The Real Time Monitoring Tests with the Stability of U.S. Monetary Transmission Mechanism

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A. Motivation

I. Looking for an easy to monitor the real time pattern of asset prices and macroeconomics variables.

II. Controlling the Size Distortion

In contrast to the classical one-shot type tests for detecting a structural break within a sample of data (e.g. Hansen, 1992; Andrews, 1993) , (Chu et al. 1996) provide a procedure to control asymptotic size well.

III. The real time monitoring procedure consists in checking whether the incoming data are consistent with the estimated model.

I. Limiting distribution of our monitoring tests follow the a Brownian Bridge, when the data generating processes (DGPs) is a unit root process with deterministic trend, a stationary time series with uncorrelated or dependent errors, or the linear regression model under general mixing assumptions on the regressors and the errors.

II. The limiting Distribution does not depend on the nature of the error process.

B2. Contributions

III. No need to use bootstrap to control the size distortion.

IV. Testing persistence change from I(0) to I(1) or vice versa with assuming known direction and location.

V. Illustration with the stability of U.S monetary transmission mechanism.

VI. Our test statistics are very useful tool in supplementing policy maker's information set not only ex-post, but also in real time, i.e., at the time of their decisions.

C1. Models and Hypothesis

Model 1 (With I(0) errors):

$$y_t = \begin{cases} \mu_0 + \alpha_0 \cdot trend + u_t & \text{for} \quad t = 1, ..., m, \\ \mu_t + \alpha_t \cdot trend + u_t & \text{for} \quad t = m+1, ..., m+q. \end{cases} \tag{1}$$

where μ and α are regression coefficients, u_t is n.i.i.d with $(0, \sigma_t^2)$ or a weakly dependent process, $ARMA(p,q)$.

C2. Models and Hypothesis

Model 2 (with I(1) errors):

$$y_t = \begin{cases} \mu_0 + \alpha_0 \cdot trend + v_t & \text{for} \quad t = 1, ..., m, \\ \mu_t + \alpha_t \cdot trend + v_t & \text{for} \quad t = m+1, ..., m+q. \end{cases} \tag{2}$$

where μ and α are regression coefficients, $v_t = v_{t-1} + u_t$ for $t = 1, ..., m$, and u_t satisfies $ARMA(p, q)$ for $t = 1, ..., m$.

C3. Models and Hypothesis

Model 3 (General Mixing Conditions):

$$y_t = \begin{cases} \beta_0 \cdot x_t + u_t & \text{for} \quad t = 1, ..., m, \\ \beta_t \cdot x_t + u_t & \text{for} \quad t = m+1, ..., m+q. \end{cases} . \tag{3}$$

where β are regression coefficients.

The regressor x_t and the errors u_t satisfy the conditions used by Deng and Perron (2008).

C4. Models and Hypothesis

$$H_0 : \mu_t = \mu_0, \alpha_t = \alpha_0 \qquad \text{for all} \quad t = m+1, ..., m+q$$

and the distribution of $(u_{m+1}, ..., u_{m+q})$ is the same as that of $(u_j, ..., u_{j+q-1}), \quad j = 1, ..., m-q+1$

$$H_1 : \mu_t \neq \mu_0 \quad \text{and/or} \quad \alpha_t \neq \alpha_0 \quad \text{for some} \quad t = m+1, ..., m+q$$

and/or the distribution of $(u_{m+1}, ..., u_{m+q})$ differs from that of $(u_j, ..., u_{j+q-1})$ for $j = 1, ..., m-q+1$

D. Monitoring Procedure

Given an initial fixed training sample of size m, the monitoring scheme needs a stopping device, determined by a detecting statistics (detector), T_n and a boundary function, $g(n/m)$.

To monitor whether the breaks occur after the data being increased to $n (n \geq m)$. More precisely, under the hypothesis of no break, T_n may cross a boundary function $g(n/m)$, for some $n \geq m$, with certain probability, say 0.05 0.10.

If a break indeed occurs, we expect T_n to cross $g(n/m)$ with a large probability. Operationally, the null hypothesis is rejected when $|T_n| \geq g(n, m)$ for some $n > m$. Conversely, the monitoring process keeps us running until a break is observed.

E1. The AR Approximation of I(0) and I(1)

(I.) AR Approximation of I(0) processes :

Berk (1974) shows that an I(0) process, u_t, can be approximated by an AR model

$$\widehat{u}_t = \sum_{t=1}^{k} \beta_j \widehat{u}_{t-j} + \widehat{e}_{tk}. \tag{4}$$

Thus, under the assumption of lag length $k = o_p(T^{1/3})$, the estimation error in autoregressive approximations is shown to be of order $O_p(T^{-1/2})$.

E2. The AR Approximation of I(0) and I(1)

(II.) AR Approximation of I(1) processes :

Bauer and Wagner (2008) proved that an I(1) process, v_t can be approximated by an AR model under mild summability constraints

$$\widehat{v}_t = \sum_{t=1}^{k} \beta_j \widehat{v}_{t-j} + \widehat{e}_{tk}. \tag{5}$$

Thus, under the assumption of lag length $k = o_p((T/logT)^{1/2})$, the estimation error in autoregressive approximations is shown to be of order $O_p((logT/T)^{1/2})$.

F1. Statistics and Results

$$m^{-1/2}\widehat{S}_n = 2^{-1/2}m^{-1/2}\sum_{i=k+1}^{n}\left(\frac{\widehat{e}_i^2}{\widehat{\sigma}_{tk}^2} - \frac{m}{m-2}\right) = m^{-1/2}S_n + o_p(1).(6)$$

$$T_n = m^{-1/2}\widehat{S}_{[mt]} \Longrightarrow W^0(t).$$

where " $\Longrightarrow$ " denotes weak convergence and where $W^0 = W(t) - tW(1)$, with W being a standard Wiener Process.

F2. Statistics and Results

THEOREM 1 (For I(0) case). Let u_t and y_t be the processes given by (1). Under the null hypothesis of Assumption 2, as $k, T \to \infty$, the following functional central limit (FCLT) holds for MCUSQAR test T_n:

$$T_n \Rightarrow W^0(t) = W(t) - tW(1)$$

where W and W^0 are the (one-dimensional) Brownian motion and Brownian bridge such as

$$lim_{m \to \infty} P\{|T_n| \geq g(t), \text{for some} \quad t \geq 1\} = 2[1 - \Phi(a) + a\Phi(a)].$$

F3. Statistics and Results

We choose $g(t) = [t(a^2 + ln(t))]^{1/2}$ to be our boundary function, then as shown in (13) in Chu *et al.* (1996),

$$P\{|W^0(t)| = (t-1)^{-1}|W(t)| \geq t(a^2 + \ln(t))]^{1/2}\} = 2[1 - \Phi(a) + a\Phi(a)]$$

for some $t \geq 1$. When $a^2 = 7.78$, the crossing probabilities is 5%.

F4. Statistics and Results

$$m^{1/2}\widehat{S}_n^* = 2^{-1/2} m^{-1/2} \sum_{i=k+1}^{n} \left(\frac{\frac{\widehat{e}_i^2}{(1-\sum_{i=1}^{k} \widehat{b}_i)^2}}{\frac{\widehat{\sigma}_{tk}^2}{(1-\sum_{i=1}^{k} \widehat{b}_i)^2}} - \frac{m}{m-2} \right) = m^{-1/2} S_n^* + o_p(1)$$

$$T_n^* = m^{-1/2} \widehat{S}_{[mt]}^* \Longrightarrow W^0(t).$$

F5. Statistics and Results

THEOREM 2 (For I(0) case) . Let u_t and y_t be the processes given by (1). Under the null hypothesis, as $k, T \rightarrow \infty$, the following functional central limit (FCLT) holds for MCUSQAR test T_n^*:

$$T_n^* \Rightarrow W^0(t) = W(t) - tW(1)$$

where W and W^0 are the (one-dimensional) Brownian motion and Brownian bridge such as

$$lim_{m \rightarrow \infty} P\{|T_n^*| \geq g(t), \text{for some} \quad t \geq 1\} = 2[1 - \Phi(a) + a\Phi(a)]. \quad (7)$$

F6. Statistics and Results

THEOREM 3 (For I(1)case) Let u_t and y_t be the processes given by (2) . Under the null hypothesis, as $k, T \rightarrow \infty$ and $k = o_p((T/logT)^{1/2})$, the following functional central limit (FCLT) holds for MCUSQAR test T_n:

$$T_n \Rightarrow W^0(t) = W(t) - tW(1)$$

where W and W^0 are the (one-dimensional) Brownian motion and Brownian bridge such as

$$lim_{m\rightarrow\infty} P\{|T_n| \geq g(t), \text{for some} \quad t \geq 1\} = 2[1 - \Phi(a) + a\Phi(a)]. \quad (8)$$

F7. Statistics and Results

THEOREM 4 (For I(1)case) Let u_t and y_t be the processes given by (2). Under the null hypothesis, as $k, T \to \infty$ and $k = o_p((T/logT)^{1/2})$, the following functional central limit (FCLT) holds for MCUSQAR test T_n^*:

$$T_n^* \Rightarrow W^0(t) = W(t) - tW(1)$$

where W and W^0 are the (one-dimensional) Brownian motion and Brownian bridge such as

$$lim_{m\to\infty} P\{|T_n^*| \geq g(t), \text{for some} \quad t \geq 1\} = 2[1 - \Phi(a) + a\Phi(a)]. \quad (9)$$

F8. Statistics and Results

THEOREM 5 (For General Mixing Condition case) Let y_t and u_t be the processes given by (3), respectively. Under the null hypothesis of Assumption 5 and Theorem 2 of Berk (1974), as $k, n \to \infty$ and $k = o(n^{1/3})$, the MCUSQAR test T_n^* and T_n has the same limiting distribution as in Theorem 2.

G.1. Example of Tests Against Changes From I(0) to I(1) or vice versa

We assume a change in persistence from I(0) to I(1) behavior, H_{01} and from I(1) to I(0) behavior, H_{10}.

Under H_{01} and H_{10}, we now suppose that v_t in (10) is generated by

$$v_t = \begin{cases} u_t, & \text{for} \quad t = 1, 2, \cdots, m \\ v_{t-1} + u_t, & \text{for} \quad t = m+1, \cdots, T \end{cases}.$$

and

$$v_t = \begin{cases} v_{t-1} + u_t, & \text{for} \quad t = 1, 2, \cdots, m \\ u_t, & \text{for} \quad t = m+1, \cdots, T \end{cases},$$

respectively, where u_t satisfies $ARMA(p,q)$.

G.2. Examples of Tests Against Changes From I(0) to I(1) or vice versa

THEOREM 6. Let u_t and v_t be the processes given by (2)), then

1. Under H_{01}, $T_n^* = O_p(k/k'T^{-1})$

2. Under H_{10}, $T_n^* = O_p(k'T^{-1}/k)$

H.1. Simulation Studies

I. $u_t = e_t$

II. $(1 - 0.7L)u_t = (1 + 0.5L)\epsilon_t$

III. $(1 - 0.375L)u_t = e_t$

IV. $(1 - L)u_t = e_t$

V. $(1 + 0.97L)u_t = (1 + 0.65L)\epsilon_t$

where, we set $e_t \sim N(0,1)$ and $\epsilon_t \sim N(0,3)$. $t = 1.1 \times m$. μ_t shifts from 2 to 2.8 and α_t shift from 0.2 to 0.6

H2. Simulation Studies

Table 1. Empirical Size and Power of T_n, When u_t are (a)-(e)

DGP	m	q	(a)	(b)	(c)	(d)	(e)
$size$	50	$2m$	4.3	4.9	5.5	5.9	6.0
		$3m$	5.2	5.2	5.8	6.2	6.2
		$4m$	5.7	5.8	6.2	6.4	6.4
	100	$2m$	4.5	4.9	4.9	4.8	4.9
		$3m$	4.6	4.7	5.0	5.2	5.2
		$4m$	4.9	4.9	5.3	5.4	5.7
	200	$2m$	4.0	4.2	4.3	4.5	4.6
		$3m$	4.1	4.3	4.2	4.5	4.7
		$4m$	4.2	4.4	4.5	4.7	4.7

H3. Simulation Studies

Table 1. Empirical Size and Power of T_n, When u_t are (a)-(e)

DGP	m	q	(a)	(b)	(c)	(d)	(e)
$Power$	50	$2m$	11.8	19.3	17.7	18.1	22.4
		$3m$	15.7	22.1	20.1	23.5	24.9
		$4m$	17.1	24.1	22.1	25.0	27.0
	100	$2m$	15.6	21.1	20.1	23.1	24.9
		$3m$	20.5	26.8	24.2	30.1	32.5
		$4m$	22.5	28.1	26.3	33.1	34.7
	200	$2m$	19.1	34.4	26.6	28.5	30.1
		$3m$	24.2	32.5	30.6	33.6	35.3
		$4m$	25.8	37.6	33.1	41.1	45.2

H4. Simulation Studies

Table 3. Breakpoint Specifications by Experiments (EX)

EX	$DGP(I)$	$DGP(II)$	μ_1	μ_2	α_1	α_2
1	(a)	(b)	0.0	0.0	0.0	0.0
2	(a)	(c)	2.0	2.8	0.2	0.6
3	(a)	(d)	2.0	2.8	0.2	0.6
4	(b)	(d)	2.0	2.8	0.2	0.6
5	(b)	(e)	0.0	0.0	0.0	0.0
6	(c)	(b)	2.0	2.8	0.2	0.6
7	(d)	(e)	2.0	2.8	0.2	0.6

H5. Simulation Studies

Table 4. The Size Performance of T_n test for Experiments Defined in Table 3.

	m	q	EX 1	2	3	4	5	6	7
$Size$	50	$2m$	5.1	5.2	5.3	5.6	5.9	5.3	6.0
		$3m$	5.3	5.4	5.6	5.6	6.1	5.5	6.2
		$4m$	5.8	5.9	5.8	6.1	6.6	5.8	6.7
	100	$2m$	4.9	5.5	5.3	5.6	5.7	5.0	5.7
		$3m$	5.0	5.9	5.6	5.7	6.1	5.1	5.9
		$4m$	5.0	6.3	6.0	5.5	6.1	5.4	6.1
	200	$2m$	4.2	5.4	5.2	5.2	5.3	4.7	5.3
		$3m$	4.4	5.6	5.2	5.0	5.4	4.9	5.6
		$4m$	4.8	5.9	5.6	5.0	5.6	5.1	5.6

H6. Simulation Studies

Table 4. The Power Performance of T_n test for Experiments Defined in Table 3.

	m	q	EX 1	2	3	4	5	6	7
$Power$	50	$2m$	25.7	13.8	12.4	30.1	56.4	44.1	59.2
		$3m$	29.6	15.2	13.2	29.9	63.8	47.2	64.2
		$4m$	34.1	17.1	16.9	42.4	65.2	52.7	67.1
	100	$2m$	29.4	17.1	14.2	35.2	39.1	50.1	60.7
		$3m$	32.9	19.5	17.1	37.1	62.7	59.4	67.6
		$4m$	39.1	20.0	19.3	48.2	70.3	67.9	72.2
	200	$2m$	31.7	19.1	17.2	37.9	64.7	59.1	67.2
		$3m$	38.2	20.7	20.1	44.3	69.6	64.8	69.7
		$4m$	44.1	22.2	20.2	51.2	77.5	74.2	75.1
	300	$2m$	38.5	21.8	19.2	26.1	72.7	77.4	86.1
		$3m$	47.9	26.5	22.2	65.1	92.2	85.4	93.6
		$4m$	52.9	26.5	24.4	70.1	99.7	89.0	100

H7. Simulation Studies

Table 5. The Size and Power Performance of T_n^* test for Experiments Defined in Table 3.

	m	q	EX 1	2	3	4	5	6	7
$Size$	50	$2m$	5.2	5.5	5.2	5.9	6.1	5.3	6.6
		$3m$	5.6	5.9	5.6	5.9	6.2	5.5	6.8
		$4m$	6.0	6.2	6.0	6.3	6.5	5.8	7.0
	100	$2m$	5.1	5.5	5.3	5.6	6.0	5.0	6.0
		$3m$	5.2	5.9	5.6	5.7	6.0	5.1	6.3
		$4m$	5.2	6.3	6.0	5.9	6.4	5.4	6.7
	200	$2m$	4.7	5.4	5.2	5.2	5.3	4.7	5.4
		$3m$	5.2	5.6	5.2	5.5	5.4	4.9	5.6
		$4m$	5.2	5.9	5.6	5.7	5.6	5.1	5.7

H8. Simulation Studies

Table 5. The Size and Power Performance of T_n^* test for Experiments Defined in Table 3.

	m	q	EX 1	2	3	4	5	6	7
Power	50	$2m$	27.9	32.1	36.1	40.2	58.4	49.1	60.2
		$3m$	33.6	37.2	41.8	45.9	64.8	50.1	69.2
		$4m$	35.1	40.1	43.9	52.1	67.2	54.6	69.1
	100	$2m$	29.9	28.1	32.2	36.1	60.2	54.1	63.7
		$3m$	34.2	31.5	30.7	39.1	68.7	60.2	70.6
		$4m$	38.7	39.0	42.3	54.2	72.3	73.2	74.2
	200	$2m$	35.5	34.7	36.2	47.9	63.7	61.2	69.1
		$3m$	39.7	46.1	44.5	60.5	70.6	65.7	72.7
		$4m$	41.0	50.2	52.3	73.7	82.5	79.2	86.2
	300	$2m$	40.8	52.7	58.3	64.2	78.7	74.4	86.6
		$3m$	46.1	66.2	69.2	76.2	97.0	90.4	93.3
		$4m$	59.8	69.3	77.9	87.3	100	96.8	100

I.1. The Stability of the U.S. Monetary Transmission Mechanism

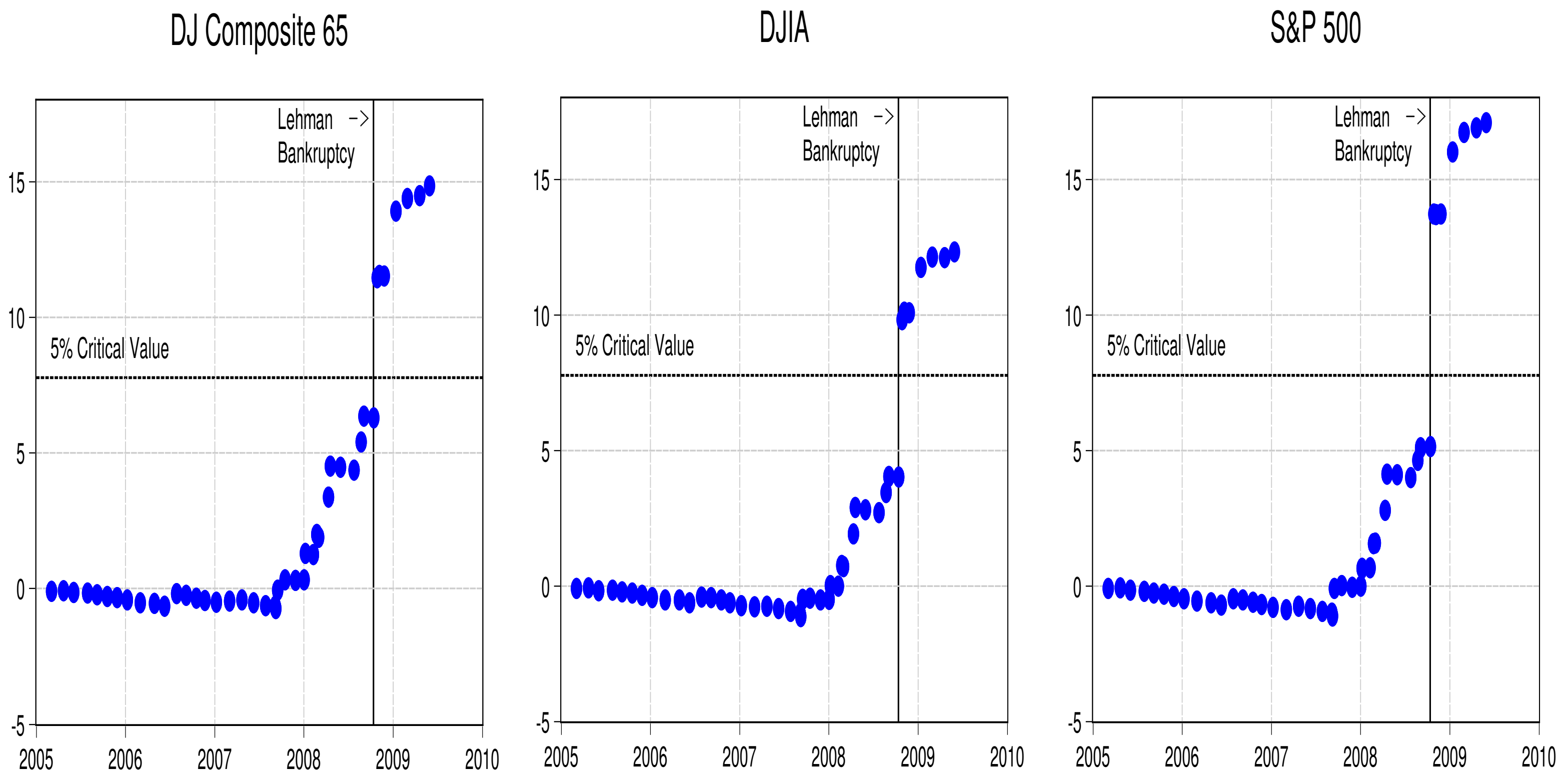
$$R_{t+1} = \alpha + \beta \cdot MPS_t + \varepsilon_{t+1} \tag{10}$$

where

$$R_{t+1} = log(P_{t+1}/P_t),$$
$$MPS_{t+1} = PR_{t+1} - \mathbf{E}_t[PR_{t+1}]$$

and PR is policy rate.

J.1. Figures



DJ Composite 65
DJIA
S&P 500
Lehman ->
Bankruptcy
5% Critical Value
15
10
5
0
-5
2005
2006
2007
2008
2009
2010

K. Concluding Remarks

I. We develop a new and improved method for monitoring the real time pattern of macroeconomics variables and asset prices.

II. The limiting distribution of our monitoring tests does not depend on the nature of error processes.

III. No need to use bootstrap to control the size distortion.

IV. Our test statistics are very useful tool in supplementing policy maker's information set not only ex-post, but also in real time, i.e., at the time of their decisions.