Partial cross ownership and tacit collusion under cost asymmetries

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Abstract

We examines the effects that passive investments in rival firms have on the incentives to collude when firms have asymmetric marginal costs. We first show that unilateral investments by the most efficient firm in rivals may not only facilitate collusion but also raise the collusive price. We also show that the most efficient firm prefers to invest in its most efficient rival and only if this investment is insufficient to sustain collusion will it begin to invest in less efficient rivals. We then consider multilateral passive investments in rivals and show that an increase in such investments never hinders tacit collusion and we establish necessary and sufficient conditions for such investments to strictly facilitate tacit collusion.

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1 Introduction

There are many cases in which firms acquire their rivals' stock as passive investments that give them a share in the rivals' profits but not in the rivals' decision making. These investments are often multilateral; examples of industries that feature complex webs of partial cross ownerships are the Japanese and the U.S. automobile industries (Alley, 1997), the global airline industry (Airline Business, 1998), the Dutch Financial Sector (Dietzenbacher, Smid, and Volkerink, 2000), the Nordic power market (Amundsen and Bergman, 2002), and the global steel industry (Gilo, Moshe, and Spiegel, 2006). While horizontal mergers are subject to substantial antitrust scrutiny and are often opposed by antitrust authorities, passive investments in rivals were either granted a de facto exemption from antitrust liability or have gone unchallenged by antitrust agencies in recent cases (Gilo, 2000).[1] This lenient approach towards passive investments in rivals stems from the courts' interpretation of the exemption for stock acquisitions "solely for investment" included in Section 7 of the Clayton Act.

In an earlier paper (Gilo, Moshe, and Spiegel, 2006) we began to investigate the merits of this lenient approach of courts and antitrust agencies towards passive investments in rivals. We showed that partial cross ownership (PCO) arrangements can facilitate tacit collusion among rival firms though cases exist in which such investments have no effect on the incentive of firms to collude. In particular we shows that when firm r increases its stake in a rival firm s, then collusion is never hindered, and that it will be surely facilitated if and only if (i) each firm in the industry holds a stake in at least one rival, (ii) the *maverick firm*

[1] For example, to the best of our knowledge, Microsoft's investments in the nonvoting stocks of Apple and Inprise/Borland Corp. were not challenged by antitrust agencies while Gillette's 22.9% stake in Wilkinson Sword was approved by the DOJ after the DOJ was assured that this stake would be passive (see *United States v. Gillette* Co. 55 Fed. Reg. at 28,312). The FTC approved TCI's 9% stake in Time Warner which at the time was TCI's main rival in the cable TV industry and even allowed TCI to raise its stake in Time Warner to 14.99% in the future, after being assured that TCI's stake would be completely passive (see *Re Time Warner Inc.,* 61 FR 50301, 1996). The FTC also agreed to a consent decree approving Medtronic Inc.'s almost 10% passive stake in SurVivaLink, one of the only two rivals of Medtronic's subsidiary in the automated External Defibriallators market (In *Re Medtronic, Inc.*, FTC File No. 981-0324, 1998).

in the industry (the firm with the strongest incentive to deviate from a collusive agreement)[2] has a direct or an indirect stake in firm r,[3] and (iii) firm s is not the industry maverick. These results were established however under the assumption that firms are symmetric and have the same marginal cost functions. In the current paper, we relax this assumption and examine the effect of PCO on the incentives of asymmetric firms to collude. This is obviously an important question since most industries feature cost asymmetries among firms.

To address this question we posit an infinitely repeated Bertrand oligopoly model in which firms have asymmetric marginal costs and they acquire some of their rivals' (non-voting) shares. This simple setting allows us to deal with the complexity generated by multilateral PCO. This complexity arises since in general, multilateral PCO arrangements create multiplier effects so the profit of each firm, both under collusion as well as under deviation from collusion, depends on the whole set of PCO in the industry and not only on the firm's own stake in rivals. Another advantage of this model is that PCO does not affect the equilibrium in the one shot case and therefore does not have any unilateral competitive effects. This allows us to focus on the effect of PCO on the ability of firms to engage in tacit collusion. We say that PCO arrangements facilitate tacit collusion if they expand the range of discount factors for which tacit collusion can be sustained.

In the first part of the paper we consider the case where only the most efficient firm in the industry invests in rivals. We show that even unilateral PCO by this firm may facilitate a market-sharing scheme in which all firms charge the same collusive price and divide the market between them. Unlike the case where firms have the same marginal costs, here firms have different monopoly prices on which they wish to collude. We assume that the collusive price is a compromise between the monopoly prices of the different firms. We show that when the most efficient firm invests in rivals, the collusive price would increase relative to

[2] The Horizontal Merger Guidelines of the US Department of Justice and FTC define maverick firms as "firms that have a greater economic incentive to deviate from the terms of coordination than do most of their rivals," see www.usdoj.gov/atr/public/guidelines/horiz_book/hmg1.html. For an excellent discussion of the role that the concept of maverick firms plays in the analysis of coordinated competitive effects, see Baker (2002).

[3] Firm i has an indirect stake in firm r if it either has a stake in a firm that has a stake in firm r, or if it has a stake in a firm that has a stake in a firm that has a stake in firm r, and so on.

the case where there are no PCO arrangements. Moreover, we show that the most efficient firm in the industry prefers to first invest in its most efficient rival both because this is the most effective way to promote tacit collusion and because such investment leads to a collusive price that is closer to the most efficient firm's monopoly price. Only if investment in the most efficient rival is insufficient to sustain a market-sharing scheme will the most efficient firm begin to invest in less efficient rivals. Less efficient firms do not wish to invest in rivals since such investments raise their collusive profits, thus implying that it is sufficient to give these firms smaller market shares in order to induce them to collude.

In the second part of the paper, we turn to multilateral PCO arrangements. In that case, cost asymmetries raise the complexity of the analysis considerably because he most efficient firm earns a positive profit even after the collusive agreement breaks down. Consequently, an increase in a firm i's direct or indirect stake in the most efficient firm has conflicting effects on firm i's incentive to collude. On one hand, a larger (direct or indirect) stake in the most efficient firm makes firm i less eager to deviate from collusion, because firm i obtains a larger share in the collusive profit of the most efficient firm. But on the other hand, the increased stake of firm i in the most efficient firm also gives it a larger share in the profit of the most efficient firm once the collusive agreement breaks down. This second effect weaken the incentive of firm i to collude.

Despite these complications, we are able to show that an increase in the stake of firm r in firm s never hinders collusion and it will strictly facilitate collusion if and only if (i) the industry maverick has a direct or indirect stake in firm r, and (ii) firm s is not the industry maverick. When either (i) or (ii) fails to hold, the increase in firm r's stake in firm s does not affect tacit collusion. These results extend our earlier results in Gilo, Moshe, and Spiegel (2006) and show that the results when firms have symmetric cost functions generalize to the asymmetric costs case.

Apart from our earlier paper (Gilo, Moshe, and Spiegel, 2006), we are aware of only one other paper, Malueg (1992), that studies the coordinated effects of PCO. His paper differs from ours in several ways as he considers a repeated symmetric Cournot game in which firms hold identical stakes in one another, and moreover, in his paper, it is effectively the controllers rather than the firms that hold stakes in rivals. This difference is important because investments by controllers do not feature the complex chain-effect interaction between the profits of rival firms which is a main focus of our paper. Other papers that look at the competitive effects of PCO include Reynolds and Snapp (1986), Bolle and Güth (1992), Flath (1991, 1992), Reitman (1994), and Dietzenbacher, Smid, and Volkerink (2000). These paper however examine the unilateral effects of PCO arrangements in the context of static oligopoly models.[4]

The rest of the paper is organized as follows: Section 2 examines the effect of PCO on the ability of firms to achieve the fully collusive outcome in the context of an infinitely repeated Bertrand model with asymmetric firms. Section 3 examines the case where only the most efficient firm in the industry invests in rivals. Section 4, examines multilateral PCO arrangements. We conclude in Section 4. Technical proofs are in the Appendix.

2 Tacit collusion absent PCO

We examine the coordinated competitive effects of PCO in the context of an infinitely repeated Bertrand oligopoly model with $n \geq 2$ firms. We assume that the n firms produce a homogenous product using a constant returns to scale technology and face a downward sloping demand function $Q(p)$. In every period, the n firms simultaneously choose prices and the lowest price firm captures the entire market. In case of a tie, the set of lowest price firms get equal shares of the total sales. The firms however have different marginal costs: let c_i be the (constant) marginal cost of firm i and assume $c_1 < c_2 < ... < c_n$. That is, higher indices represent higher cost firms. The profit of firm i when it serves the entire market at a price p is given by

$$y_i(p) = Q(p)(p - c_i).$$

We shall make the following assumptions on $y_i(p)$:

[4]See also Bresnahan and Salop (1986) and Kwoka (1992) for a related analysis of static models of horizontal joint ventures. Alley (1997) and Parker and Röller (1997) provide empirical evidence on the effect of PCO on collusion. Alley (1997) finds that failure to account for PCO leads to misleading estimates of the price-cost margins in the Japanese and U.S. automobile industries. Parker and Röller (1997) find that cellular telephone companies in the U.S. tend to collude more in one market if they have a joint venture in another market.

Assumption 1: $y_i(p)$ has a unique global maximizer, p_i^m.

Assumption 2: $p_1^m > c_n$ and $y_1(c_2) > \frac{y_1(c_j)}{j-1}$ for all $j = 3, ..., n$.

Assumption 1 is standard and holds whenever the demand function is either concave or not too convex. Since $c_1 < c_2 < ... < c_n$, then $p_1^m < p_2^m < ... < p_n^m$, where $p_i^m \equiv \arg\max_p y_i(p)$ is the monopoly price from firm i's point of view.[5] That is, higher cost firms prefer higher monopoly prices. The first part of Assumption 2 ensures that all firms are effective competitors because it states that the monopoly price of the most efficient firm exceeds the marginal cost of the least efficient firm. The second part of Assumption 2 implies that in a static Bertrand game, firm 1 will prefer to set a price slightly below c_2 and capture the entire market than share the market with firm 2 at a price slightly below c_3, or share the market with firms 2 and 3 at a price slightly below c_4, and so on. Given this assumption, it is clear that absent collusion, firm 1 will prefer to monopolize the market by charging a price slightly below c_2.

When the stage game is infinitely repeated, firms may be able to engage in tacit collusion. The fact that different firms have different monopoly prices raises the obvious question of which price would they coordinate on in a collusive equilibrium? If side payments were possible, firms would clearly let firm 1, which is the most efficient firm, serve the entire market at a price p_1^m (e.g., firms $2, ..., n$ would all set prices above p_1^m and would make no sales). The firms will then use side payments to share the monopoly profit

$$y_1^m \equiv Q(p_1^m)(p_1^m - c_1). \tag{1}$$

We rule out this possibility by assuming that side payments are not feasible, say due to the fear of antitrust prosecution.

Instead, we consider a collusive scheme led by firm 1. According to this scheme, firm 1 sets a price $\widehat{p}$, which is some compromise between the monopoly prices of the various firms,

[5] By revealed preferences, the fact that $y_i(\cdot)$ has a unique maximizer implies that $Q(p_i^m)(p_i^m - c_i) > Q(p_j^m)(p_j^m - c_i)$, and $Q(p_j^m)(p_j^m - c_j) > Q(p_i^m)(p_i^m - c_j)$. Summing up the two inequalities and simplifying, yields $Q(p_i^m)(c_j - c_i) > Q(p_j^m)(c_j - c_i)$. Assuming without a loss of generality that $j > i$, and noting that $Q'(\cdot) < 0$, it follows that $p_j^m > p_i^m$.

i.e., $p_1^m \leq \widehat{p} \leq p_n^m$. All firms adopt $\widehat{p}$ and consumers randomize between them.[6] Consequently, each firm i serves $\frac{1}{n}$ of the market and its profit in every period is $\frac{\widehat{y}_i}{n}$, where

$$\widehat{y}_i \equiv Q(\widehat{p})(\widehat{p} - c_i), \qquad i = 1, \ldots, n. \tag{2}$$

Although $\widehat{p}$ can exceed firm 1's monopoly price, p_1^m, it cannot exceed it by too much. To see why, note that firm 1 can always ensure itself a profit of $y_1(c_2)$ by setting a price slightly below c_2 and capturing the entire market. Hence, to ensure that firm 1 has an incentive to collude at $\widehat{p}$, it must be the case that $\frac{\widehat{y}_1}{n} \geq y_1(c_2)$. Since by Assumption 2, $c_2 < p_1^m \leq \widehat{p}$, it follows that $\widehat{p}$ is bounded from above by $\overline{p}$, where $\overline{p}$ is implicitly defined by $\frac{y_1(\overline{p})}{n} = y_1(c_2)$ (see Figure 1). If this is not the case, i.e., if $\widehat{p} > \overline{p}$, then firm 1 would be better off deviating to c_2 and capturing the entire market than colluding at $\widehat{p}$. Before proceeding, we add the following assumption which is illustrated in Figure 1:

Assumption 3: $\overline{p} < p_2^m$, where $\overline{p}$ is implicitly defined by $\frac{y_1(\overline{p})}{n} = y_1(c_2)$.

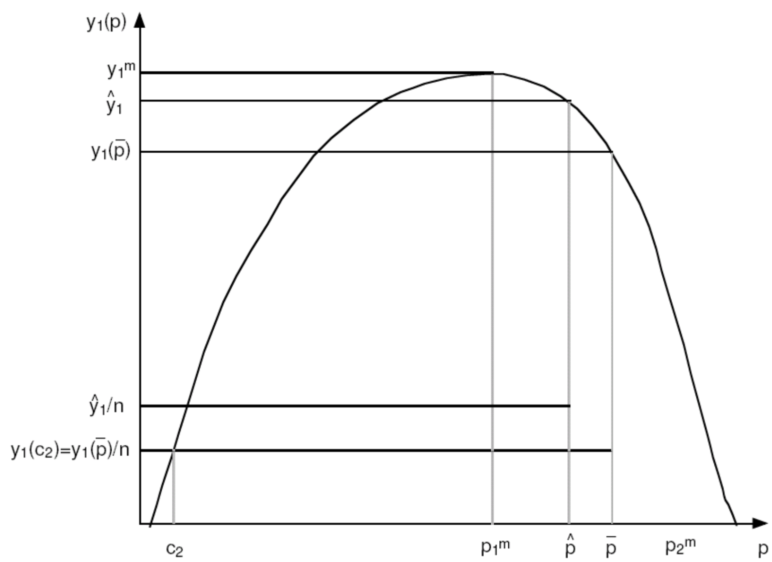

Figure 1: illustrating Assumption 3

[6] That is, we study "pure" price fixing. A more elaborate collusive scheme might also involve market division in which case the market shares need not be equal. Such a scheme however will be in general much harder to enforce and easier for antitrust authorities to detect.

Recalling that $p_1^m < p_2^m < ... < p_n^m$, Assumption 3 implies that $\overline{p} < p_i^m$ for all $i = 2, ..., n$.[7] Since $\widehat{p} \leq \overline{p}$, it follows that $\widehat{p} < p_i^m$ for all $i = 2, ..., n$: the collusive price is below the monopoly prices of all firms but 1. This implies in turn that the optimal deviation for firm $i = 2, ..., n$ is to set a price slightly below $\widehat{p}$, while the optimal deviation for firm 1 is to set a price p_1^m. Following any deviation from the collusive scheme (including a deviation by firm 1), firm 1 charges a price slightly below c_2 forever after and captures the entire market.

We assume that the pricing decisions of each firm are effectively made by its controller (i.e., a controlling shareholder) whose ownership stake is γ_{ii}. We are now interested in finding conditions that will ensure that in a subgame perfect equilibrium of the infinitely repeated game, every controller will set $\widehat{p}$ in every period.

Using δ to denote the intertemporal discount factor, the condition that ensures that the controller of firm $i = 2, ..., n$ does not wish to deviate from the collusive scheme is given by

$$\gamma_{ii} \frac{\widehat{y}_i}{n(1-\delta)} \geq \gamma_{ii} \widehat{y}_i, \qquad i = 2, ..., n. \tag{3}$$

The left-hand side of (3) is the infinite discounted payoff of firm i's controller which consists of his share in firm i's collusive profit. The right-hand side of (3) is the controller's share in the one-time profit that firm i earns in the period in which it undercuts its rivals slightly and captures the entire market. Condition (3) can be rewritten as

$$\delta \geq \widehat{\delta} \equiv 1 - \frac{1}{n}.$$

That is, the controllers of firms $2, ..., n$ have an incentive to participate in the collusive scheme provided that they are sufficiently patient. This condition is identical to the well-known condition for tacit collusion in the context of an infinitely repeated Bertrand model with n identical firms (see e.g., Tirole, 1988, Ch. 6.3.2.1).

As for firm 1, then its controller does not wish to deviate from the collusive scheme

[7] To illustrate, suppose that $Q(p) = A - p$. Then, $p_i^m = \frac{A+c_i}{2}$ and $\overline{p} = A + c_1 - c_2$, so Assumption 3 is satisfied if $A < 3c_2 - 2c_1$ (this ensures that $\overline{p} < p_2^m$). Note however that A cannot be too low since Assumption 2 requires that $A > 2c_n - c_1$.

provided that

$$\gamma_{11}\frac{\widehat{y}_1}{n\,(1-\delta)} \geq \gamma_{11}\left(y_1^m + \frac{\delta y_1(c_2)}{1-\delta}\right), \tag{4}$$

where y_1^m is the one-time profit of firm 1 in the period in which it deviates and captures the entire market while charging p_1^m, and $y_1(c_2)$ is the per-period profit of firm 1 in all subsequent periods. Condition (4) can be rewritten as

$$\delta \geq \widehat{\delta}_1\left(\widehat{p}\right) \equiv \frac{y_1^m - \frac{\widehat{y}_1}{n}}{y_1^m - y_1(c_2)}. \tag{5}$$

Note that

$$\widehat{\delta}_1\left(\widehat{p}\right) > \frac{y_1^m - \frac{\widehat{y}_1}{n}}{y_1^m} \geq 1 - \frac{1}{n} \equiv \widehat{\delta},$$

where the weak inequality follows because $y_1^m \geq \widehat{y}_1$. Since $\widehat{\delta}_1\left(\widehat{p}\right) > \widehat{\delta}$, it is clear that if firm 1 wishes to collude then all other firms surely wish to collude. That is, firm 1 is the *maverick firm* in the industry, i.e., the firm with the strongest incentive to deviate from a collusive agreement. Hence, (5) is a necessary and sufficient condition for the collusive scheme led by firm 1 to be sustained as a subgame perfect equilibrium of the infinitely repeated game. Moreover, since $\widehat{p} \geq p_1^m$, it follows that $\widehat{y}_1$ increases as $\widehat{p}$ is lowered towards p_1^m. As a result, firm 1's controller would prefer to set $\widehat{p} = p_1^m$ and thereby maximize his infinite discounted stream of collusive profits while relaxing constraint (5). Hence,

Proposition 1: *Absent PCO by firms, firm* 1 *is the industry maverick and its controller would like to set the collusive price equal to* p_1^m*. Collusion at* p_1^m *can be sustained as a subgame perfect equilibrium of the infinitely repeated game provided that* $\delta \geq \widehat{\delta}_1(p_1^m)$*.*

3 Tacit collusion with unilateral PCO by firm 1

In this section we will only examine the competitive effects of unilateral PCO investments by firm 1 in rival firms. The competitive effects of multilateral PCO arrangements are considered in Section 4. We will now use $\widehat{\delta}_1(p_1^m)$ (the critical discount factor above which the collusive scheme characterized above can be sustained) as our measure of the ease of collusion and accordingly, will say that PCO facilitates tacit collusion if it lowers $\widehat{\delta}_1(p_1^m)$, and will say that PCO hinders tacit collusion if raises $\widehat{\delta}_1(p_1^m)$.[8]

Specifically, assume that firm 1 invests in rivals and let $\alpha_{12}, ..., \alpha_{1n}$ be its ownership stakes in firms $2, ..., n$. Since the collusive profit of each firm i is $\frac{\widehat{y}_i}{n}$, it follows that firm 1's infinite discounted stream of profits under collusion is

$$\frac{\widehat{y}_1 + \sum_{i \neq 1} \alpha_{1i} \widehat{y}_i}{n(1-\delta)}.$$

If firm 1's controller deviates from the collusive scheme, all rivals make zero profits, so firm 1's payoff is

$$y_1^m + \frac{\delta y_1(c_2)}{1-\delta},$$

exactly as in the absence of PCO. Consequently, the condition that ensures that firm 1's controller does not wish to deviate from the collusive scheme is now given by

$$\gamma_{11}\left(\frac{\widehat{y}_1 + \sum_{i \neq 1} \alpha_{1i} \widehat{y}_i}{n(1-\delta)}\right) \geq \gamma_{11}\left(y_1^m + \frac{\delta y_1(c_2)}{1-\delta}\right), \tag{6}$$

or

$$\delta \geq \widehat{\delta}_1^{po}(\widehat{p}) \equiv \frac{y_1^m - \frac{\widehat{y}_1 + \sum_{i \neq 1} \alpha_{1i} \widehat{y}_i}{n}}{y_1^m - y_1(c_2)}. \tag{7}$$

Notice that $\widehat{\delta}_1^{po}(\widehat{p})$ is decreasing with each α_{1i}: the larger are the stakes of firm 1 in rival firms, the weaker is firm 1's incentive to collude. Clearly, firm 1 does not have an incentive to invest in rivals up to the point where $\widehat{\delta}_1^{po}(\widehat{p})$ drops below $\widehat{\delta}$ since then firm 1 is no longer the industry maverick, so firm 1's stakes in rivals no longer facilitate tacit collusion. Hence, we shall assume in the rest of this section that firm 1 remains an industry maverick even when it holds PCO stakes in rivals. A sufficient condition for that to be the case is that firm 1's profit when the collusive agreement breaks down, $y_1(c_2)$, is at least as large as firm 1's average stake in the profits of rival firms under collusion, $\frac{\sum_{i \neq 1} \alpha_{1i} \widehat{y}_i}{n-1}$, because then,

$$\widehat{\delta}_1^{po}(\widehat{p}) \geq \frac{y_1^m - \frac{y_1^m + \sum_{i \neq 1} \alpha_{1i} \widehat{y}_i}{n}}{y_1^m - y_1(c_2)} \geq \frac{y_1^m - \frac{y_1^m + (n-1) y_1(c_2)}{n}}{y_1^m - y_1(c_2)} = 1 - \frac{1}{n} \equiv \widehat{\delta}.$$

Assuming then that firm 1 is the industry maverick, firm 1's controller selects $\widehat{p}$ to maximize the infinite discounted sum of firm 1's collusive profits given by the left-hand side of (6) subject to (7).

[8] Of course, the infinitely repeated game admits multiple subgame perfect equilibria. We restrict attention to the most collusive equilibrium and focus on $\widehat{\delta}_1(p_1^m)$ because this is a standard way to capture the notion of "ease of collusion."

Proposition 2: *Suppose that firm 1 invests in rivals but still remains the industry maverick. Using $\widehat{p}^*$ to denote the optimal collusive price from firm 1's perspective, the following holds:*

(i) *$\widehat{p}^*$ is increasing with each α_{1i} and is above firm 1's monopoly price: $\widehat{p}^* > p_1^m$.*

(ii) *$\widehat{\delta}_1^{po}(\widehat{p}^*)$ is decreasing with each α_{1i} and is below $\widehat{\delta}_1(p_1^m)$ which is the critical discount factor above which collusion can be sustained absent PCO.*

(iii) *PCO in an efficient rival raises $\widehat{p}^*$ by less and lowers $\widehat{\delta}_1^{po}(\widehat{p}^*)$ by more than a similar PCO in a less efficient rival.*

Proof: (i) Firm 1 chooses $\widehat{p}$ to maximize the left-hand side of (6). Given Assumption 1 and recalling that $p_1^m < p_2^m < ... < p_n^m$, it follows that $\widehat{p}^*$ is increasing with each α_{1i} and is above p_1^m.

(ii) Absent PCO, the critical discount factor above which collusion can be sustained is $\widehat{\delta}_1(p_1^m)$. Using (5) and (7) it is clear that,

$$\widehat{\delta}_1(p_1^m) > \frac{y_1^m - \frac{y_1^m + \sum_{i \neq 1} \alpha_{1i} y_i(p_1^m)}{n}}{y_1^m - y_1(c_2)} \geq \frac{y_1^m - \frac{\widehat{y}_1^* + \sum_{i \neq 1} \alpha_{1i} \widehat{y}_i^*}{n}}{y_1^m - y_1(c_2)} \equiv \widehat{\delta}_1^{po}(\widehat{p}^*),$$

where $\widehat{y}_i^* \equiv Q(\widehat{p}^*)(\widehat{p}^* - c_i)$ and where the weak inequality follows because by revealed preferences, $\widehat{y}_1^* + \sum_{i \neq 1} \alpha_{1i} \widehat{y}_i^* \geq y_1^m + \sum_{i \neq 1} \alpha_{1i} y_i(p_1^m)$. To complete the proof, note that by the envelope theorem,

$$\frac{d\widehat{\delta}_1^{po}(\widehat{p}^*)}{d\alpha_{1i}} = -\frac{\frac{\widehat{y}_i}{n}}{y_1^m - y_1(c_2)} < 0.$$

(iii) Since $c_2 < ... < c_n$, it follows that $\widehat{y}_2^* > ... > \widehat{y}_n^*$, implying that PCO by firm 1 in an efficient rival raises $\widehat{p}^*$ by less and lowers $\widehat{\delta}_1^{po}(\widehat{p}^*)$ by more than does a similar investment in a less efficient rival. ■

Proposition 2 implies that investments by firm 1 in rivals do not only facilitate tacit collusion by lowering the critical discount factor above which tacit collusion can be sutained but also lead to a higher collusive price. The latter result arises because, due to it investment in rivals, firm 1 is interested in maximizing a weighted average of its own profit and the profits of the firms it invests in. The higher firm 1's investments in rivals, the higher the weight that firm 1's assigns to the rivals' profits in its objective function. Maximizing the rivals' profits requires a higher monopoly price than the monopoly price from firm 1's own perspective.

The proposition suggests that to the extent that firm 1 invests in rivals, it always prefers to invest in its most efficient rival first since this leads to a collusive price that is closer to firm 1's monopoly price and also expands the range of discount factors above which collusion can be sustained. Only if investment in the most efficient rival is not sufficient to sustain collusion, does firm 1 begin to invest in the next efficient rival.

It is also worth noting that firm 1 will have an incentive to minimize its investments in rivals subject to being able to facilitate tacit collusion. The reason for this is as follows: when the capital market is perfectly competitive, firm 1 pays a fair price for its rivals' shares and therefore just breaks even on these shares. Hence the change in the payoff of firm 1's shareholders from investing in rivals is simply equal to the change in firm 1's direct profit (i.e., excluding firm 1's share in rivals' profits). But since $\widehat{p} > p_1^m$, the direct profit of firm 1 decreases following investment in rivals, so firm 1 will prefer to invest as little as possible in rivals subject to ensuring that the collusive scheme can be sustained.

4 Tacit Collusion with multilateral PCO

In this section we turn to the case where all firms potentially invest in rivals. To this end, let α_{ij} be firm i's partial cross ownership stake in firm j and define the following $n \times n$ PCO matrix:

$$A = \begin{pmatrix} 0 & \alpha_{12} & \cdots & \alpha_{1n} \\ \alpha_{21} & 0 & \cdots & \alpha_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \alpha_{n1} & \alpha_{n2} & \cdots & 0 \end{pmatrix}.$$

Row i in the matrix A specifies the stakes that firm i has in all rival firms, while column j in the matrix A specifies the stakes that rival firms hold in firm j. Since apart from rival firms each firm is also held by its controller and possibly by outside stakeholders, the sum of each column of A is strictly less than 1. Obviously, all entries on the diagonal of the matrix

A are equal to 0.

4.1 The accounting profits under PCO

When firms hold stakes in each other, the profit of each firm potentially depends on the profits of *all* other firms in the industry. For instance, firm 1 may get a share α_{12} of firm 2's profit which may reflect firm 2's share, α_{23}, in the profit of firm 3, which in turn may reflect firm 3's share, α_{31}, in the profit of firm 1. Hence, we potentially have a multiplier effect that drives a wedge between the direct profit of each firm and its overall profit which also includes the firm's share in the profits of rival firms. Therefore, before characterizing the conditions that ensure that a collusive scheme can be supported as a subgame perfect equilibrium of the infinitely repeated game, we first need to express the profit of each firm under collusion and following a deviation from collusion.

Under collusion, all firms charge the same price, $\widehat{p}$. Since the products are homogenuous, consumers chooses which firm to buy from at random so the market share of each firm is $\frac{1}{n}$. Hence, the direct profit of each firm i (excluding its share in the profits of rivals) is $\frac{\widehat{y}_i}{n}$, where $\widehat{y}_i$ is given by equation (2). Since by assumption, $c_1 < c_2 < ... < c_n$, we have $\widehat{y}_1 > \widehat{y}_2 > ... > \widehat{y}_n$: the direct profit of firm 1 exceeds that of firm 2, which in turn exceeds that of firm 3, and so on. In addition to its direct profit, each firm i also gets a share in its rivals' profits due to its cross ownership stake in these firms. Hence, the (column) vector of collusive profits, $\widehat{\pi} = (\widehat{\pi}_1, \widehat{\pi}_2, ..., \widehat{\pi}_n)'$, is given by the solution to the following system of n equations:

$$\widehat{\pi} = \frac{\widehat{y}}{n} + A\widehat{\pi}, \tag{8}$$

where $\widehat{y} \equiv (\widehat{y}_1, ..., \widehat{y}_n)'$.

Next, we consider what happens when the controller of firm i deviates from the collusive scheme. If $i \neq 1$, then firm i will slightly undercut $\widehat{p}$, so the direct profit of all firms but i will be 0, while the direct profit of firm i will be arbitrarily close to $\widehat{y}_i$. Consequently, the vector of current profits, $\pi^{d_i} = (\pi_1^{d_i}, \pi_2^{d_i}, ..., \pi_n^{d_i})'$, is defined by the solution to the following system:

$$\pi^{d_i} = \widehat{y}^{d_i} + A\pi^{d_i}, \qquad i = 2, ..., n, \tag{9}$$

where $\widehat{y}^{d_i} \equiv (0, ..., 0, \widehat{y}_i, 0, ..., 0)'$ is an n-dimensional (column) vector with $\widehat{y}_i$ in the i-th entry and 0's elsewhere.

If the deviant is $i = 1$, then firm 1 will charge p_1^m and its profit in the current period will be y_1^m (see equation (1)). The current direct profits of all other firms will be 0. Hence, the vector of profits in period in which firm 1's controller deviates, $\pi^{d_1} = (\pi_1^{d_1}, \pi_2^{d_1}, ..., \pi_n^{d_1})'$, is defined by the solution to the following system:

$$\pi^{d_1} = \widehat{y}^{d_1} + A\pi^{d_1}, \tag{10}$$

where $\widehat{y}^{d_1} \equiv (y_1^m, 0, ..., 0)'$ is an n-dimensional (column) vector with y_1^m in the first entry and 0's elsewhere.

Once the collusive agreement breaks down, firm 1 will charge a price slightly below c_2 in every period and will capture the entire market. Hence the vector of profits following a break down of the collusive agreement, $\pi^f = (\pi_1^f, \pi_2^f, ..., \pi_n^f)'$, is defined by the solution to the following system:

$$\pi^f = y^f(c_2) + A\pi^f, \tag{11}$$

where $y^f(c_2) \equiv (y_1(c_2), 0, ..., 0)'$ is an n-dimensional (column) vector with $y_1(c_2)$ in the first entry and 0's elsewhere.

To solve systems (8)-(11), note that since the PCO matrix, A, is nonnegative and the sum of each of its columns is strictly less than 1, systems (8)-(11) are Leontief systems and have unique nonnegative solutions (see Berck and Sydsæter, Ch. 21.1 - 21.22, p. 111) defined by

$$\begin{aligned} \pi(\widehat{p}; A) &= B\frac{\widehat{y}}{n}, \\ \pi^{d_i}(\widehat{p}; A) &= B\widehat{y}^{d_i}, \qquad i = 1, \ldots, n, \\ \pi^f(c_2; A) &= By^f(c_2), \end{aligned} \tag{12}$$

where $B \equiv (I - A)^{-1}$ is an inverse Leontief matrix that specifies the aggregate imputed shares of "real" equityholders (i.e., outside equityholders that are not part of the n firms) in the accounting profits of the n firms.[9] That is, the ij-th entry in the matrix B, denoted b_{ij}, is the aggregate imputed share that the real equityholders of firm i have in the accounting profit of

[9] The terminology "imputed shares" is due to Dorofeenko et al (2008).

firm j. Equation (12) implies that the accounting collusive profit of firm $i \neq 1$ is $\pi_i(\widehat{p}; A) = \frac{1}{n}\sum_{j=1}^{n} b_{ij}\widehat{y}_j$, its one-time profit in the period in which it deviates from the collusive scheme is $\pi_i^{d_i}(\widehat{p}; A) = b_{ii}\widehat{y}_i$, and its profit in any subsequent period is $\pi_i^f(c_2; A) = b_{i1}y_1(c_2)$. The corresponding accounting profits of firm 1 are $\pi_1(\widehat{p}; A) = \frac{1}{n}\sum_{j=1}^{n} b_{1j}\widehat{y}_j$, $\pi_1^{d_1}(\widehat{p}; A) = b_{11}y_1^m$, and $\pi_1^f(c_2; A) = b_{11}y_1(c_2)$.

Given the important role that the aggregate imputed shares matrix, B, plays in our analysis, we state the following result whose proof appears in Gilo, Moshe, and Spiegel (2006).

Lemma 1: *The aggregate imputed shares matrix B has the following properties:*

(i) $b_{ii} \geq 1$ *for all* i, *and* $0 \leq b_{ij} < b_{ii}$ *for all* i *and all* $j \neq i$.

(ii) *Let i and j be two distinct firms. Then, $b_{ij} = 0$ if and only if firm i does not have a direct or an indirect stake in firm j.*[10]

(iii) $b_{ii} > 1$ *if and only if firm i has a direct or an indirect stake in some firm j which in turn has a direct or an indirect stake in firm i (i.e., $b_{ij} > 0$ and $b_{ji} > 0$).*

(iv) $\widehat{b}_i \equiv \sum_{j=1}^{n}\left(1 - \sum_{k\neq j}\alpha_{kj}\right) b_{ji} = 1$ *for all* i.

To interpret Lemma 1, recall that b_{ij} is the aggregate imputed share that the real equityholders of firm i have in the accounting profit of firm $j \neq i$ through the direct or indirect cross ownership of firm i in firm j and b_{ii} is the aggregate imputed share that the real equityholders of firm i have in the accounting profit of their own firm. Part (i) of Lemma 1 says that $b_{ij} < b_{ii}$ for all i and all $j \neq i$. Part (ii) of the lemma says that the real equityholders of firm i will get a share in the profit of a rival firm j if and only if firm i has a direct or indirect stake in firm j. Part (iii) of the lemma says that if firm i has a direct or an indirect stake in some rival firm j and this firm in turn has a direct or an indirect stake in firm i, then the aggregate imputed share that a real equityholder of firm i will have in firm i will exceed 1. In other words, a 1% stake in firm i will give a "real" equityholder of

[10] We will say that firm i has no direct or indirect stake in firm j, and has no stake in a firm that has a stake in firm j, and has no stake in a firm that has a stake in a firm that has a stake in firm j and so on.

firm i more than a 1% share in the firm's profit. The reason for this surprising property is that multilateral cross ownership arrangements create a multiplier effect that results in an overstatement of the firms' cash flows.[11] Part (iv) of the lemma ensures however that the aggregate imputed shares of "real" equityholders in each firm i sum up to 1. Hence, while the accounting profits of firms will overstate the total cash flows, the aggregate payoff of all real equityholders will sum up exactly to the total cash flows.

4.2 Collusion with multilateral PCO

Given the accounting profits of the n firms under collusion and following a deviation from the fully collusive scheme, the condition that ensures that the collusive outcome can be sustained as a subgame perfect equilibrium is

$$\frac{\gamma_{ii}\pi_i(\widehat{p};A)}{1-\delta} \geq \gamma_{ii}\left(\pi_i^{d_i}(\widehat{p};A) + \frac{\delta\pi_i^f(c_2;A)}{1-\delta}\right), \qquad i = 1,\ldots,n. \tag{13}$$

The left-hand side of (13) is the infinite discounted payoff of firm i's controller under collusion, consisting of the controller's share in firm i's collusive profit. The right-hand side of (13) is the controller's share in the profit that firm i earns when it undercuts its rivals slightly (the one-time profit $\pi_i^{d_i}(\widehat{p};A)$ in the period in which firm i deviates and $\pi_i^f(c_2;A)$ in all subsequent periods). If (13) holds, no controller wishes to unilaterally deviate from the fully collusive scheme.

Recalling that $\pi_i(\widehat{p};A) = \frac{1}{n}\sum_{j=1}^n b_{ij}\widehat{y}_j$ and $\pi_i^f(c_2;A) = b_{i1}y_1(c_2)$ for all i, $\pi_1^{d_1}(\widehat{p};A) = b_{11}y_1^m$, and $\pi_i^{d_i}(\widehat{p};A) = b_{ii}\widehat{y}_i$ for all $i \neq 1$, and using $z_{ij} \equiv \frac{b_{ij}}{b_{ii}}$ to denote the relative imputed share that the equityholders of firm i have in firm j (relative to their imputed share in their "own" firm i), the necessary condition (13) for collusion can be rewritten as

$$\delta\left(y_1^m - y_1(c_2)\right) \geq y_1^m - \frac{1}{n}\sum_{j=1}^n z_{1j}\widehat{y}_j, \tag{14}$$

and

$$\delta\left(\widehat{y}_i - z_{i1}y_1(c_2)\right) \geq \widehat{y}_i - \frac{1}{n}\sum_{j=1}^n z_{ij}\widehat{y}_j, \qquad i = 2,\ldots,n. \tag{15}$$

[11] See Dietzenbacher, Smid, and Volkerink (2000) and Dorofeenko et al (2008) for additional discussion of this effect of PCO.

Notice that by definition, $y_1^m > \frac{\widehat{y}_1}{n} \geq y_1(c_2)$ (see Figure 1) and recall that $\widehat{y}_1 > \widehat{y}_2 ... > \widehat{y}_n$. Since, part (i) of Lemma 1 implies that $z_{ii} = 1$ for all i and $z_{ij} < 1$ for all i and all $j \neq i$, it follows that both sides of (14) are positive. Moreover, $\frac{\widehat{y}_1}{n} \geq y_1(c_2)$ implies that $\frac{1}{n}\sum_{j=1}^{n} z_{ij}\widehat{y}_j = z_{i1}\frac{\widehat{y}_1}{n} + \frac{1}{n}\sum_{j\neq 1}^{n} z_{ij}\widehat{y}_j \geq z_{i1}y_1(c_2)$, with strict inequality when $z_{ij} > 0$ for some j. Hence, $\widehat{y}_i - z_{i1}y_1(c_2) \geq \widehat{y}_i - \frac{1}{n}\sum_{j=1}^{n} z_{ij}\widehat{y}_j$. Before proceeding we will now impose the following assumption on $\widehat{y}_i$:

Assumption 4: $\widehat{y}_i > \frac{1}{n}\sum_{j=1}^{n} z_{ij}\widehat{y}_j$ for all i.

Assumption 4 implies that each firm i earns more money when it unilaterally deviates from a collusive scheme than it earns under collusion. This assumption ensures that both sides of (15) are positive. Notice that in the presence of PCO this need not be the case because under collsuion, firm i gets a share in the profits of its rivals, while under deviation it does not. If firm i is relatively inefficient, then its profit under collusion may exceed its profit when it deviates even though in the latter case the firm serves the entire market while under collusion it serves only $\frac{1}{n}$ of the market (but it gets a share in the profits of its rivals).

With Assumption 4 in place, (14) and (15) imply the following result:

Lemma 2: *Let $z_{ij} \equiv \frac{b_{ij}}{b_{ii}}$ be the relative imputed share that the equityholders of firm i have in firm j (relative to their imputed share in their "own" firm i). Then, the fully collusive outcome can be sustained as a subgame perfect equilibrium of the infinitely repeated game provided that*

$$\delta \geq \widehat{\delta}^{po}(A) \equiv \max\left\{\widehat{\delta}_1(A), ..., \widehat{\delta}_n(A)\right\},$$

where

$$\widehat{\delta}_1(A) \equiv \frac{y_1^m - \frac{1}{n}\sum_{j=1}^{n} z_{1j}\widehat{y}_j}{y_1^m - y_1(c_2)}, \tag{16}$$

and

$$\widehat{\delta}_i(A) \equiv \frac{\widehat{y}_i - \frac{1}{n}\sum_{j=1}^{n} z_{ij}\widehat{y}_j}{\widehat{y}_i - z_{i1}y_1(c_2)}, \qquad i = 2, ..., n. \tag{17}$$

4.3 A firm increases its stake in a rival firm by buying shares from outsider or from the rival's controller

Now, suppose that firm r increases its stake in firm s by ω. The resulting new PCO matrix is A^{ω} which is obtained by increasing the rs-th entry in A. Our main question is whether $\widehat{\delta}_i(A^{\omega})$ is higher or lower than $\widehat{\delta}_i(A)$. To address this question, note from equation (16) that $\frac{\partial \widehat{\delta}_1(A)}{\partial z_{1j}} < 0$ for all j, and note from equation (17) that $\frac{\partial \widehat{\delta}_i(A)}{\partial z_{ij}} < 0$ for all $i \neq 1$ and all $j \neq 1$. Moreover, note from equation (17) that

$$\begin{aligned}\frac{\partial \widehat{\delta}_i(A)}{\partial z_{i1}} &= \frac{-\frac{\widehat{y}_1}{n}\left(\widehat{y}_i - z_{i1}y_1(c_2)\right) + y_1(c_2)\left(\widehat{y}_i - \frac{1}{n}\sum_{j=1}^{n} z_{ij}\widehat{y}_j\right)}{\left(\widehat{y}_i - z_{i1}y_1(c_2)\right)^2} \\ &= \frac{-\widehat{y}_i\left(\frac{\widehat{y}_1}{n} - y_1(c_2)\right) + \frac{y_1(c_2)}{n}\left(z_{i1}\widehat{y}_1 - \sum_{j=1}^{n} z_{ij}\widehat{y}_j\right)}{\left(\widehat{y}_i - z_{i1}y_1(c_2)\right)^2} \\ &= -\frac{\widehat{y}_i\left(\frac{\widehat{y}_1}{n} - y_1(c_2)\right) + \frac{y_1(c_2)}{n}\sum_{j\neq 1}^{n} z_{ij}\widehat{y}_j}{\left(\widehat{y}_i - z_{i1}y_1(c_2)\right)^2} < 0,\end{aligned}$$

where the inequality follows because by assumption, $\frac{\widehat{y}_1}{n} \geq y_1(c_2)$ (otherwise firm 1 has no incentive to collude) and $\sum_{j\neq 1}^{n} z_{ij}\widehat{y}_j = z_{ii}\widehat{y}_i + \sum_{j\neq 1,i}^{n} z_{ij}\widehat{y}_j \geq \widehat{y}_i > 0$ (recall that $z_{ii} = 1$). Hence,

Lemma 3: *An increase in z_{ij} boosts the incentive of each firm i to collude while a decrease in z_{ij} weakens it. If z_{ij} does not change than the incentive to collude is not affected.*

Lemma 3 implies that in order to determine the effect of the increase in firm r's stake in firm s by ω, we only need to examine how it affects the matrix Z whose characteristic element is z_{ij}. To this end, note from Lemma A1 in Gilo, Moshe, and Spiegel (2006) that

$$z_{ij}^{\omega} \equiv \frac{b_{ij}^{\omega}}{b_{ii}^{\omega}} = \frac{b_{ij} + \varepsilon_i b_{sj}}{b_{ii} + \varepsilon_i b_{si}}, \qquad \varepsilon_i = \frac{\omega b_{ir}}{1 - \omega b_{sr}}. \tag{18}$$

Straightforward differentiation establishes that

$$\frac{\partial z_{ij}^{\omega}}{\partial \omega} = \frac{b_{sj}b_{ii} - b_{si}b_{ij}}{\left(b_{ii} + \varepsilon_i b_{si}\right)^2} \times \frac{b_{ir}}{\left(1 - \omega b_{sr}\right)^2}. \tag{19}$$

Theorem 1 in Zeng (2000) ensures that $b_{ii}b_{sj} \geq b_{ij}b_{si}$ for all j with strict inequality for $j = s$. The intuition behind this result is that b_{si} represents firm s's imputed share in firm i and

b_{ij} represents i's imputed share in firm j. Hence, $b_{si}b_{ij}$ is firm s's indirect stake in firm j via its stake in firm i. But if firm s has a direct stake in j, then b_{sj} will be even larger than $b_{si}b_{ij}$, so $b_{sj} \geq b_{si}b_{ij}$. Since $b_{ii} \geq 1$, we get $b_{ii}b_{sj} \geq b_{si}b_{ij}$. We can now prove the following main result which generalizes Theorem 1 in Gilo, Moshe, and Spiegel (2006) to the case of asymmetric firms:

Theorem 1: *Starting with a PCO matrix A, suppose that firm r increases its stake in firm s by some $\omega > 0$, so that the new PCO matrix A^{ω} differs from A only with respect to the rs-th entry which is increased by ω. Then,*

(i) $\widehat{\delta}_s(A^{\omega}) = \widehat{\delta}_s(A)$,

(ii) $\widehat{\delta}_i(A^{\omega}) = \widehat{\delta}_i(A)$ *if* $b_{ir} = 0$, *and*

(iii) $\widehat{\delta}_i(A^{\omega}) < \widehat{\delta}_i(A)$ *for all* $i \neq s$, *provided that* $b_{ir} > 0$ *(firm i has a direct or an indirect stake in the acquiring firm r).*

Proof: (i) If $i = s$ (firm i is the target firm s), then equation (19) shows that $\frac{\partial z_{sj}^{\omega}}{\partial \omega} = 0$ for all j. Hence, by Lemma 3, $\widehat{\delta}_s(A^{\omega}) = \widehat{\delta}_s(A)$.

(ii) If $b_{ir} = 0$ (firm i has no direct or indirect stake in firm r) then equation (19) shows that $\frac{\partial z_{ij}^{\omega}}{\partial \omega} = 0$ for all j. Hence, by Lemma 3, $\widehat{\delta}_i(A^{\omega}) = \widehat{\delta}_i(A)$.

(iii) Now suppose that $i \neq s$ and $b_{ir} > 0$. Then equation (19) shows that $\frac{\partial z_{ij}^{\omega}}{\partial \omega} \geq 0$ for all j with strict inequality for $j = s$. Hence, by Lemma 3, $\widehat{\delta}_i(A^{\omega}) < \widehat{\delta}_i(A)$ for all $i \neq s$. ■

Theorem 1 implies that an increase in firm r's stake in firm s does not affect collusion if either firm s is the industry maverick, or if the industry maverick has no direct or indirect stake in firm r. In all other cases, the increase facilitates collusion by lowering the critical discount factor above which collusion can be supported.

The proof of Theorem 1 provides a simpler proof for Theorem 1 in Gilo, Moshe, and Spiegel (2006). To see why, note that in the special case where all firms have the same marginal cost (the case considered in Gilo, Moshe, and Spiegel, 2006), $\widehat{y}_1 = \ldots = \widehat{y}_n = y_1^m$

and $y_1(c_2) = 0$. Hence, equations (16) and (17) imply that $\frac{\partial \widehat{\delta}_i(A)}{\partial z_{ij}} < 0$ for all i and all j. Then, the proof of Theorem 1 implies immediately that $\widehat{\delta}_i(A) \geq \widehat{\delta}_i(A^{\omega})$ with strict equality if and only if $i \neq s$ and $b_{ir} > 0$.

Theorem 1 has the following important implication:

Corollary 1: *An increase in firm r's cross ownership stake in firm s never hinders tacit collusion and surely facilitates it if and only if (i) each industry maverick has a direct or an indirect stake in firm r, and (ii) firm s is not an industry maverick.*

4.4 A firm increases its stake in a rival firm by buying shares from another rival firm

Theorem 1 assumes implicitly that when firm r increases its stake in firm s, it buys additional shares from the outside investors or the controller of firm s. However, cases exist in which one firm buys shares in a rival firm from a third rival. A case in point is a recent transaction in the global steel industry where Luxemburg-based Arcelor has increased its stake in the Brazilian steelmaker CST from 18.6% to 27.95% by buying shares from Acesita which is also based in Brazil.[12] To examine the effect of such ownership transfers on the incentives to collude, suppose that firm r increases its stake in firm s by buying an ownership stake ϕ from firm k. The resulting PCO matrix A^{ϕ} is obtained from the original PCO matrix A by increasing the rs-th entry in A by ϕ and lowering the ks-th entry by ϕ. Equation (2) in Zeng shows that in this case,

$$z_{ij}^{\phi} \equiv \frac{b_{ij}^{\phi}}{b_{ii}^{\phi}} = \frac{b_{ij} + \varepsilon_i^{\phi} b_{sj}}{b_{ii} + \varepsilon_i^{\phi} b_{si}}, \qquad \varepsilon_i^{\phi} \equiv \frac{\phi\,(b_{ir} - b_{ik})}{1 - \phi\,(b_{sr} - b_{sk})}. \tag{20}$$

Note that if firm k's stake remains unchanged, then $\varepsilon_i^{\phi} = \varepsilon_i$. Hence, the expression we used earlier, z_{ij}^{ω}, is a special case of (20). The main difference is that while $\varepsilon_i \geq 0$, now $\varepsilon_i^{\phi} \gtreqless 0$ as $b_{ir} \gtreqless b_{ik}$.

[12] Acesita sold its entire 18.7% stake in CST to Arcelor and to CVRD which is a large Brazilian miner of iron and ore. In addition to its stake in CST, Arcelor also owns stakes in Acesita and in Belgo-Mineira, which is another Brazilian steelmaker (see "CVRD, Arcelor Team up for CST," *The Daily Deal*, December 28, 2002, M&A; "Minister: Steel Duties Still Under Study - Brazil," *Business News Americas*, April 8, 2002.)

Using (20) yields

$$\frac{\partial z_{ij}^{\phi}}{\partial \phi} = \frac{b_{ii}b_{sj} - b_{ij}b_{si}}{\left(b_{ii} + \varepsilon_i^{\phi} b_{si}\right)^2} \times \frac{b_{ir} - b_{ik}}{\left(1 - \phi\left(b_{sr} - b_{sk}\right)\right)^2}. \tag{21}$$

Repeating the same steps as in Theorem 1, we obtain the following result:

Theorem 2: *Starting with a PCO matrix A, suppose that firm r buys a stake ϕ in firm s from firm k, so that the new PCO matrix A^{ϕ} is obtained from A by increasing the rs-th entry by ϕ and decreasing the ks-th by ϕ. Then,*

(i) $\widehat{\delta}_s(A^{\phi}) = \widehat{\delta}_s(A)$,

(ii) $\widehat{\delta}_i(A^{\phi}) = \widehat{\delta}_i(A)$ *if* $b_{ir} = b_{ik}$ *(firm i has the same imputed share in firms r and k), and*

(iii) $\widehat{\delta}_i(A^{\phi}) \lesseqgtr \widehat{\delta}_i(A)$ *for all* $i \neq s$ *as* $b_{ir} \gtreqless b_{ik}$.

Theorem 2 implies the following result:

Corollary 2: *A transfer of partial cross ownership in firm s from firm k to firm r does not affect tacit collusion if the industry maverick is firm s or if, at the outset, the industry maverick has the same imputed share in firms k and r. Otherwise, the transfer of partial cross ownership facilitates tacit collusion if the industry maverick has a larger imputed share in firm r (the acquirer) than in firm k (the seller) but hinders tacit collusion if the reverse holds.*

Proposition 3 in Gilo, Moshe, and Spiegel (2006) also considered the effects of a transfer of partial cross ownership in firm s from one firm to another but under the special assumption that at the outset all firms hold the exact same ownership stakes in one another. In this case, the matrix B is symmetric in the sense that its diagonal terms are all the same and its off-diagonal terms are all equal to each other. In particular, $b_{ir} = b_{ik}$ for all $i \neq r, k$, so part (ii) of Theorem 2 shows that $\widehat{\delta}_i(A^{\phi}) = \widehat{\delta}_i(A)$ for all $i \neq r, k$. Part (i) of Theorem 2 shows in addition that $\widehat{\delta}_s(A^{\phi}) = \widehat{\delta}_s(A)$. As for firms r and k, then part (i) of Lemma 1 implies that $b_{rr} > b_{rk}$ and $b_{kr} < b_{kk}$. Hence, equation (21) shows that $\frac{\partial z_{rj}^{\phi}}{\partial \phi} \geq 0$

and $\frac{\partial z_{kj}^{\phi}}{\partial \phi} \leq 0$ for all j with strict inequality for $j = s$. Hence, by Lemma 3, $\widehat{\delta}_r(A^{\phi}) < \widehat{\delta}_r(A)$ and $\widehat{\delta}_k(A^{\phi}) > \widehat{\delta}_k(A)$, implying that the transfer of partial cross ownership in firm s from firm r to firm k strengthen the incentive of firm r to collude, weaken the incentive of firm k to collude and has no effect on the incentives of other firms to collude. In the symmetric case considered by Gilo, Moshe, and Spiegel (2006), $\widehat{\delta}_1(A) = \ldots = \widehat{\delta}_n(A)$, so the incentives of all firms to collude before the transfer of ownership are the same. Hence the transfer of partial ownership turns firm k (the seller) into a maverick firm and since $\widehat{\delta}_k(A^{\phi}) > \widehat{\delta}_k(A)$, tacit collusion is hindered.

In the present case where firms have asymmetric marginal costs, any firm can potentially be the maverick firm `[We need to check this claim]`. In particular, Corollary 2 shows that collusion is hindered when the maverick is firm k and is facilitated if the maverick is firm r.

4.5 Conditions for firm 1 to be the maverick

Recall from Section 3 that when only firm 1 invests in rival, firm 1 is the industry maverick. In this section, we provide sufficient (but not necessary) conditions that ensure that firm 1 continues to be the industry maverick even in the presence of multilateral PCO arrangements. To this end, note form (16) and (17) that

$$\begin{aligned}
\widehat{\delta}_1(A) - \widehat{\delta}_i(A) &= \frac{\left(y_1^m - \frac{1}{n}\sum_{j=1}^n z_{1j}\widehat{y}_j\right)\left(\widehat{y}_i - z_{i1}y_1(c_2)\right) - \left(\widehat{y}_i - \frac{1}{n}\sum_{j=1}^n z_{ij}\widehat{y}_j\right)\left(y_1^m - y_1(c_2)\right)}{\left(\widehat{y}_i - z_{i1}y_1(c_2)\right)\left(y_1^m - y_1(c_2)\right)} \\
&= \frac{\left(\widehat{y}_i - z_{i1}y_1^m\right)y_1(c_2) - \frac{1}{n}\sum_{j=1}^n z_{1j}\widehat{y}_j\left(\widehat{y}_i - z_{i1}y_1(c_2)\right) + \frac{1}{n}\sum_{j=1}^n z_{ij}\widehat{y}_j\left(y_1^m - y_1(c_2)\right)}{\left(\widehat{y}_i - z_{i1}y_1(c_2)\right)\left(y_1^m - y_1(c_2)\right)} \\
&= \frac{-\left(\widehat{y}_i - z_{i1}y_1^m\right)\left(\frac{\widehat{y}_1}{n} - y_1(c_2)\right) - \frac{1}{n}\sum_{j\neq 1} z_{1j}\widehat{y}_j\left(\widehat{y}_i - z_{i1}y_1(c_2)\right)}{\left(\widehat{y}_i - z_{i1}y_1(c_2)\right)\left(y_1^m - y_1(c_2)\right)} \\
&\quad + \frac{\frac{1}{n}\sum_{j\neq 1} z_{ij}\widehat{y}_j\left(y_1^m - y_1(c_2)\right)}{\left(\widehat{y}_i - z_{i1}y_1(c_2)\right)\left(y_1^m - y_1(c_2)\right)},
\end{aligned}$$

where the last equality follows because by definition, $z_{11} = 1$. Adding and subtracting $\frac{1}{n}\sum_{j\neq 1}^n z_{ij}\widehat{y}_j\left(\widehat{y}_i - z_{i1}y_1(c_2)\right)$ and $\frac{1}{n}\sum_{j\neq 1} z_{ij}\widehat{y}_j\left(\widehat{y}_1 - ny_1(c_2)\right)$ to the numerator and rearranging terms,

$$
\begin{aligned}
\widehat{\delta}_1(A)-\widehat{\delta}_i(A) &= \frac{\left(z_{i1}y_1^m+\sum_{j\neq 1} z_{ij}\widehat{y}_j-\widehat{y}_i\right)\left(\frac{\widehat{y}_1}{n}-y_1(c_2)\right)+\frac{1}{n}\sum_{j\neq 1}(z_{ij}-z_{1j})\,\widehat{y}_j\,(\widehat{y}_i-z_{i1}y_1(c_2))}{(\widehat{y}_i-z_{i1}y_1(c_2))\,(y_1^m-y_1(c_2))} \\
&\quad+\frac{\frac{1}{n}\sum_{j\neq 1} z_{ij}\widehat{y}_j\left((y_1^m-y_1(c_2))-(\widehat{y}_i-z_{i1}y_1(c_2))-(\widehat{y}_1-ny_1(c_2))\right)}{(\widehat{y}_i-z_{i1}y_1(c_2))\,(y_1^m-y_1(c_2))} \\
&= \frac{\left(z_{i1}y_1^m+\sum_{j\neq 1} z_{ij}\widehat{y}_j-\widehat{y}_i\right)\left(\frac{\widehat{y}_1}{n}-y_1(c_2)\right)+\frac{1}{n}\sum_{j\neq 1}(z_{ij}-z_{1j})\,\widehat{y}_j\,(\widehat{y}_i-z_{i1}y_1(c_2))}{(\widehat{y}_i-z_{i1}y_1(c_2))\,(y_1^m-y_1(c_2))} \\
&\quad+\frac{\sum_{j\neq 1} z_{ij}\widehat{y}_j\left(\frac{y_1^m-\widehat{y}_1}{n}+\frac{(n-1+z_{i1})y_1(c_2)}{n}-\frac{\widehat{y}_i}{n}\right)}{(\widehat{y}_i-z_{i1}y_1(c_2))\,(y_1^m-y_1(c_2))},
\end{aligned}
$$

Recalling that $z_{ii}=1$, it follows that $\sum_{j\neq 1} z_{ij}\widehat{y}_j>\widehat{y}_i$. Moreover, $\frac{\widehat{y}_1}{n}\geq y_1(c_2)$ (see Figure 1). Hence, the first term is positive. The following proposition provides sufficient (but not necessary) conditions for the other two terms to be nonnegative, which ensures that firm 1 is the maverick firm in the industry in the sense that $\widehat{\delta}_1(A)>\widehat{\delta}_i(A)$ for all $i\neq 1$.

Proposition 4: *Sufficient (but not necessary) conditions for firm* 1 *(the most efficient firm in the industry) to be the industry maverick is that (i)* $z_{1j}\leq z_{ij}$ *for all* $i,j\neq 1$ *(the relative imputed share of firm* 1 *in each firm* j *is no greater than the relative share of any other firm in firm* j*), and (ii)* $\frac{\widehat{y}_i}{n}\leq\frac{(n-1+z_{i1})y_1(c_2)}{n}$ *for all* $i\neq 1$ *(the collusive profit of each firm* $i\neq 1$ *is small relative to the profit that firm* 1 *earns once the collusive scheme breaks down).*

5 PCO by controllers

We now turn to the case where controllers directly acquire (passive) ownership stakes in rival firms. The U.S. car rental industry in the mid 1990's is a case in point, as GM, which was the controlling shareholder in National Car Rental, held a passive ownership stake of 25% in Avis, while Ford, which was the controlling shareholder in Hertz, acquired 100% of the preferred nonvoting stock of Budget Rent a Car. The question is what effect, if any, such investments have on tacit collusion, above and beyond the effect that we have already identified in the previous section.

To address this question, let γ_{ij} be the stake that firm i's controller holds in firm $j\neq i$, in addition to his controlling stake in firm i, γ_{ii}. To avoid triviality, we assume that γ_{ij} represents a completely passive investment (e.g., non-voting shares) that gives the controller a share γ_{ij} of firm j's profit but no control over its actions. Moreover, we assume that γ_{ii} is sufficiently large relative to γ_{ij} for all i and all j so that the controller of each firm i is better off maximizing firm i's profit than sacrificing firm i's profit in order to boost the profits of rival firms in which the controller has stakes. With these assumptions in place, condition (13) which ensures that collusion can be sustained becomes

$$\frac{\sum_{j=1}^{n}\gamma_{ij}\pi_j(\widehat{p};A)}{1-\delta} \geq \sum_{j=1}^{n}\gamma_{ij}\left(\pi_j^{d_i}(\widehat{p};A)+\frac{\delta\pi_j^f(c_2;A)}{1-\delta}\right), \qquad i=1,...,n. \tag{22}$$

Condition (22) generalizes condition (13) to the case where controllers hold direct stakes in rival firms. Each controller then may get a share in the profit of potentially every firm in the industry. The left-hand side of (22) is the infinite discounted payoff of firm i's controller under collusion, while the right-hand side of (22) is the controller's infinite discounted payoff under deviation.

Recalling that $\pi_i(\widehat{p};A)=\frac{1}{n}\sum_{j=1}^{n}b_{ij}\widehat{y}_j$ and $\pi_i^f(c_2;A)=b_{i1}y_1(c_2)$ for all i, $\pi_1^{d_1}(\widehat{p};A)=b_{11}y_1^m$, and $\pi_i^{d_i}(\widehat{p};A)=b_{ii}\widehat{y}_i$ for all $i\neq 1$, using $z_{ij}\equiv\frac{b_{ij}}{b_{ii}}$ to denote the relative imputed share that the equityholders of firm i have in firm j, and rearranging terms, the critical discount factors above which each firm i wishes to collude are given by:

$$\delta \geq \widehat{\delta}_1^c(A) \equiv \frac{\sum_{j=1}^{n}\gamma_{1j}\left(b_{j1}y_1^m-\frac{1}{n}\sum_{k=1}^{n}b_{jk}\widehat{y}_k\right)}{\sum_{j=1}^{n}\gamma_{1j}b_{j1}\left(y_1^m-y_1(c_2)\right)},$$

and

$$\delta \geq \widehat{\delta}_i^c(A) \equiv \frac{\sum_{j=1}^{n}\gamma_{ij}\left(b_{jj}\widehat{y}_j-\frac{1}{n}\sum_{k=1}^{n}b_{jk}\widehat{y}_k\right)}{\sum_{j=1}^{n}\gamma_{ij}b_{jj}\left(\widehat{y}_j-z_{j1}y_1(c_2)\right)}, \qquad i=2,...,n.$$

Straightforward differentiation reveals that

$$\begin{aligned}
\frac{\partial\widehat{\delta}_1^c(A)}{\partial\gamma_{1\ell}} &= \frac{\left(b_{\ell 1}y_1^m-\frac{1}{n}\sum_{k=1}^{n}b_{\ell k}\widehat{y}_k\right)\sum_{j=1}^{n}\gamma_{1j}b_{j1}-b_{\ell 1}\sum_{j=1}^{n}\gamma_{1j}\left(b_{j1}y_1^m-\frac{1}{n}\sum_{k=1}^{n}b_{jk}\widehat{y}_k\right)}{\left(y_1^m-y_1(c_2)\right)\left(\sum_{j=1}^{n}\gamma_{1j}b_{j1}\right)^2} \\
&= \frac{\frac{1}{n}b_{\ell 1}\sum_{j=1}^{n}\gamma_{1j}\left(\sum_{k=1}^{n}b_{jk}\widehat{y}_k\right)-\frac{1}{n}\sum_{j=1}^{n}\gamma_{1j}b_{j1}\left(\sum_{k=1}^{n}b_{\ell k}\widehat{y}_k\right)}{\left(y_1^m-y_1(c_2)\right)\left(\sum_{j=1}^{n}\gamma_{1j}b_{j1}\right)^2} \\
&= \frac{\frac{1}{n}\sum_{j=1}^{n}\gamma_{1j}b_{j1}b_{\ell 1}\left(\sum_{k=1}^{n}\left(\frac{b_{jk}}{b_{j1}}-\frac{b_{\ell k}}{b_{\ell 1}}\right)\widehat{y}_k\right)}{\left(y_1^m-y_1(c_2)\right)\left(\sum_{j=1}^{n}\gamma_{1j}b_{j1}\right)^2}.
\end{aligned}$$

[To be continued]

6 Conclusion

Acquisitions of one firm's stock by a rival firm have been traditionally treated under Section 7 of the Clayton Act which condemns such acquisitions when their effect "may be substantially to lessen competition." However, the third paragraph of this section effectively exempts passive investments made "solely for investment." As argued in Gilo (2000), antitrust agencies and courts, when applying this exemption, did not conduct full-blown examinations as to whether such passive investments may substantially lessen competition.[13]

In this paper we showed that although there are cases in which passive investments in rivals have no effect on the ability of firms to engage in tacit collusion, an across the board lenient approach towards such investments may be misguided. This is because passive investments in rivals may well facilitate tacit collusion, especially when these investments are multilateral and in firms that are not industry mavericks. In addition, we showed that direct investments by firms' controllers in rivals may either substitute investments by the firms themselves or facilitate collusion further, especially when the controllers have small stakes in their own firms. We believe that antitrust courts and agencies should take account of these factors when considering cases involving passive investments among rivals.

Throughout the paper we have focused exclusively on the effect of PCO on the ability of firms to engage in (tacit) price fixing. However, if in addition to price fixing firms can also divide the market among themselves, then they would clearly be able to sustain collusion for a larger set of discount factors since they would have more instruments (the collusive price and the market shares). In particular, it would be possible to relax the incentive constraints of maverick firms by increasing their market shares at the expense of firms with nonbinding incentive constraints. This suggests in turn that in the presence of market sharing schemes, firms may have an incentive to become industry mavericks in order to receive a larger share of the market. As our analysis shows, one way to become an industry maverick is to avoid

[13] We are aware of only two cases in which the ability of passive investments to lessen competition was acknowledged: the FTC's decision in *Golden Grain Macaroni Co.* (78 F.T.C. 63, 1971), and the consent decree reached with the DOJ regarding US West's acquisition of Continental Cablevision (this decree was approved by the district court in United states v. US West Inc., 1997-1 Trade cases (CCH), ¶71,767, D.C., 1997).

investing in rivals.[14] Interestingly, this implies that beside the fact that market sharing schemes are harder to enforce (firms need to commit to ration their sales) and are more susceptible to antitrust scrutiny, they have another drawback, which is that they provide firms with a disincentive to invest in rivals and thereby facilitate tacit collusion.

[14]Indeed, in a previous version of the paper, we showed that under market sharing scehems and cost asymmetries, only the most efficient firm in the industry has an incentive to invest in rivals to sustain collusion while all other firms find it optimal to not invest in rivals.

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