + K_2^n) - p^o. \tag{23}$$

Analogously, for the new product we obtain the demand function

$$\tilde{\tilde{Q}}^n(p_n, K_1^o, K_2^o) = \frac{1 - \eta(K_1^o + K_2^o) - p^n}{\gamma}, \tag{24}$$

if $\tilde{Q}^o(p^o, p^n) > K_1^o + K_2^o$. It is straightforward but cumbersome to list the entire demand system, while taking into account unequal prices and all possible capacity constraint constellations. Therefore, we refrain from presenting the complete demand system. It will turn out that in equilibrium prices are always identical and capacity constraints for a given product bind either for none or both firms.

A.3 Price Equilibria in Case of Large Capacities on the Old Market

Here we first review existence and properties of equilibria in the pricing and capacity choice stage if the capacities of the firms in the old market are large.

We first consider the case where each of the two firms has sufficient capacity to serve the entire old market demand at a price of zero, i.e

$$K_1^o > K_2^o \geq Q^o(0) = 1, i = 1, 2.$$

The standard Bertrand argument learns that there exists an equilibrium with $p^o = 0$ regardless of capacities chosen on the new market. For the new market we are left with a standard model of capacity choice and price competition (Kreps and Scheinkman (1983)). Equilibrium capacities correspond to Cournot quantities based on the residual demand (see (21) with $p^o = 0$)

$$\tilde{Q}^n(0, p^n) = \frac{1-\eta}{\gamma - \eta^2} - \frac{1}{\gamma - \eta^2} p^n.$$

This gives equilibrium capacities of

$$K_i^{n*} = \max\left[0, \frac{1-\eta-c}{3(\gamma-\eta^2)}\right],$$

which are fully exploited at the equilibrium price

$$p^{n*} = \frac{1}{3}\left(1 - \eta + 2c\right),$$

whenever K_i^{n*} is positive. Production on the old market is given by

$$Q^{o*} = 1 - 2\eta K_i^{*n} = \min\left[1, \frac{1}{3} + \frac{2}{3}\frac{\gamma - \eta(1-c)}{\gamma - \eta^2}\right].$$

In cases where the sum of the capacities on the old market is sufficiently large to drive the price to zero, but at least firm 2 does not have sufficient capacity to serve the whole demand on the old market at a price of zero by itself, there exists no pure strategy equilibrium at the pricing stage. To argue why, we exploit a result of Proposition 8 (see section A.4) that states that in a pure strategy equilibrium both firms set an identical price for each separate product. Assume that a pure strategy equilibrium exists. Then the equilibrium price on the old market must either be zero or positive. However, a zero price is not an equilibrium, because, since firm 2's capacity falls short of serving the whole market, firm 1 can profitably deviate by marginally increasing its price. But also a positive price is not an equilibrium, because the firm that is not capacity constrained can gain by undercutting.[3]

A.4 Backward Induction Analysis of the Game

We further concentrate on the case where capacities on the old market are sufficiently small such that in equilibrium positive prices on both markets evolve. We treat the game in reverse order, implying that we start analyzing the pricing stage where the prices are determined for given capacity levels. This is followed by the analysis of the capacity investment stage where each innovating firm has to fix its capacity for the innovative product. Then we study the innovation stage in which the firms have to decide whether to innovate or not. The proofs of all propositions are given in Appendix B.

A.4.1 Analysis of the pricing stage

At this stage both firms have to fix output prices for the old product, and for the new product once they have entered the new market. In case a firm did not enter, the firm's capacity for the new product is zero, implying that the firm does not have to take any actual pricing decision for the new product. For notational convenience we set this price equal to the price for the new product chosen by the competitor. The next proposition shows that in equilibrium firms set identical prices for each separate product, while the corresponding capacities are always binding in case these prices are positive.

Proposition 8 *For any subgame characterized by capacities $K_1^o, K_2^o, K_1^n, K_2^n$ the following three statements hold:*

(i) In any subgame-perfect equilibrium in pure strategies both firms set an identical price for each separate product.

(ii) In any subgame where both firms have positive capacities for a product, the equilibrium prices are such that the demand for that product is strictly positive.

(iii) In any subgame where both firms have positive capacities for a product, the equilibrium price for that product can only be positive if capacities of both producers are binding.

In addition to this proposition it holds that a profile that induces a price of zero on the old market can only be an equilibrium profile if each of the two competitors has sufficient capacity to serve the entire market demand at a price

[3] In a symmetric product differentiation market in which each firm offers one different product, Vives (1999, pp.165-166) shows that a symmetric pure strategy equilibrum need not exist in a similar situation, i.e. where at equilibrium excess aggregate capacity of the competitors of any firm is smaller than individual firm production.

of zero.[4] Furthermore, it is easy to understand that building up capacities on the new market that allow to serve the entire market demand arising with $p^n = 0$ is never rational. Therefore, we can conclude that there are two different types of equilibria: one where the price on the old market equals zero, which occurs when both firms have enough capacity to serve the whole market, and one where the price on the old market is positive. In what follows we assume that capacities for the old product are not so large that the price on that market becomes zero if capacities are fully employed. This ensures existence of a pure strategy equilibrium at the pricing stage where all prices are positive.

The following proposition provides the conditions that the capacity levels have to fulfill in order for such an equilibrium to exist.

Proposition 9 *Consider pure-strategy equilibria of the price-setting game in a subgame characterized by capacities* $(K_1^o, K_2^o, K_1^n, K_2^n)$ *with* $K_1^n, K_2^n > 0$. *Then an equilibrium with* $p^o > 0, p^n > 0$ *exists if and only if*

$$\begin{array}{lrcl} (a) & 2K_1^o + 2\eta K_1^n + K_2^o + \eta K_2^n & \leq & 1, \\ (b) & K_1^o + \eta K_1^n + 2K_2^o + 2\eta K_2^n & \leq & 1, \\ (c) & 2\eta K_1^o + 2\gamma K_1^n + \eta K_2^o + \gamma K_2^n & \leq & 1, \\ (d) & \eta K_1^o + \gamma K_1^n + 2\eta K_2^o + 2\gamma K_2^n & \leq & 1. \end{array}$$

Next we turn to the case where only one of the firms has entered the new market. The following proposition provides existence conditions in terms of capacity levels.

Proposition 10 *Consider a subgame characterized by capacities* $(K_1^o, K_2^o, K_1^n, K_2^n)$ *with* $K_i^n = 0$.

i) If $0 < K_j^n \leq K^*, j \neq i$ *with*

$$K^* = \frac{1-\eta}{2(\gamma - \eta^2)}, \tag{25}$$

then there exists a pure strategy equilibrium with $p_1^o = p_2^o > 0, p_j^n > 0$ *if and only if*

$$2K_j^o + 2\eta K_j^n + K_i^o < 1. \tag{26}$$

In this equilibrium it holds that $q_1^o = K_1^o, q_2^o = K_2^o, q_j^n = K_j^n$.

ii) If $K_j^n > K^*, j \neq i$ *then there exists a pure strategy equilibrium equilibrium with* $p_1^o = p_2^o > 0, p_j^n > 0$ *if and only if*

$$2K_j^o + K_i^o < \frac{\gamma - \eta}{\gamma - \eta^2}. \tag{27}$$

In this equilibrium it holds that $q_1^o = K_1^o, q_2^o = K_2^o, q_j^n = K^* < K_j^n$.

[4] In Appendix A.3 we state the reason why no pure strategy equilibrium with a price of zero exists if capacity of one of the firms is not sufficient to serve the whole market.

Proposition 10 shows conditions for existence of pricing equilibria in case only one of the firms has entered the new market. However, in general these equilibria are not unique. Instead, there exists a whole continuum of price vectors that are Nash equilibria. For these price vectors it holds that the prices for the old product are such that they fall below the equilibrium price of the old product when the new product would not exist. In what follows we select the equilibrium with the largest price of the old product, implying that this price is closest to the equilibrium price of the old product in absence of the new product price levels.

A.4.2 Analysis of the Capacity Investment Stage

A subgame at the capacity investment stage is characterized by the capacities on the old market and the entry decision at the first stage. Here the firms that entered the new market have to choose the capacity levels necessary to produce the new product. First, consider the case where both firms have entered. The following proposition gives the equilibrium levels of capacities on the new market, the resulting equilibrium payoffs, and an existence condition depending on capacities on the old market.[5]

Proposition 11 *In any subgame* (K_1^o, K_2^o, Y, Y) *there exists a subgame perfect equilibrium with* $K_1^n, K_2^n \geq 0, p^o > 0, p^n > 0$ *if conditions (6) and (7) hold. The equilibrium capacity of both firms for the new product reads*

$$K_i^{n*} = K_{i,YY}^n := \frac{1 - c - 3\eta K_i^o}{3\gamma}, \tag{28}$$

and the equilibrium profits are given by

$$\pi_{i,YY} = \frac{(1-c)^2}{9\gamma} + K_i^o \left(1 - \frac{\eta}{\gamma}(1-c)\right) - K_i^o(K_i^o + K_j^o)\frac{\gamma - \eta^2}{\gamma}. \tag{29}$$

The following proposition presents the innovative capacity levels, payoffs, and existence conditions for the subgames where only one firm has the option to produce the new product.

Proposition 12 *In any subgame* (K_1^o, K_2^o, D_1, D_2) *with* $D_i = N, D_j = Y$ *there exists a subgame perfect equilibrium with* $K_j^n \geq 0, p^o > 0, p^n > 0$ *if* $2K_j^o + K_i^o \leq \frac{1-c}{\eta}, i = 1, 2$ *and (7) hold. The equilibrium capacity of firm j for the new product*

[5] In order for the statement '$(\tilde{K}_1^n, \tilde{K}_2^n)$ are equilibrium capacity levels at Stage 2' to be meaningful, we must make sure that for the given capacities of the old product an equilibrium at the pricing stage exists for any $(K_1^n, K_2^n) \in [0, \bar{K}_1^n] \times [0, \bar{K}_2^n]$. In that respect this statement has the precise meaning that there exist some $\bar{K}_1^n, \bar{K}_2^n$ with $\bar{K}_i^n > \tilde{K}_i^n$ such that an equilibrium at the pricing stage exists for any $(K_1^n, K_2^n) \in [0, \bar{K}_1^n] \times [0, \bar{K}_2^n]$ and $(\tilde{K}_1^n, \tilde{K}_2^n)$ are equilibrium choices in the subgame perfect equilibrium of the subgame (K_1^o, K_2^o, Y, Y) of that game. Similar statements apply to the cases where only one firm has decided to carry out the product innovation project.

reads

$$K_j^{n*} = K_{j,D_1D_2}^n := \begin{cases} \frac{1-c-\eta(2K_j^o+K_i^o)}{2\gamma} & 2K_j^o + K_i^o \geq \frac{\gamma-\eta}{\gamma-\eta^2} - \frac{1}{\eta}c \\ K^* & 2K_j^o + K_i^o < \frac{\gamma-\eta}{\gamma-\eta^2} - \frac{1}{\eta}c, \end{cases} \tag{30}$$

and the equilibrium profits are given by

$$\pi_{j,D_1D_2} = \begin{cases} \frac{(1-c)^2}{4\gamma} + K_j^o\left(1-(1-c)\frac{\eta}{\gamma}\right) & \\ -K_i^o\frac{\eta}{2\gamma}\left(1-c-\frac{\eta}{2}K_i^o\right) - \frac{\gamma-\eta^2}{\gamma}K_j^o(K_j^o+K_i^o) & 2K_j^o + K_i^o \geq \frac{\gamma-\eta}{\gamma-\eta^2} - \frac{1}{\eta}c \\ \frac{\gamma-\eta^2-\eta^2(1+\gamma-2\eta)}{4(\gamma-\eta^2)^2} - \frac{\eta(1-\eta)}{2(\gamma-\eta^2)}(K_i^o+K_j^o) & \\ +K_j^o\left(1-\frac{\eta(1-\eta)}{2(\gamma-\eta^2)} - (K_i^o+K_j^o)\right) - \frac{1\eta}{2(\gamma-\eta^2)}c & 2K_j^o + K_i^o < \frac{\gamma-\eta}{\gamma-\eta^2} - \frac{1}{\eta}c, \end{cases} \tag{31}$$

$$\pi_{i,D_1D_2} = \begin{cases} \left(\frac{2\gamma-\eta(1-c)}{2\gamma} - \frac{\gamma-\eta^2}{\gamma}K_j^o - \frac{2\gamma-\eta^2}{2\gamma}K_i^o\right)K_i^o & 2K_j^o + K_i^o \geq \frac{\gamma-\eta}{\gamma-\eta^2} - \frac{1}{\eta}c \\ \left(1-\frac{\eta(1-\eta)}{2(\gamma-\eta^2)} - (K_i^o+K_j^o)\right)K_i^o & 2K_j^o + K_i^o < \frac{\gamma-\eta}{\gamma-\eta^2} - \frac{1}{\eta}c, \end{cases} \tag{32}$$

where $D_j = Y, D_i = N$.

Finally, if no firm has the option to introduce the new product, profits correspond to the standard expression for capacity constrained price competition taking place on the old market. The result is presented in the last proposition of this section.

Proposition 13 *In any subgame* (K_1^o, K_2^o, N, N) *there exists a unique subgame perfect equilibrium with positive profits if and only if the condition*

$$K_1^o + K_2^o < 1$$

holds. Equilibrium profits are given by

$$\pi_{i,NN} = (1-(K_i^o+K_j^o))K_i^o. \tag{33}$$

A.4.3 Analysis of the innovation stage

Based on these results the following conditions can be derived which induce exsitence of a subgame perfect equilibrium of the full game.

Proposition 14 *For any set of parameter values satisfying the conditions*

(i) $K_1^o < \frac{1-c}{3\eta}$,

(ii) $2K_1^o + K_2^o < \frac{\gamma-\eta(1-c)}{\gamma-\eta^2}$,

(iii) $K_1^o + K_2^o < 1$.

there exists a pure strategy subgame perfect equilibrium of the game.

B Proofs

B.1 Proof of Proposition 1

This proposition summarizes the results of Propositions 8 and 10 - 14.

B.2 Proof of Proposition 2

To show parts (i) and (ii) of the proposition we use the expressions of the payoffs presented in Propositions 11, 12, and 13 to obtain

$$\Delta\pi_{i,Y} = \begin{cases} \frac{(1-c)^2}{9\gamma} - \frac{\eta}{2\gamma}((1-c) - \eta K_i^o)K_i^o - I & K_i^o + 2K_j^o \geq \frac{\gamma-\eta}{\gamma-\eta^2} - \frac{1}{\eta}c \\ \frac{(1-c)^2}{9\gamma} + \frac{\eta(1-\eta)}{2(\gamma-\eta^2)}K_i^o & \\ -\frac{\eta}{\gamma}(1-c-\eta(K_i^o+K_j^o))K_i^o - I & K_i^o + 2K_j^o < \frac{\gamma-\eta}{\gamma-\eta^2} - \frac{1}{\eta}c \end{cases}, \tag{34}$$

$$\Delta\pi_{i,N} = \begin{cases} \frac{1}{4\gamma}((1-c) - \eta(2K_i^o + K_j^o))^2 - I & 2K_i^o + K_j^o \geq \frac{\gamma-\eta}{\gamma-\eta^2} - \frac{1}{\eta}c \\ \frac{1-\eta}{4(\gamma-\eta^2)^2}(\gamma(1+\eta-2c) - 2\eta^2(1-c) & \\ -2\eta(\gamma-\eta^2)(2K_i^o+K_j^o)) - I & 2K_i^o + K_j^o < \frac{\gamma-\eta}{\gamma-\eta^2} - \frac{1}{\eta}c \end{cases}. \tag{35}$$

Parts (i) and (ii) follow directly from these expressions, where the condition $K_i^o \leq \frac{1-c}{3\eta}$ has to be used to show monotonicity of $\Delta\pi_{i,Y}$.

To prove part (iii) consider an arbitrary pair $(\tilde{K}_1^o, \tilde{K}_2^o), \tilde{K}_1^o \geq \tilde{K}_2^o$ of capacities for the old product. If $\tilde{K}_1^o + 2\tilde{K}_2^o \geq \frac{\gamma-\eta}{\gamma-\eta^2} - \frac{c}{\eta}$ we have

$$\Delta\pi_{1,Y} - \Delta\pi_{2,Y} = \frac{\eta}{2\gamma}((1-c-\eta\tilde{K}_2^o)\tilde{K}_2^o - (1-c-\eta\tilde{K}_1^o)\tilde{K}_1^o),$$

which is negative due to $K_1^o > K_2^o$ and the condition $K_1^o \leq \frac{1-c}{3\eta}$. To show that this inequality also holds for $\tilde{K}_1^o + 2\tilde{K}_2^o < \frac{\gamma-\eta}{\gamma-\eta^2} - \frac{c}{\eta} < 2\tilde{K}_1^o + \tilde{K}_2^o$, we observe that for such a parameter constellation we have $K_{1,YN}^n = \frac{1-c-\eta(2\tilde{K}_1^o+\tilde{K}_2^o)}{2\gamma}$ and $K_{2,NY}^n = K^* < \frac{1-c-\eta(2\tilde{K}_1^o+\tilde{K}_2^o)}{2\gamma}$. Obviously, $\Delta\pi_{2,Y}$ is constant in $K_{2,NY}^n$ whereas $\Delta\pi_{1,Y}$ increases with increasing $K_{2,NY}^n$. Accordingly, $\Delta\pi_{1,Y} - \Delta\pi_{2,Y}$ increases if $K_{2,NY}^n = K^*$ is replaced by $K_{2,NY}^n = \frac{1-c-\eta\left(2\tilde{K}_1^o+\tilde{K}_2^o\right)}{2\gamma}$. For the latter expression we know from the calculations above that $\Delta\pi_{1,Y} - \Delta\pi_{2,Y} < 0$ and therefore this must also hold for the equilibrium value $K_{2,NY}^n = K^*$. Finally, if $2\tilde{K}_1^o + \tilde{K}_2^o \leq \frac{\gamma-\eta}{\gamma-\eta^2} - \frac{c}{\eta}$ inspection of the corresponding terms directly shows that again $\Delta\pi_{1,Y} - \Delta\pi_{2,Y} < 0$.

Analogous arguments establish that $\Delta\pi_{1,N} - \Delta\pi_{2,N} < 0$.

B.3 Proof of Proposition 3

Assume that for a given parameter set there exists a subgame perfect equilibrium where firm 1 carries out the innovation project. If in that equilibrium both firms choose $D_i = Y$ the claim of the proposition is trivially true. If in that equilibrium $D_1 = Y, D_2 = N$, we must have $\Delta\pi_{1,N} > 0$ and $\Delta\pi_{2,Y} < 0$. Using Proposition 2 this implies $\Delta\pi_{2,N} > 0$ and $\Delta\pi_{1,Y} < 0$. Therefore, there exists also a Nash equilibrium of the entry stage game with $D_1 = N, D_2 = Y$.

B.4 Proof of Proposition 8

(i) To prove that firms set identical prices, assume that for one of the two products it holds that $p_1^h \neq p_2^h$. Without loss of generality assume $p_1^h < p_2^h$. We consider three cases. If $p_1^h = 0$, then the current profit of firm 1 generated by product h is zero. If the firm increases its price by some amount $\epsilon < p_2^h$, then its profit generated by the product h becomes positive and at the same time the demand for the other product is not lowered, which implies that the total profit of firm 1 increases. So, $p_1^h = 0$ cannot be an equilibrium choice for firm 1 if $p_2^h > 0$.

The second case to consider is $p_1^h > 0$, where capacity of firm 1 for product h is binding in the sense that the demand for product h under price p_1^h is strictly larger than K_1^h. Here firm 1 can increase profits by increasing p_1^h by an $\epsilon > 0$ that is sufficiently small to ensure that the capacity constraint remains binding. As in the first case, this will not lower demand for the other product.

In the third case we have $p_1^h > 0$ and the capacity of firm 1 is larger or equal than the demand under this price. Here the sales of firm 2 for product h are equal to zero. Without loss of generality let us assume that $h = n$. If firm 2 would alter its price to $\tilde{p}_2^n = p_1^n - \epsilon > 0$ for some small $\epsilon > 0$, its profits generated from selling product n would increase by $\Delta\pi_2^n = \tilde{p}_2^n \min\left[K_2^n, q_1^n + \frac{\epsilon}{\gamma-\eta^2}\right]$. Due to our convention that firms with capacity zero set the price equal to their competitor, $p_1^n < p_2^n$ implies $K_2^n > 0$. Therefore $\Delta\pi_2^n > 0$. If the capacity constraint of firm 2 for the old product is binding, profits there do not change and therefore total profits of firm 2 increase due to this change in price. If firm 2's capacity constraint for the old product is not binding and firm 2 sells a positive quantity q_2^o of the old product under the original price, then profits of Firm 2 from the old product decrease by an amount less than or equal to $\Delta\pi_2^o = p_2^o \frac{\epsilon\eta}{\gamma-\eta^2}$. Accordingly, total profit of firm 2 changes by

$$\Delta\pi_2 = \Delta\pi_2^n - \Delta\pi_2^o \geq (p_1^n - \epsilon)\min\left[K_2^n, q_1^n + \frac{\epsilon}{\gamma - \eta^2}\right] - p_2^o \frac{\epsilon\eta}{\gamma - \eta^2},$$

and it is easy to see that $\Delta\pi_2 > 0$ for sufficiently small ϵ.

(ii) Assume that there exists an equilibrium in the pricing stage where $p_1^o = p_2^o = p^o$ is such that demand for the old product is zero. Without loss of generality we consider the price where any decrease yields positive demand. Note that $p^o > 0$. From part (i) of this proposition we know that $p_1^n = p_2^n = p^n$. Now consider the following deviation of prices set by firm 1: $\tilde{p}_1^o = p^o - \xi, \tilde{p}_1^n = p^n - \epsilon$.

If ϵ is sufficiently small compared to ξ, this implies that

$$\tilde{q}_1^o = -\frac{\eta}{\gamma - \eta^2}\epsilon + \frac{\gamma}{\gamma - \eta^2}\xi > 0,$$
$$\tilde{q}_1^n \geq q_1^n.$$

Accordingly, this deviation induces a change in profit being equal to

$$\begin{aligned}
&(\tilde{\pi}_1^o + \tilde{\pi}_1^n) - (\pi_1^o + \pi_1^n) \\
&= (p^o \tilde{q}_1^o + \tilde{p}^n \tilde{q}_1^n) - (p^o q_1^o + p^n q_1^n) \\
&= p^o \tilde{q}_1^o + (p^n(\tilde{q}_1^n - p^n q_1^n) - \tilde{q}_1^n \epsilon \\
&\geq 0,
\end{aligned}$$

where the last inequality follows from the fact that the last term can be made arbitrarily small. Therefore, there exists a profitable deviation and the considered profile cannot be an equilibrium. The same arguments imply that there cannot be an equilibrium where demand for the new product is zero.

(iii) From the first statement we know that in equilibrium it holds that $p_1^h = p_2^h = p^h$. Assume that $p^h > 0$ and $q_1^h < K_1^h$, where q_1^h denotes the sales of firm 1 for product h given all capacities and prices. Due to $K_i^h > 0$, and our assumption that firms equally share demand in case of equal prices, it follows from statement (ii) that $q_i^h > 0, i = 1, 2$. Using a standard argument in the analysis of Bertrand competition, we know that firm 1 can gain sales by the amount of $\min\left(K_1^h - q_1^h, q_2^h\right)$, if it reduces its price by a marginal amount. Obviously, this deviation is profitable for firm 1 and therefore $q_1^h < K_1^h$ cannot hold in equilibrium. Analogous arguments hold for the second firm.

B.5 Proof of Proposition 9

Note first that it follows from Proposition 8 that in any equilibrium with $p^o > 0, p^n > 0$, capacities of both firms for both products must be binding. From (23) and (24) we obtain that with binding capacities prices are given by

$$p^o = 1 - (K_1^o + K_2^o) - \eta(K_1^n + K_2^n),$$
$$p^n = 1 - \eta(K_1^o + K_2^o) - \gamma(K_1^n + K_2^n).$$

Accordingly, the conditions

$$(K_1^o + \eta K_1^n) + (K_2^o + \eta K_2^n) < 1,$$
$$(\eta K_1^o + \gamma K_1^n) + (\eta K_2^o + \gamma K_2^n) < 1$$

result in positive prices. It is easy to see that the first condition is implied by (a) and the second by (d).

It is obvious that, since capacities are binding, it is not profitable for a firm to lower the price. Therefore, all that has to be checked is whether it pays for a firm to increase both prices. Since the analysis for firm 2 is analogous, it is sufficient to focus on deviation incentives for firm 1. First, we show that under conditions (a) - (d) it is never profitable for Firm 1 to increase both prices in a way that its capacity constraints for both products are no longer binding. If firm 1 raises both prices to $\tilde{p}_1^o > p_2^o, \tilde{p}_1^n > p_2^n$ such that both capacity constraints do not bind, then, while taking into account that capacity constraints for firm 2 are still binding, we obtain the following sales for firm 1 (cf. (21)-(22)):

$$\tilde{q}_1^o(\tilde{p}_1^o, \tilde{p}_1^n) = \frac{\gamma - \eta}{\gamma - \eta^2} - K_2^o + \frac{\eta}{\gamma - \eta^2}\tilde{p}_1^n - \frac{\gamma}{\gamma - \eta^2}\tilde{p}_1^o,$$
$$\tilde{q}_1^n(\tilde{p}_1^o, \tilde{p}_1^n) = \frac{1 - \eta}{\gamma - \eta^2} - K_2^n - \frac{1}{\gamma - \eta^2}\tilde{p}_1^n + \frac{\eta}{\gamma - \eta^2}\tilde{p}_1^o.$$

The profit of firm 1 under such a deviation reads

$$\tilde{\pi}_1(\tilde{p}_1^o, \tilde{p}_1^n) = \tilde{q}_1^o(\tilde{p}_1^o, \tilde{p}_1^n)\tilde{p}_1^o + \tilde{q}_1^n(\tilde{p}_1^o, \tilde{p}_1^n)\tilde{p}_1^n,$$

and considering the partial derivative of this profit function with respect to p_1^o at the original prices we obtain that

$$\begin{aligned}
\frac{\partial \tilde{\pi}_1(p^o, p^n)}{\partial \tilde{p}_1^o} &= K_1^o - \frac{\gamma}{\gamma - \eta^2}p^o + \frac{\eta}{\gamma - \eta^2}p^n \\
&= K_1^o + \left(K_1^o + K_2^o - \frac{\gamma - \eta}{\gamma - \eta^2}\right) \\
&= 2K_1^o + K_2^o - \frac{\gamma - \eta}{\gamma - \eta^2},
\end{aligned}$$

where we use $\tilde{q}_1^o(p^o, p^n) = K_1^o$ to derive the first and the second line. In a similar way it can also be shown that

$$\frac{\partial \tilde{\pi}_1(p^o, p^n)}{\partial \tilde{p}_1^n} = 2K_1^n + K_2^n - \frac{1 - \eta}{\gamma - \eta^2}.$$

If it is optimal for firm 1 to increase both prices in a way that both capacity constraints are not binding the gradient of $\tilde{\pi}_1$ must have a positive slope. This would imply that both partial derivatives of $\tilde{\pi}_1$ must be positive, which means that

$$2K_1^o + K_2^o > \frac{\gamma - \eta}{\gamma - \eta^2}, \tag{36}$$
$$2K_1^n + K_2^n > \frac{1 - \eta}{\gamma - \eta^2}. \tag{37}$$

Adding η times (36) to γ times (37) yields

$$2\eta K_1^o + 2\gamma K_1^n + \eta K_2^o + \gamma K_2^n > 1,$$

and this inequality contradicts (c). Accordingly, under condition (c) it is never profitable for firm 1 to increase prices such that both capacity constraints are no longer binding.

Let us now turn to potential deviations where firm 1 increases the prices in a way that one capacity constraint remains binding. If the capacity constraint for the old product remains binding we must have (cf. (22))

$$K_1^o = \frac{\gamma - \eta}{\gamma - \eta^2} - K_2^o + \frac{\eta}{\gamma - \eta^2}\tilde{p}_1^n - \frac{\gamma}{\gamma - \eta^2}\tilde{p}_1^o,$$

and from using this condition we can depict the price of the old product as a function of $\tilde{p}_1^n$:

$$\hat{p}_1^o(\tilde{p}_1^n) = \frac{\gamma - \eta}{\gamma} - \frac{\gamma - \eta^2}{\gamma}(K_1^o + K_2^o) + \frac{\eta}{\gamma}\tilde{p}_1^n.$$

The profit of firm 1 expressed as a function of $\tilde{p}_1^n$ reads

$$\hat{\pi}_1(\tilde{p}_1^n) = \hat{p}_1^o(\tilde{p}_1^n)K_1^o + \tilde{p}_1^n\tilde{q}_1^n(\hat{p}_1^o(\tilde{p}_1^n), \tilde{p}_1^n)).$$

Accordingly, the derivative of $\hat{\pi}_1$ at $\tilde{p}_1^n = p^n$ is given by

$$\begin{aligned}\frac{\partial\hat{\pi}_1(p^n)}{\partial\tilde{p}_1^n} &= \frac{\partial\hat{p}_1^o}{\partial\tilde{p}_1^n}K_1^o + K_1^n + \left(\frac{\partial\tilde{q}_1^n}{\partial\tilde{p}_1^o}\frac{\partial\hat{p}_1^o}{\partial\tilde{p}_1^n} + \frac{\partial\tilde{q}_1^n}{\partial\tilde{p}_1^n}\right)p^n \\ &= \frac{\eta}{\gamma}K_1^o + K_1^n - \frac{1}{\gamma}p^n \\ &= 2\frac{\eta}{\gamma}K_1^o + 2K_1^n + \frac{\eta}{\gamma}K_2^o + K_2^n - \frac{1}{\gamma}.\end{aligned}$$

This shows that an increase of prices such that the capacity constraint for the old market remains binding, is profitable for firm 1 if and only if condition (c) is violated. Similar calculations establish that an increase of prices such that the capacity constraint for the new market keeps binding is profitable for firm 1 if and only if condition (a) is violated. The analogous conditions for firm 2 are given by (b) and (d). Therefore, if and only if all conditions (a) -(d) are satisfied, no firm has incentives to deviate from setting prices $p_i^o = p^o$, $p_i^n = p^n$, where under p^o, p^n all capacity constraints bind. This completes the proof.

B.6 Proof of Proposition 10

We first prove part ii) of the proposition. Without restriction of generality we set $j = 1, i = 2$. Note first that analogous arguments as in the proof of Proposition 8 show that for given prices p_1^o, p_1^n such that $Q^o(p_1^o, p_1^n) = K_1^o + K_2^o$, firm 2 has no incentives to deviate from $p_2^o = p_1^o$. Considering firm 1, it is obvious that setting $p_1^o < p_2^o$ is dominated by $p_1^o = p_2^o$ for all p_1^n, p_2^o where $Q^o(p_2^o, p_1^n) = K_1^o + K_2^o$. From Proposition 8 (iii) we know that in equilibrium $Q^o(p_2^o, p_1^n) = K_1^o + K_2^o$ has to hold. Therefore, we only consider choices of prices of firm 1 with $p_1^o \geq p_2^o$. Under the assumption that capacity constraints for the new product do not bind, firm 1 solves the constrained maximization problem

$$\max_{p_1^o, p_1^n}(Q^o(p_1^o, p_1^n) - K_2^o)p_1^o + Q^n(p_1^o, p_1^n)p_1^n$$
$$\text{s.t. } p_1^o \geq p_2^o$$
$$Q^o(p_1^o, p_1^n) - K_2^o \leq K_1^o.$$

The Lagrangian of the problem is denoted by

$$L(p_1^o, p_1^n, \mu) = (Q^o(p_1^o, p_1^n) - K_2^o)p_1^o + Q^n(p_1^o, p_1^n)p_1^n + \mu(p_1^o - p_2^o) + \nu(K_1^o + K_2^o - Q^o(p_1^o, p_1^n))$$

with $\mu, \nu \geq 0$. Employing the Karush-Kuhn-Tucker conditions we get as sufficient optimality conditions:

$$\begin{aligned}
\frac{\partial L}{\partial p_1^o} &= 0, \\
\frac{\partial L}{\partial p_1^n} &= 0, \\
\mu(p_1^o - p_2^o) &= 0, \\
\nu(K_1^o + K_2^o - Q^o(p_1^o, p_1^n)) &= 0.
\end{aligned}$$

Inserting the demand expressions one directly obtains that

$$p_1^o = \frac{1 - K_2^o + \mu + \nu}{2}, \tag{38}$$

$$p_1^n = \frac{1 - \eta - \nu}{2} + \eta p_1^o. \tag{39}$$

Assuming $\mu > 0$ we must have $p_1^o = p_2^o$, so that $\mu = 2p_2^o + K_2^o - 1 - \nu$. Accordingly the condition

$$2p_2^o + K_2^o - 1 - \nu > 0 \tag{40}$$

must hold. For the price vector (p_1^o, p_2^o, p_1^n) to be an equilibrium we must have $Q^o(p_2^o, p_1^n) = K_1^o + K_2^o$. Inserting (39) and taking into account $p_1^o = p_2^o$, we obtain from this equality that

$$p_2^o = 1 - \frac{\eta(1 - \eta + \nu)}{2(\gamma - \eta^2)} - (K_1^o + K_2^o).$$

Inserting this into 40 gives the condition

$$1 - \frac{\eta(1 - \eta)}{\gamma - \eta^2} - (2K_1^o + K_2^o) - \nu\frac{\gamma + \eta - \eta^2}{\gamma - \eta^2} > 0.$$

Obviously, this inequality is satisfied for some non-negative ν if and only if it is satisfied for $\nu = 0$. Hence we set $\nu = 0$. This gives

$$p_2^o = \tilde{p}^o := 1 - \frac{\eta(1 - \eta)}{2(\gamma - \eta^2)} - (K_1^o + K_2^o), \tag{41}$$

which implies that condition (40) is equivalent to (27). Furthermore, it is easy to check that (27) implies $p_1^o = p_2^o = \tilde{p}^o > 0$, which by (39) yields $p_1^n > 0$. Altogether this shows that if (27) holds, then, if p_2^o is given by (41), it is optimal for firm 1 to choose $p_1^o = p_2^o = \tilde{p}^o$ and

$$p_1^n = \tilde{p}_1^n = 1 - \frac{\gamma(1 - \eta)}{2(\gamma - \eta^2)} - \eta(K_1^o + K_2^o). \tag{42}$$

It also holds that $Q^o(\tilde{p}_o, \tilde{p}_1^n) = K_1^o + K_2^o$. For the quantity of the new product we obtain

$$Q^n(\tilde{p}_o, \tilde{p}_1^n) = \frac{1-\eta}{2(\gamma-\eta^2)} := K^*.$$

This implies that under the conditions given in part (ii) of the proposition, $(\tilde{p}_1^o, \tilde{p}_2^o, \tilde{p}_1^n)$ is an equilibrium at the pricing stage where both prices are positive, capacities for the old product are binding, while there is idle capacity for the new product.

To establish the 'only if' part of the proposition we note that in case where (27) is violated for a best response of firm 1 to some p_2^o, which has the property $Q^o(p_1^o, p_1^n) = K_1^o + K_2^o$, we must have $p_1^o > p_2^o$. Put formally, there is no non-negative μ such that the Karush-Kuhn-Tucker conditions are satisfied for $p_1^o = p_2^o$ and $Q^o(p_1^o, p_1^n) = K_1^o + K_2^o$. We know from Proposition 8 that there are no equilibria with $p_1^o > p_2^o$ and there are no equilibria with idle capacity for the old product.

The proof of part (i) of the proposition follows the same arguments with the exception that for $K_1^n < K^*$ firm 1 has to take explicitly into account its capacity constraint for the new product. Consequently, the optimization problem firm 1 solves to determine its best response to p_2^o has the additional constraint

$$Q^n(p_1^o, p_1^n) \leq K_1^n.$$

Solving this extended optimization problem and following similar steps as in the proof of part (ii) shows that if and only if condition (26) is satisfied, there is a price p_2^o and an optimal solution of firm 1's optimization problem with the properties $p_1^o = p_2^o$ and $Q^o(p_1^o, p_1^n) = K_1^o + K_2^o$. If a solution exists it is given by

$$p_1^o = p_2^o = \tilde{p}^o := 1 - (K_1^o + K_2^o) - \eta K_1^n \tag{43}$$

$$p_1^n = \tilde{p}_1^n := 1 - \eta(K_1^o + K_2^o) - \gamma K_1^n. \tag{44}$$

B.7 Proof of Proposition 11

If both firms have positive capacities on both markets, then it follows from Proposition 8 that both firms charge identical prices for the same product and in any equilibrium with $p^o > 0, p^n > 0$ all capacities are binding. Consequently, the profit of firm i at the capacity investment stage reads

$$\pi_i(K_i^n, K_j^n) = (1-\eta(K_1^o+K_2^o)-\gamma(K_1^n+K_2^n))K_i^n+(1-(K_1^o+K_2^o)-\eta(K_1^n+K_2^n))K_i^o-cK_i^n.$$

¿From the first order condition we obtain the reaction function

$$K_i^n = \frac{1-c-\eta(2K_i^o+K_j^o)}{2\gamma} - \frac{1}{2}K_j^n,$$

which yields the equilibrium capacities

$$K_{iYY}^n = \frac{1-c-3\eta K_i^o}{3\gamma}.$$

These capacities are positive due to the condition that $K_i^o \leq \frac{1-c}{3\eta}$. Inserting these capacities into the profit function of firm i immediately yields (29).

To prove existence we show that condition (7) implies conditions (a) - (d) of Proposition 9, taking into account that $K_i^n = K_{i,YY} > 0$. To do so we substitute equation (28) into the conditions of Proposition 9. This gives for condition (a):

$$\begin{aligned} 2K_1^o + 2\eta\frac{1-c-3\eta K_1^o}{3\gamma} + K_2^o + \eta\frac{1-c-3\eta K_2^o}{3\gamma} &\leq 1 \\ 6\gamma K_1^o + 2\eta\left(1-c-3\eta K_1^o\right) + 3\gamma K_2^o + \eta\left(1-c-3\eta K_2^o\right) &\leq 3\gamma \\ 6\left(\gamma-\eta^2\right)K_1^o + 3\left(\gamma-\eta^2\right)K_2^o &\leq 3\gamma - 3\eta\left(1-c\right) \\ 2K_1^o + K_2^o &\leq \frac{\gamma-\eta\left(1-c\right)}{\gamma-\eta^2}, \end{aligned} \quad (45)$$

which is the same as (7). Performing the same exercise for condition (b) leads to

$$\begin{aligned} K_1^o + \eta\frac{1-c-3\eta K_1^o}{3\gamma} + 2K_2^o + 2\eta\frac{1-c-3\eta K_2^o}{3\gamma} &\leq 1 \\ K_1^o + 2K_2^o &\leq \frac{\gamma-\eta\left(1-c\right)}{\gamma-\eta^2}. \end{aligned}$$

Since $K_1^o \geq K_2^o$, condition (45) is tighter, so this is implied by (7). Confronting (28) with condition (c) gives

$$2\eta K_1^o + 2\gamma\frac{1-c-3\eta K_1^o}{3\gamma} + \eta K_2^o + \gamma\frac{1-c-3\eta K_2^o}{3\gamma} \leq 1,$$

which is automatically satisfied. The same holds for condition (d), i.e.

$$\eta K_1^o + \gamma\frac{1-c-3\eta K_1^o}{3\gamma} + 2\eta K_2^o + 2\gamma\frac{1-c-3\eta K_2^o}{3\gamma} \leq 1.$$

B.8 Proof of Proposition 12

We know from Proposition 10 that if the competitor has not innovated, the equilibrium quantity for the new product chosen by firm j will never exceed K^*.Consequently, additional capacity above that level will stay idle at the pricing stage. Since at the pricing stage both firms act simultaneously there is no strategic reason to hold excess capacity and therefore it can never be optimal for firm j to fix a capacity level $K_j^n > K^*$.

Assuming that $K_j^n \leq K^*$, which implies that the production capacity will be fully used, the profit of firm j at the capacity investment stage reads

$$\pi_j(K_j^n, 0) = (1-\eta(K_1^o+K_2^o)-\gamma K_j^n)K_j^n + (1-(K_1^o+K_2^o)-\eta K_j^n)K_j^o - cK_j^n.$$

¿From the first order condition we obtain the optimal capacity choice given by the first line of (30). This capacity is positive due to the condition that

$2K_j^o + K_i^o \leq \frac{1-c}{\eta}$ and below K^* if and only if $2K_j^o + K_i^o \geq \frac{\gamma-\eta}{(\gamma-\eta^2} - \frac{1}{\eta}c$. Inserting the optimal capacities into the profit function of the two firms immediately yields (31) and (32).

To prove existence of the equilibrium we show that condition (7) implies expression (26) of Proposition 10. Inserting the optimal capacity K_j^n for $2K_j^o + K_i^o \geq \frac{\gamma-\eta}{(\gamma-\eta^2} - \frac{1}{\eta}c$ into 10 gives the condition

$$2K_j^o + 2\eta \left(\frac{1 - c - \eta(K_i^o + 2K_j^o)}{2\gamma} \right) + K_j^o < 1.$$

Straightforward transformations show that this is equivalent to (7). Since for $2K_j^o + K_i^o < \frac{\gamma-\eta}{(\gamma-\eta^2} - \frac{1}{\eta}c$, the optimal value of K_j^n is smaller than the capacity level presented in the first line of (30), condition (26) always holds in such a case. Consequently, equilibrium existence at the pricing stage is guaranteed.

B.9 Proof of Proposition 14

First observe that conditions (i), (ii) imply all the conditions needed in Propositions 11, 12 and 13. Therefore, in all four subgames of the capacity investment stage pure strategy subgame perfect equilibria exist. The 2×2 game with payoff matrix (13) has no equilibrium in pure strategies if and only if either

$$\Delta\pi_{1,Y} := \pi_{1,YY} - I - \pi_{1,NY} > 0 \tag{46}$$
$$\Delta\pi_{1,N} := \pi_{1,YN} - I - \pi_{1,NN} < 0 \tag{47}$$
$$\Delta\pi_{2,Y} := \pi_{2,YY} - I - \pi_{2,YN} < 0 \tag{48}$$
$$\Delta\pi_{2,N} := \pi_{2,NY} - I - \pi_{2,NN} > 0, \tag{49}$$

or

$$\Delta\pi_{1,Y} < 0 \tag{50}$$
$$\Delta\pi_{1,N} > 0 \tag{51}$$
$$\Delta\pi_{2,Y} > 0 \tag{52}$$
$$\Delta\pi_{2,N} < 0 \tag{53}$$

hold.

The first of these options however violates the property $\Delta\pi_{1,Y} < \Delta\pi_{2,Y}$ established in Proposition 2, whereas the second option violates $\Delta\pi_{1,N} < \Delta\pi_{2,N}$ shown in the same proposition.

B.10 Proof of Proposition 4

Follows directly from the analytical expressions for π_{i,D_1D_2} taking into account that $K_j^o < \frac{1-c}{\eta}$ which follows from condition (6).

B.11 Proof of Proposition 5

For the proofs of the following results the following Lemma is useful:

Lemma 15 • *Along the curve where $\Delta\pi_{1,N} = 0$ we have*

$$\frac{dK_2^o}{dK_1^o} = -2$$

- *Along the curve where $\Delta\pi_{2,N} = 0$ we have*

$$\frac{dK_2^o}{dK_1^o} = -0.5$$

- *Define $\hat{K}$ by $\Delta\pi_{1,Y} = 0$ for $K_1^o = K_2^o = \hat{K}$. Then for any admissible set of parameters η, γ, c, I such that $\hat{K} > 0$, the isocline where $\Delta\pi_{1,Y} = 0$ is an increasing curve in the $K_1^o - K_2^o$-plane that goes through $\hat{K}$ and intersects the line $K_2^o = 0$ at some $K_1^o \in [0, \hat{K}]$.*

- *The isocline where $\Delta\pi_{2,Y} = 0$ is an increasing curve in the $K_1^o - K_2^o$-plane that goes through $\hat{K}$ and for all points on the isocline with $K_1^o \geq K_2^o$ we have $K_1^o \geq \hat{K}$.*

Proof of the Lemma: For the $\Delta\pi_{i,N}$ isoclines the claims follow directly by implicit differentiation of (35). Concerning the slope of the $\Delta\pi_{i,Y}$ isoclines we obtain from implicit differentiation of (35) that these slopes are non-negative. Furthermore, we observe that if $K_1^o + 2K_2^o \geq \frac{\gamma-\eta}{\gamma-\eta^2} - \frac{1}{\eta}c$ along the $\Delta\pi_{1,Y} = 0$ isocline, then this isocline is vertical in the $K_1^o - K_2^o$-plane. For all capacities where this condition is violated the isocline is strictly increasing. It is easy to verify that if at $\hat{K}$ the isocline is vertical, then we have $\hat{K} = \hat{K}_v$ with

$$\hat{K}_v = \frac{1}{6\eta}\left(3(1-c) - \sqrt{(1-c)^2 + 72\gamma I}\right), \tag{54}$$

which for $I = 0$ reduces to $\frac{1-c}{3\eta}$ and decreases with increasing values of I. If at $\hat{K}$ the isocline is upward sloping, then $\hat{K} \leq \hat{K}_v$. Together with $\frac{d\Delta\pi_{1,Y}}{dK_2^o} \geq 0$ this implies that for $K_1^o = \hat{K}_v, K_2^o = 0$ we must have $\Delta\pi_{1,Y} \leq 0$. Also, from the expression $\hat{K}_v$ it follows that $\hat{K} > 0$ can only hold for $I < \frac{(1-c)^2}{9\gamma}$. For any value of I in that range we have $\Delta\pi_{1,Y} > 0$ at $K_1^o = K_2^o = 0$. Due to $\frac{d\Delta\pi_{1,Y}}{dK_1^o} < 0$ this implies that there exists a unique value $\bar{K} \in [0, \hat{K}]$ such that $\Delta\pi_{1,Y} = 0$ for $K_1^o = \bar{K}, K_2^o = 0$. Accordingly, the slope of the $\Delta\pi_{1,Y} = 0$ isocline exceeds 1 at $K_1^o = K_2^0 = \hat{K}$. By symmetry we must have that the slope of the $\Delta\pi_{2,Y} = 0$ isocline is positive and smaller 1 at $K_1^o = K_2^0 = \hat{K}$. This implies the claims of the Lemma which completes the proof.

To prove the claims of Proposition 5 we define $\bar{K}'$ as the intersection of the $\Delta\pi_{2,N} = 0$ isocline with the horizontal axis. From (35) it follows after some straight forward transformations that

$$\bar{K}' = \begin{cases} K' & I \leq \bar{I} \\ K'' & I > \bar{I}, \end{cases}$$

where

$$K' = \frac{1 - c - 2\sqrt{\gamma I}}{\eta} \tag{55}$$

$$K'' = \frac{1-c}{\eta} - \frac{\gamma(1-\eta)}{2\eta(\gamma - \eta^2)} - \frac{2(\gamma - \eta^2)}{\eta(1-\eta)} I \tag{56}$$

and $\bar{I} = \frac{\gamma(1-\eta)^2}{4(\gamma-\eta^2)^2}$. It is easy to see that $\bar{K}'$ is strictly monotonously decreasing for increasing I.

Using (54) and (55) we note that the inequality $K' > \hat{K}_v$ is equivalent to

$$2\sqrt{\gamma I} < \frac{4(1-c)}{3(2-\sqrt{2})}.$$

Because K' is only positive if $2\sqrt{\gamma I} < 1-c$, we conclude that $K' > \hat{K}_v$ whenever $K' > 0$. Furthermore, direct calculation shows that for $I = \bar{I}$ we have

$$\bar{K}' = K' = K'' = \frac{1-c}{\eta} - \frac{X}{\eta},$$

where in order to simplify notation we define $X = \frac{\gamma(1-\eta)}{\gamma-\eta^2}$. If $(1-c) \leq X$ we have $\bar{K}' = K' > \hat{K}_v$ for all values of I where $\bar{K}' \geq 0$. If $(1-c) > X$, then $\bar{K}' = K''$ holds for values of I where $\bar{K}'$ is close to zero. Define $\tilde{I}$ as the value of I where $K'' = 0$ holds. Inserting $I = \tilde{I}$ into (54) and taking into account that $(1-c) > X$ yields that $\hat{K}_v < 0$ holds for $I = \tilde{I}$ if

$$(1-c) < \frac{3}{3-\sqrt{5}} X.$$

Since $X \geq 0.5$ holds due to assumption (3), the right hand side of this inequality is larger than one and therefore the inequality is always true. Hence, for all admissible values of η, γ and c there exists a range of values of I such that $\bar{K}'$ is positive and close to zero and $\hat{K}_v < \bar{K}'$. From the proof of Lemma 15 we know that $\bar{K} \leq \hat{K} \leq \hat{K}_v$, where again $\bar{K}$ is the intersection of the $\Delta\pi_{1,Y} = 0$ isocline with the $K_2^o = 0$ axis. This implies that there exists a range of I-values such that $\bar{K} < \bar{K}'$. Furthermore, within that range we can choose I such that $\bar{K}' > 0$ and $\tilde{K}_1^o = \bar{K}' - \epsilon, \tilde{K}_2^o = 0$ for small ϵ satisfies (6) - (8). Such capacities are to the right of the $\Delta\pi_{1,Y} = 0$ isocline, but to the left of the $\Delta\pi_{2,N} = 0$ isocline. It follows that for capacities $(\tilde{K}_1^o, \tilde{K}_2^o)$ there exists an equilibrium where F1 chooses not to innovate and F2 innovates. Since the slope of the $\Delta\pi_{2,N}$ isocline equals -2 a small increase of either K_1^o or K_2^o (see lines 3 and 4 of Table 1) implies transition to an equilibrium where no firm innovates.

To see that this transition is associated with a payoff jump, we observe that for any positive I the equilibrium value of K_2^n in the NY-equilibrium has to be strictly positive. Therefore the transition to the NN-equilibrium implies a downward jump of K_2^n and therefore an upward jump of p^o. Accordingly, the profit of F1, which is given by $\pi_1 = p^o K_1^o$ exhibits an upward jump.

B.12 Proof of Proposition 6

First observe that $\Delta\pi_{i,Y} > 0$ for $I = 0$ and $K_1^o = K_2^o = 0$. Furthermore, we have shown in Lemma 15 that for appropriate positive values of I the $\Delta\pi_{1,Y}$-isocline intersects the $K_2^o = 0$-line for some $K_1^o = \bar{K}$ such that (6) - (8) are satisfied. In the same lemma we also show that the $\Delta\pi_{2,Y}$-isocline does not intersect the $K_2^o = 0$-line. Together with line 1 of Table 1 this implies if K_1^o increases at a state $\tilde{K}_1^o = \bar{K}_1^o - \epsilon, K_2^o = 0$ with ϵ small there is transition from a YY-equilibrium to a NY-equilibrium. This transition implies a downward jump of K_1^n to zero and straight forward considerations show that this results in an upward jump of the profit of firm 2.

B.13 Proof of Proposition 7

Using the same arguments as in the proof of Proposition 6 we know that for appropriate values of I the $\Delta\pi_{1,Y}$-isocline intersects the $K_2^o = 0$-line for some $K_1^o = \bar{K}$ such that (6) - (8) are satisfied. Straight forward calculations using (34) show that the isocline is upward sloping but not vertical at this point if and only if (18) holds. In such a case we consider an increase of K_2^o at a state $\tilde{K}_1^o = \bar{K}_1^o + \epsilon, K_2^o = 0$ with ϵ small. It follows from line 2 of Table 1 that such an increase leads to a transition from a NY-equilibrium to an equilibrium where both firms innovate. Since I is positive, the capacity of firm 1 in the YY-equilibrium has to be strictly positive. Therefore, this transition induces an upward jump of K_1^n which results in a downward jump of the profit of firm 2.

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